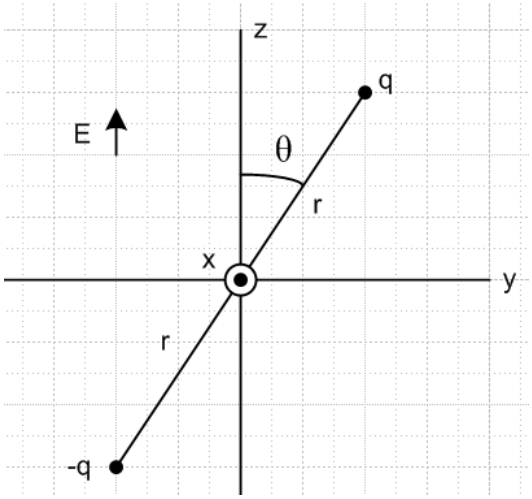
Does an electric dipole in a uniform E field precess? PhL 3.24.17

Basically no web hits on this subject, lots on mag dipole in B field.

1. What do we know about a dipole in an E field? Start in kindergarten with this picture



Upper right is 1, lower left is 2. Then

$\mathbf{F_1} = q\mathbf{E} = qE\hat{\mathbf{z}}$ $\mathbf{r_1} = r\sin\theta\,\hat{\mathbf{y}} + r\cos\theta\,\hat{\mathbf{z}}$

$\mathbf{F_2} = (-q)\mathbf{E} = -qE\hat{\mathbf{z}} = -\mathbf{F_1}$ $\mathbf{r_2} = -\mathbf{r_1}$

$\mathbf{N_1} = \mathbf{r_1} \times \mathbf{F_1} = [r\sin\theta\,\hat{\mathbf{y}} + r\cos\theta\,\hat{\mathbf{z}}] \times [qE\hat{\mathbf{y}}] = r\sin\theta qE\,\hat{\mathbf{x}}$

$\mathbf{N_2} = \mathbf{r_2} \times \mathbf{F_2} = (-\mathbf{r_1}) \times (-\mathbf{F_1}) = \mathbf{N_1} = r\sin\theta qE\,\hat{\mathbf{x}}$

We have a massless stick connecting the charges. The total torque on this dumbbell is (about origin)

$\mathbf{N} = \mathbf{N_1} + N_2 = 2\mathbf{N_1} = 2r\sin\theta qE\,\hat{\mathbf{x}}$

Now define the dipole moment p as

$p \equiv 2rq$ $\mathbf{p} \equiv 2rq\hat{\mathbf{r}}$

Then

$N = 2r\sin\theta qE\,\hat{\mathbf{x}} = pE\sin\theta\,\hat{\mathbf{x}}$

Now consider

$\mathbf{p} \times \mathbf{E} = [\, 2rq\hat{\mathbf{r}}] \times [E\hat{\mathbf{z}}] = [p\hat{\mathbf{r}}] \times [E\hat{\mathbf{z}}] = pE\, \hat{\mathbf{r}} \times \hat{\mathbf{z}} = pE \sin\theta\, \hat{\mathbf{x}}$

Therefore we have shown that

$\mathbf{N} = \mathbf{p} \times \mathbf{E}$

where torque is about the origin. You can imagine increasing q and decreasing r in a way that the product is constant, then you could have **p** very close to the origin.

So kindergarten class, we know that there is a torque on a dipole in an electric field given by $\mathbf{N} = \mathbf{p} \times \mathbf{E}$. This will be true in general, not just for the above picture.

2. Now suppose the two charges in the above picture are in motion at t = 0 where q is going into paper at v, and -q is coming out of paper at v. Then

$\mathbf{v}_1 = -v\hat{\mathbf{x}}$
$\mathbf{v}_2 = v\hat{\mathbf{x}}$

Since we are introducing "motion" into the problem, assume the two changes have same mass m. Then

$\mathbf{L}_1 = \mathbf{r}_1 \times (m\mathbf{v}_1) = r\hat{\mathbf{r}} \times (-mv\hat{\mathbf{x}}) = -rmv\, \hat{\mathbf{r}} \times \hat{\mathbf{x}} = -rmv\, [\sin\theta\, \hat{\mathbf{y}} + \cos\theta\, \hat{\mathbf{z}}] \times \hat{\mathbf{x}}$

$= -rmv\, [\, -\sin\theta\, \hat{\mathbf{z}} + \cos\theta\, \hat{\mathbf{y}}\,]$

$= rmv\, [\, \sin\theta\, \hat{\mathbf{z}} - \cos\theta\, \hat{\mathbf{y}}\,]$

$\mathbf{L}_2 = \mathbf{r}_2 \times (m\mathbf{v}_2) = (-\mathbf{r}_1) \times (-m\mathbf{v}_1) = \mathbf{L}_1$

So just for interest, the dumbbell has total angular momentum

$\mathbf{L} = \mathbf{L}_1 + \mathbf{L}_2 = = 2\mathbf{L}_1 = 2rmv\, [\, \sin\theta\, \hat{\mathbf{z}} - \cos\theta\, \hat{\mathbf{y}}\,]$

We do not have torque free motion of the dumbbell because E makes a torque on it, $\mathbf{N} = \mathbf{p} \times \mathbf{E}$.

The equation of motion is then

$\mathbf{N} = \dot{\mathbf{L}}$

Now write

$\mathbf{L} = 2\, \mathbf{r}_1 \times (m\mathbf{v}_1)$

$\dot{\mathbf{L}} = 2m\, \mathbf{r}_1 \times \dot{\mathbf{v}}_1$

We then have equation of motion

$$2m\,\mathbf{r}_1 \times \dot{\mathbf{v}}_1 = \mathbf{p} \times \mathbf{E}$$