This is a Reader Exercise in

STOP! How does this compare with

$$(\omega)_x = \cos\varphi\,\dot{\theta} + \sin\theta\sin\varphi\,\dot{\psi}$$
$$(\omega)_y = \sin\varphi\,\dot{\theta} - \sin\theta\cos\varphi\,\dot{\psi}$$
$$(\omega)_z = \dot{\varphi} + \cos\theta\,\dot{\psi}\ . \qquad \text{(H.4.12)}$$

Well, $\dot{\theta} = 0$ and $\varphi(t) = (L/I'_1)t$ and $\dot{\psi} = (-\Omega')$ so the above says

$$(\omega)_x = \sin\theta\sin\varphi\,(-\Omega') \quad =?= \quad \omega\sin(\theta-\alpha)\sin\varphi$$
$$(\omega)_y = -\sin\theta\cos\varphi\,(-\Omega') \quad =?= \quad -\omega\sin(\theta-\alpha)\cos\varphi$$
$$(\omega)_z = (L/I'_1) + \cos\theta\,(-\Omega')\ . \quad =?= \quad \omega\cos(\theta-\alpha)$$

For this to be consistent with (I.5.26) I would have to show that

$$\sin\theta(-\Omega') = \omega\sin(\theta-\alpha)$$
$$\sin\theta(-\Omega') = \omega\sin(\theta-\alpha)$$
$$(L/I'_1) + \cos\theta\,(-\Omega') = \omega\cos(\theta-\alpha)$$

I know that $\Omega' = \omega\cos\alpha\,(I'_3-I'_1)/I'_1$ so then have to show

$$-\sin\theta\omega\cos\alpha\,(I'_3-I'_1)/I'_1 = \omega\sin(\theta-\alpha)$$
$$(L/I'_1) - \cos\theta\,\omega\cos\alpha\,(I'_3-I'_1)/I'_1 = \omega\cos(\theta-\alpha)$$

But $\omega\sin\alpha/\sin\theta = L/I'_1$ so restate these equations as

$$-\sin\theta\omega\cos\alpha\,(I'_3-I'_1)/I'_1 = \omega\sin(\theta-\alpha)$$
$$\omega\sin\alpha/\sin\theta - \cos\theta\,\omega\cos\alpha\,(I'_3-I'_1)/I'_1 = \omega\cos(\theta-\alpha)$$

or

$$-\sin\theta\cos\alpha\,(I'_3-I'_1)/I'_1 = \sin(\theta-\alpha)$$
$$\sin\alpha/\sin\theta - \cos\theta\,\cos\alpha\,(I'_3-I'_1)/I'_1 = \cos(\theta-\alpha)$$

or

$$-\sin\theta\cos\alpha\,[\,(I'_3/I'_1) - 1] = \sin(\theta-\alpha) \qquad [\,(I'_3/I'_1) - 1]$$
$$\sin\alpha/\sin\theta - \cos\theta\,\cos\alpha\,[\,(I'_3/I'_1) - 1] = \cos(\theta-\alpha)$$

Good grief! Now us

$$\tan\theta = (I'_1/I'_3)\tan\alpha \quad \Rightarrow \quad (I'_1/I'_3) = \tan\theta/\tan\alpha \quad \Rightarrow \quad (I'_3/I'_1) = \tan\alpha/\tan\theta$$

so then the two equations are

$- \sin\theta\cos\alpha\ [\ \tan\alpha/\tan\theta - 1\ ] = \sin(\theta-\alpha)$ $\quad [\ (I'_3/I'_1) - 1\ ]$

$\sin\alpha/\sin\theta\ - \cos\theta\ \cos\alpha\ [\tan\alpha/\tan\theta - 1\ ] = \cos(\theta-\alpha)$

or

$- \sin\theta\cos\alpha\ [\ \tan\alpha/\tan\theta - 1\ ] = \sin(\theta-\alpha)$

$\sin\alpha\ - \sin\theta\cos\theta\ \cos\alpha\ [\tan\alpha/\tan\theta - 1\ ] = \sin\theta\cos(\theta-\alpha)$

or

$- \sin\theta\cos\alpha\ [\sin\alpha\cos\theta/\cos\alpha\sin\theta - 1\ ] = \sin(\theta-\alpha)$

$\sin\alpha\ - \sin\theta\cos\theta\ \cos\alpha\ [\sin\alpha\cos\theta/\cos\alpha\sin\theta - 1\ ] = \sin\theta\cos(\theta-\alpha)$

or

$- \sin\theta\cos\alpha\ [\sin\alpha\cos\theta - \cos\alpha\sin\theta\ ] = \sin(\theta-\alpha)\cos\alpha\sin\theta$

$\cos\alpha\sin\theta\sin\alpha\ - \sin\theta\cos\theta\ \cos\alpha\ [\sin\alpha\cos\theta - \cos\alpha\sin\theta\ ] = \cos\alpha\sin\theta\sin\theta\cos(\theta-\alpha)$

or

$- \sin\theta\cos\alpha\ \sin\alpha\cos\theta + \sin\theta\cos\alpha\cos\alpha\sin\theta = \sin(\theta-\alpha)\ \cos\alpha\ \sin\theta$

$\cos\alpha\sin\theta\sin\alpha - \sin\theta\cos\theta\ \cos\alpha\sin\alpha\cos\theta + \sin\theta\cos\theta\ \cos\alpha\ \cos\alpha\sin\theta = \cos\alpha\sin\theta\sin\theta\cos(\theta-\alpha)$

or

$- \sin\alpha\cos\theta + \cos\alpha\sin\theta = \sin(\theta-\alpha)$ // correct!!!

$\sin\alpha - \cos\theta\ \sin\alpha\cos\theta + \cos\theta\ \cos\alpha\sin\theta = \sin\theta\cos(\theta-\alpha)$ // problem Houston

$\cos\alpha\sin\theta\sin\alpha - \sin\theta\cos\theta\ \cos\alpha\sin\alpha\cos\theta + \sin\theta\cos\theta\ \cos\alpha\ \cos\alpha\sin\theta = \cos\alpha\sin\theta\sin\theta\cos(\theta-\alpha)$

$\sin\theta\sin\alpha - \sin\theta\cos\theta\ \sin\alpha\cos\theta + \sin\theta\cos\theta\ \cos\alpha\sin\theta = \sin\theta\sin\theta\cos(\theta-\alpha)$

$\sin\alpha - \cos\theta\ \sin\alpha\cos\theta + \cos\theta\ \cos\alpha\sin\theta = \sin\theta[\cos\theta\ \cos\alpha + \sin\theta\ \sin\alpha]$

$\sin\alpha - \cos\theta\ \sin\alpha\cos\theta + \cos\theta\ \cos\alpha\sin\theta = \sin\theta\cos\theta\ \cos\alpha + \sin\theta\sin\theta\ \sin\alpha$

$\sin\alpha - \cos^2\theta\ \sin\alpha + \cos\theta\ \cos\alpha\sin\theta = \sin\theta\cos\theta\ \cos\alpha + \sin^2\theta\sin\alpha$

$\sin\alpha + \cos\theta\ \cos\alpha\sin\theta = \sin\theta\cos\theta\ \cos\alpha + \sin^2\theta\sin\alpha + \cos^2\theta\ \sin\alpha$

$\sin\alpha + \cos\theta\ \cos\alpha\sin\theta = \sin\theta\cos\theta\ \cos\alpha + \sin\alpha$

$\cos\theta\ \cos\alpha\sin\theta = \sin\theta\cos\theta\ \cos\alpha$ yes !!!!