Review and Critique of April 2017 Frames Doc PhL 4.7.17

1. Section 1.

This section doubled in size from 15 to 30 pages. Section 1.1, formerly called 1 (a), went from 1/2 page to 13 pages, so this accounts for most of the increase. The short version had a proof of sorts of the Basis Theorem, not called that at the time, and it had no Dirac.

Here is what is in the new Section 1.1 which has grown so much:

Section 1.1
- orthog of basis vectors
- initial definition of R_{nm} appears in $\mathbf{e'}_n = R_{nm}\,\mathbf{e}_m$
- expansions of all basis vectors in terms of dot products
- evaluations of all basis vector components in terms of R_{ni} or δ_{ni} in 1.1.8 [was in (b)]
- statement of completeness (new)
- claim there is a "notation problem" with meaning of **a** = T**b**
- comments on how the Dirac notation is used in QM
- Dirac notation
  - bras and kets, dot products $\langle\mathbf{a}|\mathbf{b}\rangle$ and direct products $|\mathbf{a}\rangle\langle\mathbf{b}|$
  - mention of dyads and $\mathbf{a}\otimes\mathbf{b}$ notation for $|\mathbf{a}\rangle\langle\mathbf{b}|$ (pretty good I think)
  - real Hilbert space
  - imagined "operators" in the Hilbert space.
  - $T_{mn} \equiv \langle\mathbf{e}_m|\,\mathcal{T}\,|\mathbf{e}_n\rangle$ to define this matrix T
  - completeness $\mathcal{I} = |\mathbf{e'}_n\rangle\langle\mathbf{e'}_n|$
  - show how $(T)'_{mn}$ is related to T_{mn} and $(T)' = RTR^{-1}$
  - expansion of abstract tensor $\mathcal{T}$
  - first comments on Active versus Passive view and why needed.
- The Basis Theorem
  - I carefully prove this in both directions and then I state the theorem
  - shorthand notations using column vectors (will be used with Euler angles)
  - showing the R' = R matrices
- Other Dirac facts
- double concatenation
- triple concatenation justified since we are going to do Euler angles eventually

What exactly is the "notation problem" with **a** = T**b** ? I think the issues is "in what basis are the matrix elements T_{ij} that you imply? " The concatenation examples show how confusing this can be.

So I have forced the reader to invest in Dirac notation. My real motivation for this is so I can use it in the ω proof by Method 2 which I am now telling the reader to skip.

Section 1.2
- expansion of vectors on the two bases
- careful use of primes -- the key point.

Section 1.3

Active and Passive Views and associated notations like (**V**') = R**V**. (completely new!)
the idea of passive meaning "back-rotation" of basis vectors
notion of kinematic versus basis vectors (completely new)
the issue of notation overload between S' as frame, and **v**' as vector in that frame.
Passive transformation of rank-2 tensor (T)' = RTR^{-1}
dot products

So I have now summarized these new sections, and YES, there is lots of new material.

Essay: Authors generally don't even *talk* about these things, certainly not in this much detail. They just use the basis vectors. The Basis Theorem is never even mentioned. So I would like to claim that a major contribution of frames doc is that it clarifies the underlying machinery implicitly used by other authors. And I think this is a very appropriate topic to appear early in a doc named "rotating frames of reference". The title itself is a fairly rare subject to be written about on its own, and I think further justifies my document. So I guess I do not feel that most of my new Section 1.1,2,3 material is irrelevant as some reviewer might claim. It is at the core of things in fact. In the old frames doc, I was hazy about many of the details, but I feel they are not sharp and clear and brought out into the open for discussion and analysis. I think the Dirac notation really is a gold standard for vectors and matrices, though not for higher tensors. Each of my docs always ends up talking about Dirac notation, so it must be useful at least to me.

I continue now

Section 1.4 when are two vectors equal

Another topic never talked about, but I think that it is mentioned by the wedge guy Spivak! I think this is a key statement to be made early on, so not irrelevant.

Section 1.5

small rotations and rotation matrix notation, I obtain idea that d**a** = d**φ** x **a** for a small rotation.
Certainly THIS is mainline stuff for rotating frames of reference.

Section 1.6

time rate of change of a rotating vector. (d**a**/dt) = **ω** x **a**
I just divide by dt, and show WHY frame labels are needed, mainline stuff! Conical motion.

Section 1.7

apply the rule just noted to basis vectors.

Section 1.8

now we can list off the 4 velocities and 8 accelerations and defined the "natural" notations

These last four subsections are mainline burger for any doc on this subject. But I have never seen anyone list off all these velocities and accelerations. They usually just pick one or two and use them without showing this larger framework. Again, I am doing a monograph so this stuff is apropos.

Section 1.9

angular momentum.
This is definitely more advanced and it is a *whole new section* since last release in 2012.

In old frames doc these are not mentioned at all until Section 11, so I am just giving advanced warning. But this is the first appearance of my reoriented Fig 1.

I list off the four objects $\mathbf{L}^{(c)}$, $\dot{\mathbf{L}}^{(c)}$, $\mathbf{L}'^{(c')}$, $\dot{\mathbf{L}}'^{(c')}$.

Why do I do this? Because I have just shown four forms of velocity and p = mv and so L comes to mind and then if L = r x mv there must be four forms of L. But I only deal with the two natural forms of L and the time derivatives of same. I think it is reasonable to discuss L briefly at this point.

Actually, I now see that this angular momentum stuff was buried in Section 1 (h) = 1.8 with no visible heading. I decided to break it out into new section 1.9 to make it more visible. So not new after all.

Section 1.10

new label needed for time deriv of component, this is NOT new, not much changed

Section 1.11

When do operations d/dt and taking a component "commute" ?

What is new here is that I formally *prove* the claim of the theorem.

Conclusion on Section 1: So yes, Section one on "Preliminaries" has grown from 15 to 30 pages, and on review I think every bit of it is completely worthwhile. There is all this stuff going on *underneath* the Goldstein and Marion presentations, it is a formalism of basis vectors, vectors and tensors and how things are related under transformations in particular rotations. In a sense, this section alone is perhaps worth doing a "paper" on in my sense of a paper. Nothing original, just clarifying the underpinnings that everyone else just assumes. Similar to my wedge product claim of why I wrote it.

Section 2

the G rule. My proof is better and more concise, but basically section same as last release

This is VERY important to state up front and clearly, not haphazardly in passing!

Section 3

how things look to a rotating observer.

Perhaps this is a little fluffy, but want to stress that observers can "see" both frames, but can only experimentally measure time derivatives on their own frames. Only 1.5 pages so no big deal, not new.

Section 4

relationship between Frame S and Frame S'

Section 4.1

a lot of "words" describing Fig 1 which, until now, have not been clearly stated.

Section 4.2 redrawing the general picture so $\boldsymbol{\omega}$ points out of paper.

Section 4.3 details on $\dot{\mathbf{b}}_S$ and $\dot{\mathbf{b}}_{S'}$ just mechanical stuff

Section 4.4 special case #1

Section 4.5 special case #2

Section 4.6 the turntable (on which the ant shall crawl) , picture of the rotor

Section 4.7 surface of the Earth frame setup

Section 4.8 flying camera

This whole chapter really is about how you would SET UP your two frames for problems of interest, and does those special cases which are so important. My little jokes and rotor pictures I feel provide some relief to a reader who is facing all these brutal equations. The author is a real person, not a computer.

Section 4 is basically the *same as it was*, but probably lots of small edits were done. Similarly, Sections 2 and 3 are basically as they were.

Section 5 the goal

It seems stupid to have a whole Section which is 1/3 of a page long, but I do this for a reason. It is to force the reader to realized exactly what this frames doc is trying to do. I want a super clear statement of the problem. If I just state this at the end of some section, it would be easily overlooked. A chapter titled the goal and only 1 paragraph long will definitely be seen and read.

Section 6. Determination of velocities
Section 7. Determination of accelerations

These two sections are basically unchanged from the previous frames doc, and are still dry as dust.

Section 8. The Fictitious Forces

The theory is the same, but I think I have greatly improved my "interpretations" of the forces. I think the projectile example is very good because it addresses four directions all at once, usually authors just look at one direction. I did add lots of new stuff to this section.

Here is a comparison:

2012 frames doc	Sections 8.1 thru 8.5 = 10 pages
new frames doc	Sections 8.1 thru 8.5 = 10 pages

so I just rewrote pieces, did not add a lot

Section 8.6 Tethered satellites and Tidal Forces no real change here!

Section 8.7 Special Case #3

This is new and is a setup for the tides stuff. I was very unhappy with my handling of this gimbals frame description in previous frames doc. So now it is a whole special case where only b moves.

Here is a comparison on my tides treatment:

2012 frames doc Sections 8.7 thru 8.5 = 69-57 = 12 pages
new frames doc Sections 8.7 thru 8.8 = 101-75 = 26 pages

So tides more than doubled. Here is a review:

tidal forces in Cartesians and plot of the total forces on the globe (all new)
both docs have the force at the four points
both have the ellipse picture, but older one just quotes the results, new one derives results in footnote.
after rotation turned back on, old doc says nothing about how tides appear to observer
new doc figures this out for any location on Earth (ideal water world) (few lines below)
tides in polar coordinates added.
water surface treatment of Butikov presented in full (all new, not just quoted)
plots of tides which would obtain for various earth tilt angles (all new)
what is the angle θ_1 for the actual Earth (all new, involves lunar standstills and many pictures)
spring and neap (same)
tides on the real earth (all new, the fancy map)
surface by potential method (all new).
I dropped the section on the Moon-facing Earth dies on moon, was too lazy to figure it all out.

So this is a 26 page treatment of the tides which I hope is the best you can find anywhere, at my limited level of detail. I am not writing a book about the tides (they surely exist), but I am not just arm-waving either. I have taken a 26-page middle ground, including some fairly technical stuff. It is better than what I had before. This is perhaps my most major non-appendix rotating frames example, other than the ant trails.

Section 9 Marion comparison

In the 2012 doc I compared Marion to *both* swap and non-swap, which was confusing. Now I compare only to the swap notation, and I don't mention Taylor. This new swap comparison agrees with the original swap notation comparison, so at least I am not contradicting the older doc. I how have detailed tables showing the comparison, had small tables before.

Section 10 Goldstein comparison

In the old doc, I had a very confused presentation of this comparison, but now I have a much better one by assuming Special Case #4. The new is very much better I think.

Section 11. Angular Momentum and Fictitious Torques; the Reynolds Transport Theorem

This is basically unchanged, and remains a very messy section that will be hard to read. I do now connect Lai with a special case of aligned origins but I don't call it Special Case #4 for some reason.

Section 12. Summary of the Forward Problem Solution
Section 13. Summary of the Inverse Problem Solution

These are longer now because I add subsections for Special Case #4. Otherwise basically the same.

Section 14. Rotating Frames in Curvilinear Coordinates basically the same.

Section 15. Ant on Turntable Problems

Flow is the same, but I added hopefully-clarifying text here and there. I found an error in the projectile stuff and fixed it, changed nature of the plots for low projectile velocity.

Now I look at the appendices and try to compare

Appendix A: Derivation of R(ξ) and Properties of Rotation Matrices

Basically the same but I move some details like spatial derivatives and affine connection to new Appendix E. I added a proof of cross product covariance and my fancy rotation matrix rule that does not appear anywhere I know.

Now things get confusing, here is an explanation

	2012 doc	new doc	
rotation matrix stuff	Appendix A	Appendix A	no change in labeling
G Rule	Appendix C	Appendix B	
Foucault	Appendix D	Appendix C	
center of gravity		Appendix D	
spherical coordinates		Appendix E	
dumbbell satellite		Appendix F	
rotation matrices and theorems		Appendix G	
Euler angles and ω calcs		Appendix H	
Rigid body stuff		Appendix I	
tensor doc compare	Appendix B	Appendix J	

The total size of appendix D thru I is 418-227 = 191 pages

This is where the BULK of the increases size lies.

I could have made some of these appendices main line chapters, but then I had to alter the flow of the main doc a lot. As I did it, the main doc stayed fully the same in outline, Section numbers and so on, so this really could be considered an Update of same.

Old doc was 161 pages.
New doc is 427 pages. It went up by 427/161 = 2.65 so it more than doubled but less than tripled.

Conclusion: I think it is a much-improved doc in terms of the "theory" part, and it has much more "meat" in terms of actually applications. All new: center of gravity idea, dumbbell dynamics, rotation stuff and Euler and rigid body.

Why did I add all this new stuff?

Goldstein was just bothering me with the Euler angle ω calculation, it stuck in my craw, I did not like it and it was critical to rigid body. You can see how unhappy my Goldstein notes are on this subject. Of course the G Rule was the main reason I wrote the *first* version of frames doc, I just did not like how he just threw this out as if it were obvious. I realized that this topic of "rotating frames" was not one that is much monographed, probably because it is so boring, so I still feel I have "made a contribution", though nothing I do is new. I have put pieces together perhaps in a new way, especially with Dirac and the Basis theorem. I could find no treatments of the dumbbell like mine. The Foucault treatment is very much improved with mention of Airy and the full plane solution. Tides improved as well.

History of work on New Frames Doc

I have updated my physics work log now with all this stuff.

On Nov 17. 2016 I was starting my Goldstein review into his Chapter 4 on rigid body motion, and decided now would be a good time to update frames doc a bit, and work in my many accumulated errata there. This was the start of a 4.5 calendar month effort which roughly tripled the size of frames doc and added a lot more meat into the thing, as noted above. I never *dreamed* it would be this much time. I kept having "paradoxes" and the biggest one was with the initial Goldstein calculation of **ω** in the body frame. I was unable to make this produce the right results in both frame S and frame S' components. Only when I rewrote it in terms of matrices rather than operators using the Basis Theorem did it work. I could never have made it work with the Dirac notation, so that alone justified its addition to Section 1. There were many other paradoxes along the way, each one its on Myth of Sisyphus rock rolling to the bottom again. As a measure of the magnitude of this update task, I now have 206 supporting and sandbox Word docs just in the New Frames folder. I created every one of these docs. Many of them are old versions of things, or archived-out old sections, but most of them are "research" on paradox issues that I was not able to resolve without writing one or more (in a series) of docs on that subject.

Today April 7 I really think the thing is stable and I will put it up on Xmission after another PDF pass.

By the way, Goldstein's chapter 4 on rigid body motion runs p 93-185 = 92 pages which is about 1/4 of his entire book. My Appendix H and I are now 323-418 = 95 pages, so similar in length.