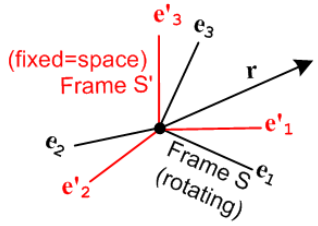
10. Notation comparison with Goldstein (1950) and Goldstein, Poole and Safko (2001)

In this Section we compare our notation for rotating-frame kinematics and non-inertial-frame physics to that of Goldstein on (1950) and Goldstein, Poole and Safko (GFS 2001).

These texts are also close to our "swap" notation. However, in neither text is the reader informed of the location of the origin of the inertial frame or of the rotating frame. In Goldstein we are told that **r** is a "vector from the origin of the terrestrial system" and that "terrestrial measurements are usually made with respect to a coordinate system fixed *in* the earth, which therefore rotates with a constant angular velocity ω relative to *the inertial system*". Having studied their rotational equations, it is our conclusion that the two frames of reference must have their origins co-sited at the center of the Earth,



(10.1)

Most of the follow-on discussion is about the Coriolis force which involves velocity and not **r**, so the location of the origin is not important.

In terms of our Fig 1, this means that **b** = 0 and so **r'** = **r** and we are simultaneously Special Case #1 and Special Case #2 since the rotation axis passes through both origins. In the summary Sections 12 and 13 below we refer to this combination as Special Case #4. For this situation, in our velocity table (1.8.4) the left and right sides are exactly the same. Similarly acceleration table (1.8.6) is identical to table (1.8.5). For example, with **b** = 0 we have,

$$\mathbf{v}_{S'} = (\partial \mathbf{r}/\partial t)_{S'} = (\partial \mathbf{r}'/\partial t)_{S'} = v'_{S'} = \mathbf{v}'$$

$$\mathbf{a}_{S'} = (\partial^2 \mathbf{r}/\partial t^2)_{S'} = (\partial^2 \mathbf{r}'/\partial t^2)_{S'} = a'_{S'} = \mathbf{a}' \,. \qquad (10.2)$$

Of course $\ddot{\mathbf{b}}_{S'} = 0$ when **b** = 0. We now gather up some of our equations in swap notation and state them for the case that **b** = 0,

$$\mathbf{r}' = \mathbf{r} \qquad // \ \mathbf{b} = 0 \qquad (6.1)_s$$

$$\mathbf{v}' = \mathbf{v} + \boldsymbol{\omega} \times \mathbf{r} \qquad (6.6b)_s$$

$$\mathbf{a}' = \mathbf{a} + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad (7.6a)_s$$

$$m\mathbf{a} = \mathbf{F}_{\mathbf{eff}} = \mathbf{F}' - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - 2m\,\boldsymbol{\omega} \times \mathbf{v} - m\dot{\boldsymbol{\omega}} \times \mathbf{r} \qquad // = \mathbf{F}' + \mathbf{F}_{\mathbf{fict}} \qquad (8.1.5)_s$$

In the above swap notation equations, Frame S' is fixed (space) and Frame S is the rotating frame (r). The four equations above appear in (12.2.8) and (12.2.9) of the Special Case #4 swap notation summary.

We start our comparison with equation $(6.5)_s$:

$\mathbf{v}' = \mathbf{v} + \boldsymbol{\omega} \times \mathbf{r}$ *(6.6b)$_s$*

$\mathbf{v}_s = \mathbf{v}_r + \boldsymbol{\omega} \times \mathbf{r}$. // Goldstein p 135 (4-104)
// GPS p 175 (4.88)

The next comparison is (we add $\dot{\boldsymbol{\omega}} \times \mathbf{r}$ to their equations),

$\mathbf{a}' = \mathbf{a} + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ *(7.6a)$_s$*

$\mathbf{a}_s = \mathbf{a}_r + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$. // Goldstein p 135 (4-105)
// GPS p 175 (4.89)

And finally,

$m\mathbf{a} = \mathbf{F}_{eff} = \mathbf{F}' - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - 2m\,\boldsymbol{\omega} \times \mathbf{v} - m\dot{\boldsymbol{\omega}} \times \mathbf{r}$ *(8.1.5)$_s$*

$m\mathbf{a}_r = \mathbf{F}_{eff} = \mathbf{F} - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - 2m\,\boldsymbol{\omega} \times \mathbf{v}_r - m\dot{\boldsymbol{\omega}} \times \mathbf{r}$. // Goldstein p 135 (4-106,7)
// GPS p 175 (4.90,1)

Based on these comparisons, we construct the following translation table:

(10.3)

our swap notation	Goldstein authors	$\mathbf{b} = 0$ so $\mathbf{r} = \mathbf{r}'$
S	r	name of the rotating frame (r = rotating or body)
S'	s	name of the fixed frame (s = space)
∂_S	$(d/dt)_r$	time derivative in the rotating frame
$\partial_{S'}$	$(d/dt)_s$	time derivative in the fixed frame
$\mathbf{r}$	$\mathbf{r}$	position in rotating frame
$\mathbf{v}$	$\mathbf{v}_r$	velocity in rotating frame
$\mathbf{a}$	$\mathbf{a}_r$	acceleration in rotating frame
$\mathbf{r}'$	$\mathbf{r}$	position in fixed frame
$\mathbf{v}'$	$\mathbf{v}_s$	velocity in fixed frame (s = space)
$\mathbf{a}'$	$\mathbf{a}_s$	acceleration in fixed frame
$\mathbf{F}'$	$\mathbf{F}$	force in fixed frame = true force in rotating frame
$\mathbf{F}_{eff}$	$\mathbf{F}_{eff}$	total effective force in the rotating frame
$\mathbf{F}_{fict}$		total fictitious force in the rotating frame
$\mathbf{b}$	0	location of the rotating frame origin (measured in the fixed frame)
$\dot{\mathbf{b}}_{S'}$	0	velocity of the rotating frame origin (measured in the fixed frame)
$\ddot{\mathbf{b}}_{S'}$	0	acceleration of the rotating frame origin (measured in the fixed frame)

Since $\mathbf{b} = 0$ is assumed, the Goldstein and GPS texts only treat a special case of the "rotating frames of reference" scenario we depict in Fig 1 in which $\mathbf{b}(t)$ is a general dynamic vector.