Tides if Earth and Moon don't rotate but are held apart by a stick.

$\mathbf{F}_{tid} = mM_1G\{ \mathbf{d}_0/d_0^3 - \mathbf{d}/d^3 \}$ // the red term needs to be removed for stick situation

$= mM_1G\{ d_0\hat{\mathbf{x}}/d_0^3 - [(d_0 + R_2\cos\theta)\hat{\mathbf{x}} + R_2\sin\theta\,\hat{\mathbf{y}}]\, d_0^{-3}[1 - 3(R_2/d_0)\cos\theta]\}$

$= (mM_1G/d_0^2)\{ \hat{\mathbf{x}} - [(1 + (R_2/d_0)\cos\theta)\hat{\mathbf{x}} + (R_2/d_0)\sin\theta\,\hat{\mathbf{y}}]\,[1 - 3(R_2/d_0)\cos\theta]\}$

$= (mM_1G/d_0^2)\{ \hat{\mathbf{x}} - \hat{\mathbf{x}} + 3(R_2/d_0)\cos\theta\,\hat{\mathbf{x}} - (R_2/d_0)\cos\theta)\,\hat{\mathbf{x}} - (R_2/d_0)\sin\theta)\,\hat{\mathbf{y}}\} + O((R_2/d_0))^2$

$\approx (mM_1G/d_0^2)\{2(R_2/d_0)\cos\theta\,\hat{\mathbf{x}} - (R_2/d_0)\sin\theta)\,\hat{\mathbf{y}}\}$

$= (mM_1G/d_0^3)\{2R_2\cos\theta\,\hat{\mathbf{x}} - R_2\sin\theta)\,\hat{\mathbf{y}}\}$

$= (mM_1G/d_0^3)(2x\,\hat{\mathbf{x}} - y\,\hat{\mathbf{y}}) - mM_1G\{ d_0\hat{\mathbf{x}}/d_0^3 \}$
// so here I manually subtract the red term to make it missing above

$= (mM_1G/d_0^3)(2x\,\hat{\mathbf{x}} - y\,\hat{\mathbf{y}}) - mM_1G/d_0^3\{ d_0\hat{\mathbf{x}}\}$

$= (mM_1G/d_0^3)(2x\,\hat{\mathbf{x}} - y\,\hat{\mathbf{y}} - d_0\hat{\mathbf{x}})$

$= (mM_1G/d_0^3)((2x - d_0)\,\hat{\mathbf{x}} - y\,\hat{\mathbf{y}})$

$= (mM_1G/d_0^3)((2x - d_0)\,\hat{\mathbf{x}} - y\,\hat{\mathbf{y}})$

$\approx (mM_1G/d_0^3)(-d_0\,\hat{\mathbf{x}} - y\,\hat{\mathbf{y}})$ // mainly to the left

****

Now trace the tidal shape argument:

We seek a potential V(x,y) which solves this equation (no minus sign in $\nabla V = \mathbf{F}$)

$$\nabla V(x,y) = \mathbf{F}_{g2} + \mathbf{F}_{tid} = -(GM_2m/r^2)\,\hat{\mathbf{r}} + (mM_1G/d_0^3)(-d_0\,\hat{\mathbf{x}} - y\,\hat{\mathbf{y}}) \qquad (8.8.63)$$

where we use (8.8.24) for $\mathbf{F}_{tid}$. The exact solution for V is the following (by inspection) :

$$V(x,y) = (GM_2m/r) + (GM_1m/d_0^3)(-d_0x - y^2/2)\,. \quad // \nabla(1/r) = -(1/r^2)\hat{\mathbf{r}},\ r > 0 \qquad (8.8.64)$$

Of interest are surfaces on which V is a constant (since we expect the water surface to be such a surface) so we let k be a constant and write

$$(M_2/r) + (M_1/d_0^3)(-d_0x - y^2/2) = 1/k$$

$(kM_2/r) + (M_1/kd_0^3)(-d_0x - y^2/2) = 1$

$(kM_2/r) = 1 - (kM_1/d_0^3)(-d_0x - y^2/2)$

$(kM_2/r)^2 = [1 - (kM_1/d_0^3)(-d_0x - y^2/2)]^2$

$(kM_2)^2 = (x^2+y^2) [1 - (kM_1/d_0^3)(-d_0x - y^2/2)]^2$

So I do the same approx done in frames doc to get

$(kM_2)^2 = (x^2+y^2) [1 - 2(kM_1/d_0^3)(-d_0x - y^2/2)$

The d_0 term in there swamps things so I guess

$(kM_2)^2 = (x^2+y^2) [1 + 2x(kM_1/d_0^2)]$

$= (x^2+y^2) [1 + 2x\varepsilon']$ $\varepsilon' = kM_1/d_0^2$

And what does this look like? Set $\varepsilon' = .05$ and $kM_2 = 1$ to get

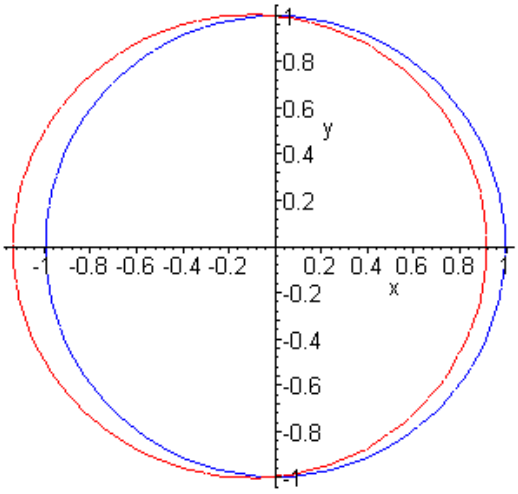
```
f := (x^2+y^2)*(1+2*x*e);
```

$$f := (x^2+y^2)(1+2xe)$$

```
e := .1;
```

$$e := .1$$

```
s1 := implicitplot(f=1,x=-2..2,y=-2..2,scaling=constrained,color=red):
s2 := implicitplot(x^2+y^2=1, x=-2..2,y=-2..2,color=blue):
display(s1,s2);
```



I would have to normalize things as before, but you can see that the water piles up on the left side of the earth facing the moon, and pulls away from the far side of the Earth. So I think my conjecture is correct.