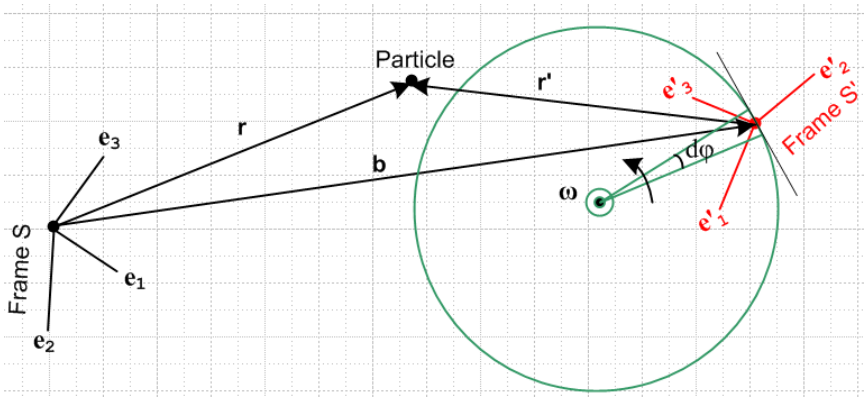
13.3 Summary of the Inverse Problem Equations (non-swap notation)

As noted above, we obtain the Inverse Problem equations by applying the Swap Rules (13.2.1) to the Forward Problem equations. But the Swap Rules can be thought of as having two steps: (1) do the swap one usually does to go from swap to non-swap notation as in (13.2.2); (2) then take **b**→ **-b** (including derivatives) and **ω**→ **-ω** (including derivatives**)**. We have step (1) already carried out in Section 12.2, so to get the equations below we need only carry out step (2) on the Section 12.2 equations. Here is a repeat of Fig (12.1.1) which of course applies to both the Forward Problem and the Inverse Problem :



(13.3.1)

r', **v'**, **a'**	position, natural velocity and natural acceleration in Frame S'	(13.3.2)
r, **v**, **a**	position, natural velocity and natural acceleration in Frame S	
ω	angular velocity of Frame S' relative to Frame S	
b	vector directed from origin of Frame S to origin of Frame S'	

$$\mathbf{r'} = -\mathbf{b} + \mathbf{r} \qquad \text{(a)}$$

$$\mathbf{v'} = \mathbf{v} - \boldsymbol{\omega} \times \mathbf{r} - \dot{\mathbf{b}}_{S'} \qquad \text{// all these equations are from (12.2.2) with } \boldsymbol{\omega} \text{ and } \mathbf{b} \text{ negated} \qquad \text{(b)}$$

$$\mathbf{v'} = \mathbf{v} - \boldsymbol{\omega} \times \mathbf{r'} - \dot{\mathbf{b}}_{S} \qquad \text{(c)}$$

$$\mathbf{a'} = \mathbf{a} - \dot{\boldsymbol{\omega}} \times \mathbf{r} - 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - \ddot{\mathbf{b}}_{S'} \qquad \text{(d)}$$

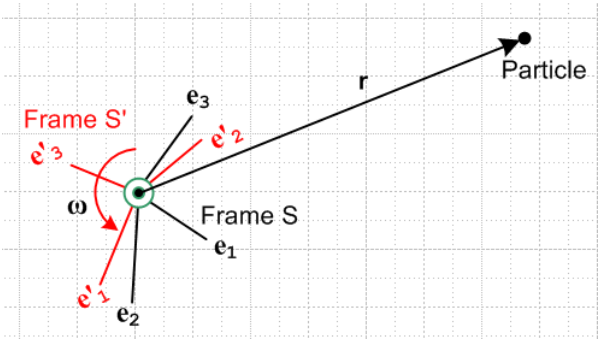
$$\mathbf{a'} = \mathbf{a} - \dot{\boldsymbol{\omega}} \times \mathbf{r'} - 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) + 2\,\boldsymbol{\omega} \times \dot{\mathbf{b}}_{S} - \ddot{\mathbf{b}}_{S} \qquad \text{(e)}$$

$$\mathbf{L'}^{(c')} = \mathbf{L}^{(c)} + m(\mathbf{r}-\mathbf{c}) \times [-(\boldsymbol{\omega} \times \mathbf{r}) - \dot{\mathbf{b}}_{S'}] \qquad \text{(f)}$$

$$\dot{\mathbf{L}}'^{(c')} = \dot{\mathbf{L}}^{(c)} + m(\mathbf{r}-\mathbf{c}) \times [-\dot{\boldsymbol{\omega}} \times \mathbf{r} - 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - \ddot{\mathbf{b}}_{S'}]$$
$$- m(\dot{\mathbf{c}} - \boldsymbol{\omega} \times \mathbf{c} - \dot{\mathbf{b}}_{S'}) \times (\mathbf{v} - \boldsymbol{\omega} \times \mathbf{r} - \dot{\mathbf{b}}_{S'}) + m\,\dot{\mathbf{c}} \times \mathbf{v} \qquad \text{(g)}$$

Special Case #4 (Inverse Problem, non-swap notation)

If the origins of Frame S and Frame S' coincide, then $\mathbf{b} = 0$. In this case one of course has $\mathbf{r} = \mathbf{r}'$ and also $\mathbf{c} = \mathbf{c}' = 0$. We then write $\mathbf{L} \equiv \mathbf{L}^{(0)}$, $\mathbf{L}' \equiv \mathbf{L}'^{(0)}$, $\mathbf{N} \equiv \mathbf{N}^{(0)}$ and $\mathbf{N}' \equiv \mathbf{N}'^{(0)}$. The new drawing is this,



(13.3.3)

We have placed the common origin in the plane of paper and are viewing things from a direction which causes the instantaneous $\boldsymbol{\omega}$ vector to point toward the viewer. Frame S' is rotating relative to Frame S at rate ω. The vector $\mathbf{r}$ in general is not in the plane of paper. Here are the simplified equations obtained from those above with $\mathbf{r} = \mathbf{r}'$, $\mathbf{b} = \dot{\mathbf{b}}_S = \dot{\mathbf{b}}_{S'} = \ddot{\mathbf{b}}_S = \mathbf{c} = \mathbf{c}' = 0$:

$\mathbf{r}, \mathbf{v}', \mathbf{a}'$	position, natural velocity and natural acceleration in Frame S'	(13.3.4)
$\mathbf{r}, \mathbf{v}, \mathbf{a}$	position, natural velocity and natural acceleration in Frame S	
$\boldsymbol{\omega}$	angular velocity of Frame S relative to Frame S'	
$\mathbf{r} = \mathbf{r}'$		(a)

$$\mathbf{v}' = \mathbf{v} - \boldsymbol{\omega} \times \mathbf{r} \qquad \text{(b) = (c)}$$

$$\mathbf{a}' = \mathbf{a} - \dot{\boldsymbol{\omega}} \times \mathbf{r} - 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad \text{(d) = (e)}$$

S' S Euler Coriolis centripetal

$$\mathbf{L}' = \mathbf{L} - m\,\mathbf{r} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad \text{(f)}$$

$$\dot{\mathbf{L}}' = \dot{\mathbf{L}} + m\mathbf{r} \times [\, -\dot{\boldsymbol{\omega}} \times \mathbf{r} - 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})\,] \qquad \text{(g)}$$