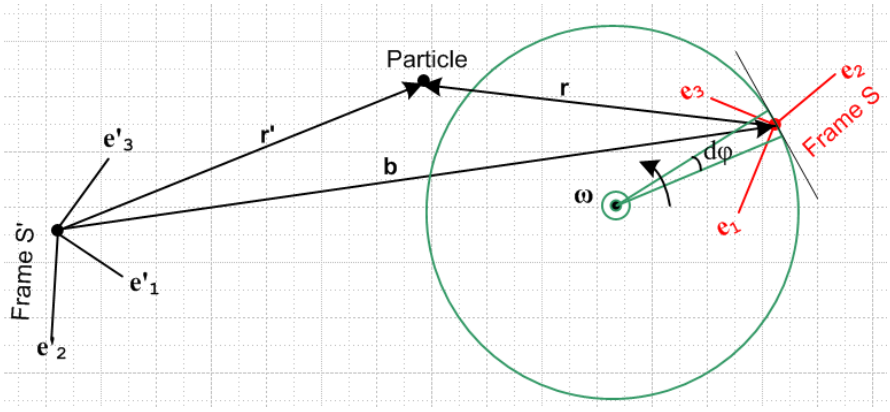
13.4 Summary of the Inverse Problem Equations (swap notation)

The results of this section are those of Section 13.3 but with S ↔ S' and **v'↔v** for all vectors except **b** and **ω**. Again, this is just a change of labeling. We repeat Fig (12.2.1) which shows the swap notation case :



(13.4.1)

r, **v**, **a**	position, natural velocity and natural acceleration in Frame S	(13.4.2)
r', **v'**, **a'**	position, natural velocity and natural acceleration in Frame S'	
ω	angular velocity of Frame S relative to Frame S'	
b	vector directed from origin of Frame S' to origin of Frame S	

$$\mathbf{r} = -\mathbf{b} + \mathbf{r}' \qquad \text{(a)}$$

$$\mathbf{v} = \mathbf{v}' - \boldsymbol{\omega} \times \mathbf{r}' - \dot{\mathbf{b}}_S \qquad \text{(b)}$$

$$\mathbf{v} = \mathbf{v}' - \boldsymbol{\omega} \times \mathbf{r} - \dot{\mathbf{b}}_{S'} \qquad \text{(c)}$$

$$\mathbf{a} = \mathbf{a}' - \dot{\boldsymbol{\omega}} \times \mathbf{r}' - 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') - \ddot{\mathbf{b}}_S \qquad \text{(d)}$$

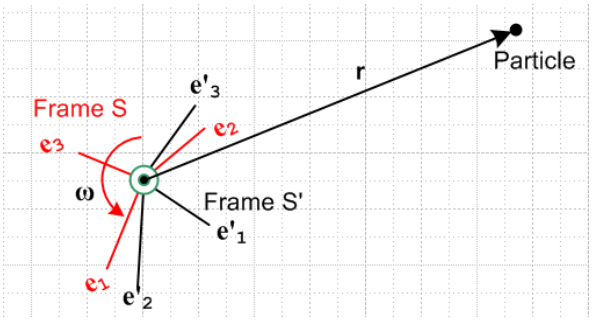
$$\mathbf{a} = \mathbf{a}' - \dot{\boldsymbol{\omega}} \times \mathbf{r} - 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\,\boldsymbol{\omega} \times \dot{\mathbf{b}}_{S'} - \ddot{\mathbf{b}}_{S'} \qquad \text{(e)}$$

$$\mathbf{L}^{(c)} = \mathbf{L}'^{(c')} + m(\mathbf{r}'-\mathbf{c}') \times [-(\boldsymbol{\omega} \times \mathbf{r}') - \dot{\mathbf{b}}_S] \qquad \text{(f)}$$

$$\dot{\mathbf{L}}^{(c)} = \dot{\mathbf{L}}'^{(c')} + m(\mathbf{r}'-\mathbf{c}') \times [-\dot{\boldsymbol{\omega}} \times \mathbf{r}' - 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') - \ddot{\mathbf{b}}_S]$$
$$- m(\dot{\mathbf{c}}' - \boldsymbol{\omega} \times \mathbf{c}' - \dot{\mathbf{b}}_S) \times (\mathbf{v}' - \boldsymbol{\omega} \times \mathbf{r}' - \dot{\mathbf{b}}_S) + m\,\dot{\mathbf{c}}' \times \mathbf{v}' \qquad \text{(g)}$$

Special Case #4 (Inverse Problem, swap notation)

If the origins of Frame S and Frame S' coincide, then $\mathbf{b} = 0$. In this case one of course has $\mathbf{r} = \mathbf{r}'$ and also $\mathbf{c} = \mathbf{c}'$ for the torque and angular momentum reference points. It is usual in this case to select $\mathbf{c} = \mathbf{c}' = 0$. We then write $\mathbf{L} \equiv \mathbf{L}^{(0)}$, $\mathbf{L}' \equiv \mathbf{L}'^{(0)}$, $\mathbf{N} \equiv \mathbf{N}^{(0)}$ and $\mathbf{N}' \equiv \mathbf{N}'^{(0)}$. The new drawing is this,



(13.4.3)

We have placed the common origin in the plane of paper and are viewing things from a direction which causes the instantaneous $\boldsymbol{\omega}$ vector to point toward the viewer. Frame S is rotating relative to Frame S' at rate ω. The vector $\mathbf{r}$ in general is not in the plane of paper. Here are the simplified equations obtained from those above with $\mathbf{r} = \mathbf{r}'$, $\mathbf{b} = \dot{\mathbf{b}}_S = \dot{\mathbf{b}}_{S'} = \ddot{\mathbf{b}}_S = \mathbf{c} = \mathbf{c}' = 0$:

$\mathbf{r}, \mathbf{v}, \mathbf{a}$	position, natural velocity and natural acceleration in Frame S	(13.4.4)
$\mathbf{r}, \mathbf{v}', \mathbf{a}'$	position, natural velocity and natural acceleration in Frame S'	
$\boldsymbol{\omega}$	angular velocity of Frame S relative to Frame S'	
$\mathbf{r} = \mathbf{r}'$		(a)

$$\mathbf{v} = \mathbf{v}' - \boldsymbol{\omega} \times \mathbf{r}' \qquad \text{(b) = (c)}$$

$$\mathbf{a} = \mathbf{a}' - \dot{\boldsymbol{\omega}} \times \mathbf{r}' - 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') \qquad \text{(d) = (e)}$$

$$\mathbf{L} = \mathbf{L}' - m\,\mathbf{r}' \times (\boldsymbol{\omega} \times \mathbf{r}') \qquad \text{(f)}$$

$$\dot{\mathbf{L}} = \dot{\mathbf{L}}' + m\mathbf{r}' \times [-\dot{\boldsymbol{\omega}} \times \mathbf{r}' - 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}')] \qquad \text{(g)}$$