Fictitious Torque Theory PhL 12.13.16

I am not a happy camper with this subject.

First review the linear theory.

1, You start with $\mathbf{F} = m\mathbf{a}$ in inertial frame S.

2. You read frames doc and you note that you can write **a** in terms of Frame S' quantities like so:

$$\mathbf{a} = \mathbf{a}' + \dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') + \ddot{\mathbf{b}}_S \qquad (7.6a)$$

and therefore

$$\mathbf{a}' = \mathbf{a} - [\,\dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') + \ddot{\mathbf{b}}_S]$$

3. You define $\mathbf{F}'_{\mathtt{eff}}$ in Frames S' like so

$$\mathbf{F}'_{\mathtt{eff}} \equiv m\mathbf{a}'$$

$$\mathbf{F}'_{\mathtt{fict}} \equiv \mathbf{F}'_{\mathtt{eff}} - \mathbf{F}$$

4. You then trivially find that

$$\mathbf{F}'_{\mathtt{eff}} \equiv m\mathbf{a}' = m\mathbf{a} - m\,[\,\dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') + \ddot{\mathbf{b}}_S]$$

$$= \mathbf{F} - m\,[\,\dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') + \ddot{\mathbf{b}}_S]$$

5. You conclude then that

$$\mathbf{F}'_{\mathtt{fict}} = -\,m\,[\,\dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') + \ddot{\mathbf{b}}_S]$$

This methodology seems bulletproof and each term has a reasonable interpretation.

In Frame S one can write $\mathbf{F} = \partial_S \mathbf{p} = m\,\partial_S \mathbf{v} = m\mathbf{a}$. These equations are true only because Frame S' is an inertial frame.

Question #1: We know that in Frame S can write $\mathbf{p} = m\mathbf{v}$. Can you write $\mathbf{p}' = m\mathbf{v}'$ in Frame S' ?

Answer: I would regard these equations as *definitions* of **p** and **p'** so yes both are true. But Newton's Law is not a definition of force, so you cannot write **F'** = m**a'** in Frame S' where things are a simple rotated frame scenario. You can only write $\mathbf{F'}_{\mathtt{eff}} \equiv$ m**a'** in Frame S'.

Now what happens for the angular theory?

1, You start with **N** = I**α** in inertial frame S. But these things need a reference point, so I then write this as

$$\mathbf{N}^{(c)} = \mathrm{I}^{(c)}\boldsymbol{\alpha}^{(c)}$$

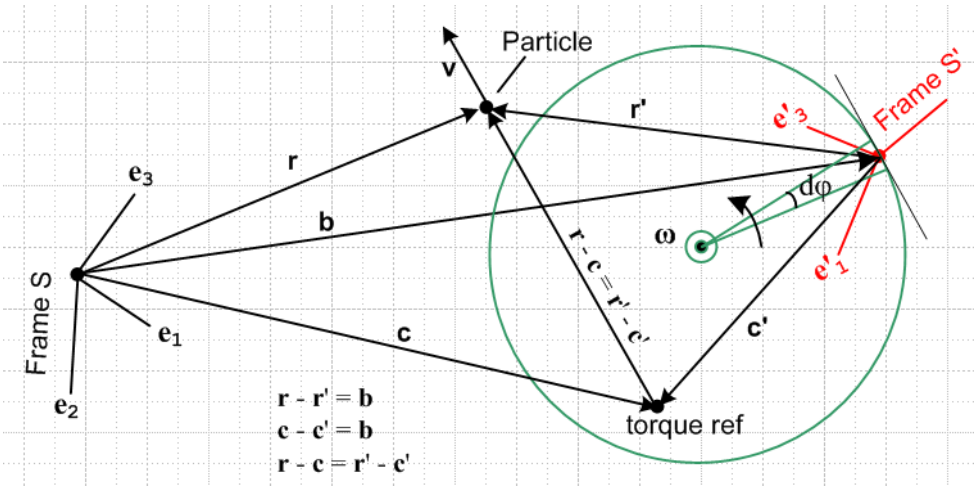
But right off the bat things are hazy. What exactly are these three objects? I am more comfortable with this starting point for an individual point particle,

$$\mathbf{N} = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (m\mathbf{a}) \qquad \mathbf{L} = \mathbf{r} \times (m\mathbf{v})$$

This is by default a statement about your selected origin O. If you go to some other reference point, you can write

$$\mathbf{N}^{(c)} = (\mathbf{r}\text{-}\mathbf{c}) \times \mathbf{F} = (\mathbf{r}\text{-}\mathbf{c}) \times (m\mathbf{a}) \qquad \mathbf{L}^{(c)} = (\mathbf{r}\text{ -}\mathbf{c}) \times (m\mathbf{v})$$

I don't have any inertia or angles yet! Frames doc opens this topic at (1.8.11) with this picture



We don't really have any "angles" associated with the motion of the particle. The angle dφ is about the motion of the Frame S' origin, not about some circular motion of the Particle.

Equations like $\mathbf{N}^{(c)} = \mathrm{I}^{(c)}\boldsymbol{\alpha}^{(c)}$ I think are specific to "circular motion". The above picture does not show such circular motion of mass m, even instantaneously. So I think in a general theory, you cannot be talking about things like $\boldsymbol{\alpha}^{(c)}$. I think you have to do it as I have done it in frames doc where there are no such angle things.

I have updated frames doc with this little clarification that in general writing $\mathbf{N}^{(c)} = I^{(c)}\boldsymbol{\alpha}^{(c)}$ is only useful it you are talking circular motion.

I further reviewed the frames doc torque stuff and I think it is correct. The final result is that

$$\mathbf{N'}^{(c')}_{\mathbf{fict}} = \dot{\mathbf{L}}'^{(c')} - \dot{\mathbf{L}}^{(c)}$$

$$\begin{aligned}\mathbf{N'}^{(c')}_{\mathbf{fict}} &= -(\mathbf{r'}-\mathbf{c'}) \times [\, m\ddot{\mathbf{b}}_S + m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) + 2m\,\boldsymbol{\omega} \times \mathbf{v'} + m\dot{\boldsymbol{\omega}} \times \mathbf{r'}] \\ &\quad + m(\dot{\mathbf{c}}' + \boldsymbol{\omega} \times \mathbf{c'} + \dot{\mathbf{b}}_S) \times (\mathbf{v'} + \boldsymbol{\omega} \times \mathbf{r'} + \dot{\mathbf{b}}_S) - m\dot{\mathbf{c}}' \times \mathbf{v'} \quad . \end{aligned} \tag{11.3.10}$$

It is just plain complicated and *impossible* to interpret, there is just too much stuff. Recall that

$$\dot{\mathbf{L}}'^{(c')} - \dot{\mathbf{L}}^{(c)} = (\mathbf{r'}-\mathbf{c'}) \times \mathbf{F}_{\mathbf{fict}} + m(\dot{\mathbf{c}}' + \boldsymbol{\omega} \times \mathbf{c'} + \dot{\mathbf{b}}_S) \times (\mathbf{v'} + \boldsymbol{\omega} \times \mathbf{r'} + \dot{\mathbf{b}}_S) - m\dot{\mathbf{c}}' \times \mathbf{v'} \tag{11.2.16}$$

$$\mathbf{F'}_{\mathbf{fict}} = -m\ddot{\mathbf{b}}_S - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) - 2m\,\boldsymbol{\omega} \times \mathbf{v'} - m\dot{\boldsymbol{\omega}} \times \mathbf{r'} \quad . \tag{8.1.8}$$

I am pretty sure I have done this right.