Pendulum with no ODEs ? PhL 1.30.17

Before continuing into App F, I have a **critique**. In the ant trails discussion, I was critical of those who wrote and solved ODE's to find solutions, when they could have just grabbed one of my formulas. So why have I done all this work on the pendulum and dumbbell satellite when maybe I could just grab solutions from those same formulas??

OK, let's review the ant section.

15.1 Problem 1: In Frame S' shown in Fig 15.1.1 an ant starts at some $\mathbf{r'}_0$ and crawls to the Frame S' origin at a constant speed V.

In the rotating frame S' I write $\mathbf{v'} = -V\hat{\mathbf{r}}'$, integrate to get $r' = r'_0 - Vt$ with $\theta' = \theta'_0$ at all times. I then have

$$\mathbf{r'} = \mathbf{r'}_0 - Vt\,\hat{\mathbf{r}}'$$
$$\mathbf{v'} = -V\,\hat{\mathbf{r}}' = -V\cos(\theta'+\varphi)\,\hat{\mathbf{x}} - V\sin(\theta'+\varphi)\,\hat{\mathbf{y}}$$
$$\mathbf{a'} = 0\ . \qquad (15.1.7)$$

I want to know r(t) in Frame S and I use "my equation" $\mathbf{r} = \mathbf{b} + \mathbf{r'}$ and that gives $\mathbf{r}(t)$ without doing any ODE's. But I guess I did do $\mathbf{v'} = \dot{\mathbf{r}}'$ and $\mathbf{a'} = \dot{\mathbf{v}}'$ implicitly. Next I get $\mathbf{v}(t)$ by just quoting one of "my" formulas. Finally I get a from my fanciest formula with all the fictitious accelerations shown. In the end I have found r(t), v(t) and a(t) without rally dealing with any ODE's. I did not do $\mathbf{F'}_{\mathbf{eff}} = m\mathbf{a'}$ in the rotating frame. I then plot the resulting paths. In the case b = 0 the Frame S' origin is at the turntable center so you can imagine what that plots would look like. I quote the results for the b = 0 case at the end of Sec 15.1.

15.2 Problem 2: Ant spirals in at constant V and Ω to the Origin of Frame S'

Here I use that last quoted result adding primes so that NOW in frame S' the ant spirals into the Frame S' origin, and we want to know his path in Frame S. I state this initial trajectory description in Frame S' and then I change from the Prime unit vectors to the Unprime ones in (15.2.6). I am then ready to determine the objects $\mathbf{r}(t)$,$\mathbf{v}(t)$ and $\mathbf{a}(t)$. I use the exact same $\mathbf{r} = \mathbf{b} + \mathbf{r'}$ and I have $\mathbf{r}(t)$, done. And I use the same equation for v and for a, all done. I then make plots. So once again, I have done no ODE's.

The idea is that if you KNOW the $\mathbf{r'}$,$\mathbf{v'}$,$\mathbf{a'}$ in Frame S', you just turn my crank and you have $\mathbf{r}$,$\mathbf{v}$,$\mathbf{a}$ in Frame S just using my simple formulas. Plots are interesting. So far no ODE's .

15.3 Problem 3: Inverse Problem: Ant flies in Frame S at constant velocity V

Here Frame S ant flies in a line over the turntable, and we ask what the path looks like in Frame S'. Now I use r' = r-b and out pops what I want. Suddenly I am using $\mathbf{e}_i$ type basis vectors instead of $\hat{\mathbf{x}}$ type ones, why did I switch? I do comment on the primes.

Disgression: Go back to Section 1.1 and the use of $\mathbf{e}_i$, I think these are the obvious $\mathbf{e}_1 = \hat{\mathbf{x}}$ etc so maybe I should add an early comment to that effect. // OK I added (C.1.5) and that is cleared up! Back to problem 3.

The ant in Frame S does $\mathbf{v} = V\hat{\mathbf{n}}$ in some direction $\hat{\mathbf{n}}$. Once again we start from $\mathbf{r}' = \mathbf{r}-\mathbf{b}$ and the solution for $\mathbf{r}'(t)$ is then (15.3.6). I also read off $\mathbf{v}'(t)$ from one of my formulas. I then state $\mathbf{a}'(t)$ but I don't pursue the various terms. No ODE's ! I then make plots and compare with Thornton and there is a discrenancy in the numbers but not in the plots. I do not claim here that Thornton uses ODE's.

OK, so I was wrong about the ODE claim. It is in Problem 4 that I make this claim, but it was good to review those ant problems.

15.4 Problem 4: The Projectile Problem of Section 8.3

The picture is (8.3.1). Frame S is fixed and inertial. Frame S' is glued to the rotating platter. I claim that the hard way is to use Newton's Law with fictitious forces and get these coupled ODE's

$$\ddot{v}_x - 2\omega\dot{v}_y - \omega^2 v_x = 0$$
$$\ddot{v}_y + 2\omega\dot{v}_x - \omega^2 v_y = 0 \quad . \qquad (15.4.5)$$

"**But**, we don't *have* to solve this coupled system of differential equations because we already know the solution, and we didn't have to even *look* at a differential equation to find it! The solution is (15.3.7) which we quote " I then quote the result of the ant overflight Problem 3.

OK, so there is that claim I made: why do coupled ODE's when you don't have to !!

Big Question: Why can't I do this same thing with pendulums and with dumbbell satellites?

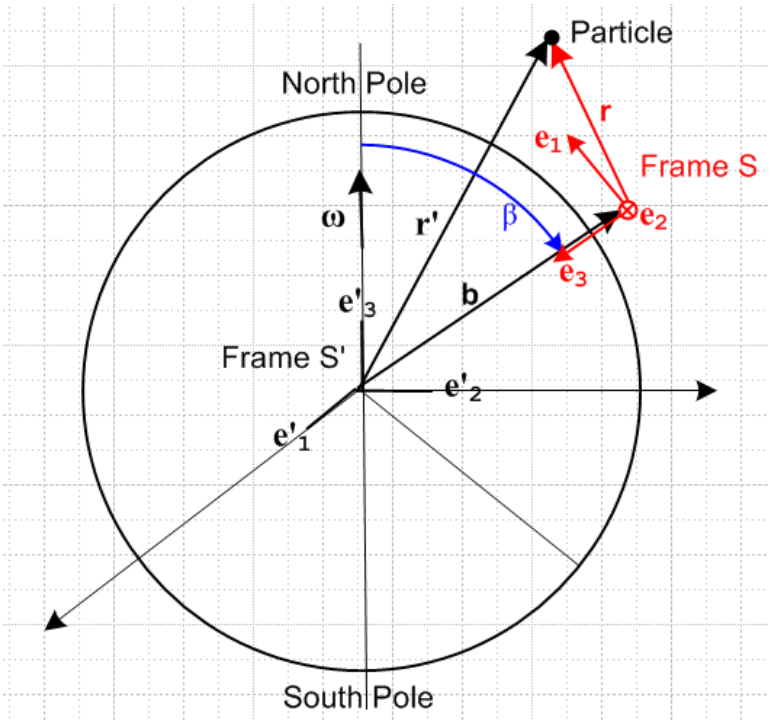
In my no-ODE case above I used the solution to the ant-overflight problem to solve the four-projectile problem. Each projectile is really just an ant overflight.

My Hard Way is Newton's Law and I guess I could have made this argument back in the ant overflight problem rather than here in the 4-projectile problem. How do we view the flying ant in Frame S' as a Newton's Law problem. From Frame S' see the ant with $\mathbf{r}'$ and $\mathbf{v}'$ and $\mathbf{a}'$ doing a complicated motion. We explain this as Newton's Law with

$$\mathbf{F}'_{\mathtt{fict}} = \underset{\text{centrifugal}}{m\omega^2\mathbf{r}'} \quad \underset{\text{Coriolis}}{- 2m\,\boldsymbol{\omega} \times \mathbf{v}'} \quad . \qquad (8.1.13)$$

and we solve that problem which is the pair of ODE's.

OK, now think spherical pendulum and you want to view the swing from two Frames and then just use my little formulas. What would those two frames be?? I want one frame to be inertial as in the ant overflight. OK, in my setup I have



So yes, Frame S' is inertial. If I knew the trajectory of the pendulum in Frame S'. I could easily write it in rotating Frame S using my little formulas.

In Frame S' we have a very complicated problem. The pendulum swings in its mount, but the mount is moving and in fact rotating. The mount point is the origin of Frame S' and so I know that the pendulum mount point has this trajectory in Frame S' :

$\mathbf{b}(t) = R_{z'}(\omega t)\,\mathbf{b}(0)$ // the attachment point is moving

$\hat{\mathbf{z}}(t) = R_{z'}(\omega t)\,\hat{\mathbf{z}}(0)$ // the down direction is moving

In Frame S I had Newton's Law with just one fictitious force (Coriolis).

$$m\mathbf{a} = mg\hat{\mathbf{z}} + \mathbf{T} - 2m\,\boldsymbol{\omega} \times \mathbf{v} \qquad \mathbf{T} = -T\hat{\mathbf{r}} \tag{C.3.3}$$

In Frame S' we have instead

$$m\mathbf{a}' = mgR_{z'}(\omega t)\,\hat{\mathbf{z}}(0) + \mathbf{T}'$$

So now all the complexity is inside tension **T'**. The only forces on mass m are the string and gravity which is changing its direction all the time.

Suppose we have some solution **r'**(t) for the mass m in inertial Frame S'. Then we know that

$$\mathbf{T}' = T'\,[\,\mathbf{b}(t) - \mathbf{r}'(t)\,]\,/\,|\,\mathbf{b}(t) - \mathbf{r}'(t)\,| = T'\,[\,\mathbf{b}(t) - \mathbf{r}'(t)\,]\,/\,\ell$$

and then at least I have a scalar function T(t) to solve for. So then we have

$$m\mathbf{a}' = mgR_{\mathbf{z}'}(\omega t)\,\hat{\mathbf{z}}(0) + T'\,[\,\mathbf{b}(t) - \mathbf{r}'(t)\,]/\ell$$

This is three scalar equations where the unknown functions are x',y',z',T' The fourth equation I think is this

$$x^2+y^2+z^2 = \ell^2$$

BUT you have to express this in Frame S' coordinates. We have $\mathbf{r}' = \mathbf{b} + \mathbf{r}$ so

$$x' = \mathbf{b}(t) \bullet \hat{\mathbf{x}}' + x = b_{\mathbf{x}'}(t) + x \qquad \text{etc}$$

so the fourth equation is something like

$$(x' - \mathbf{b}(t) \bullet \hat{\mathbf{x}}')^2 + (y' - \mathbf{b}(t) \bullet \hat{\mathbf{y}}')^2 + (z' - \mathbf{b}(t) \bullet \hat{\mathbf{z}}')^2 = \ell^2$$

So there you are, four equations in four unknowns x',y',z',T' . Three of the equations are ODE's.

Conclusion: this is a much harder problem to solve "Find r'(t) of the pendulum mass in Frame S'"

So now I can answer the original question:

In the ant overflight problem, I know the trajectory of the ant in the inertial frame S. So it was then a simple matter to express this path in Frame S' coordinates using my "little formulas".

In the pendulum problem I do NOT know the trajectory of the pendulum in inertial Frame S', so I cannot just translate it to some other frame. In fact, I don't know the trajectory of the pendulum in ANY frame.

In the ant overflight problem, the inertial frame was a trivial problem with a trivial path.

In the pendulum problem, the inertial frame problem is an even worse problem that in the rotating frame!

In order to use the conversion little formulas, you have to KNOW something about the path in one of the two frames that are related by rotation. In the pendulum problem I don't know the path in any frame when I start out.

I DO know the pendulum **r**(t) in a frame on the Earth's surface when the Earth is not rotating, it is that sn function business. That is yes an inertial frame. If the problem were simply to *view* that known path from a rotating frame, I could use the conversions. But that is not the problem. The problem is that the pendulum itself is in that rotating frame where you are doing the viewing. I cannot take the rotating Earth pendulum problem and view it from some inertial frame in which it is the plane pendulum problem.

OK enough! It was a reasonable critique and I now know the resolution.

You cannot avoid the ODE's for the full Foucault pendulum problem by going to some other inertial frame.