Notation Tests 2_25_17 PhL 2.25.17

Scenario 1

In Frame S we have **p** = m**v** for a particle.
In Frame S' we have **p'** = m**v'** for that same particle.
Both frames are inertial, they are not rotating frames, just static frames.
Frames are Special Case #4 with co-sited origins.

What is the meaning of this equation: **p'** = R**p** ?

In the passive view, there is only one true vector **p** and you observe it from two frames. When you write the above equation, you really mean this

$(p)'_i = R_{ij}p_j$ or in vector notation $(\mathbf{p})' = R\mathbf{p}$

This equation tells you the components of **p** in the Frame S'.

You could define a new vector **p'** ≡ R**p** in Frame S, and then it happens that the components of this vector in Frame S are the same as the components of **p** in Frame S', since

$(p)'_i = (p')_i$

I guess when I write **p'** = R**p** , I have to think of **p'** as an actively rotated vector in Frame S. And so when I write **p'** = m**v'** I am thinking of two actively rotated vectors in Frame S.

Fact: When you even talk about the equation **p'** = m**v'** , you are taking the active view and these are new vectors within Frame S which are obtained by **p'** = R**p** and **v'** = R**v** .
ok to here
Question: How does the above discussion relate to the frames doc Special Case #1 equation

$\mathbf{v} = \mathbf{v}' + \boldsymbol{\omega} \times \mathbf{r}$ Special Case #1 only *(6.6a)* ?

First of all, this latter equation assumes that the two frames are not static but are rotating relative to each other at ω. It we take the limit that they become static, then this equation says

$\mathbf{v} = \mathbf{v}'$

You see the danger! So here for two static frames I say **v** = **v'** but above I say for two static frames that **v'** = R**v**. This does seem paradoxical! How shall I "splain" the distinction here?

Answer 1: If we take ONLY the passive view in the static frames case, there is only one vector **v** and you can obtain its components in either frame $(v)_i$ or $(v)'_i$. There exists no vector **v'**. The equation

$\mathbf{v} = \mathbf{v}' + \boldsymbol{\omega} \times \mathbf{r}$ is entirely in the passive view and each v is "natural". If you turn off ω, there is only one vector **v**. The meaning of the symbols is different in $\mathbf{v} = \mathbf{v}' + \boldsymbol{\omega} \times \mathbf{r}$ and $\mathbf{v}' = R\mathbf{v}$ but the symbols look exactly the same! In the passive view, there is no equation of the form $\mathbf{p}' = m\mathbf{v}'$.

Note added 2.26.17 In our frame application where we have equations like $\mathbf{v} = \mathbf{v}' + \boldsymbol{\omega} \times \mathbf{r}$ and $\mathbf{r} = \mathbf{r}' + \mathbf{b}$. the prime symbol is used to indicate the "natural" time derivative like $\mathbf{v}'$ and we have separated origins so each needs a vector name. Thus we have already "used up" the prime symbol as a notation tool. Therefore, we are not allowed to simply *define* vectors $\mathbf{r}' \equiv R\mathbf{r}$ and $\mathbf{v}' \equiv R\mathbf{v}$ in as needed in the Active View program. The symbols $\mathbf{r}'$ and $\mathbf{v}'$ are already used !!!!!! However, in the Passive View we are still OK if we write things like $(\mathbf{r})' = R\mathbf{r}$ and $(\mathbf{r}')' = R\mathbf{r}'$ and $(\mathbf{v})' = R\mathbf{v}$ and $(\mathbf{v}')' = R\mathbf{v}'$ for vectors $\mathbf{r},\mathbf{r}',\mathbf{v},\mathbf{v}'$. It would be wrong to write $\mathbf{r}' \equiv R\mathbf{r}$!!!!! So we NEED the notation like $(\mathbf{r})' = R\mathbf{r}$ if we want to express the component sum idea in "vector notation" AND be compatible with our other prime meaning.
ok to here

Scenario 2

In Frames doc I have $\mathbf{e}_n = R\mathbf{e}'_n$ which is the back-rotated basis vector deal that we always use when we are taking the passive view!

$(\mathbf{e}_n)_i = [R\mathbf{e}'_n]_i = R_{ij}\,(e'_n)_j$ Frame S components

$(\mathbf{e}_n)'_i = [R\mathbf{e}'_n]'_i = R'_{ij}\,(e'_n)'_j$ Frame S' components // note prime on R'_{ij} (1.1.33)

This $\mathbf{e}_n = R\mathbf{e}'_n$ has the general form $\mathbf{a} = T\mathbf{b}$ which I mention in frames doc as a notation that needs to be clarified with the Dirac notation. Note that $\mathbf{L} = I\boldsymbol{\Omega}$ has just this form. So in Dirac I guess we could say

$$|\mathbf{L}\rangle = I\,|\boldsymbol{\Omega}\rangle = |\mathbf{e}_i\rangle\langle\mathbf{e}_i|\,I\,|\mathbf{e}_j\rangle\langle\mathbf{e}_j|\,\boldsymbol{\Omega}\rangle = I_{ij}\,(\Omega)_j\,|\mathbf{e}_j\rangle$$

Close to get

$$L_k = \langle\mathbf{e}_k|\mathbf{L}\rangle = I_{ij}\,(\Omega)_j\,\langle\mathbf{e}_k|\mathbf{e}_j\rangle = I_{ij}\,(\Omega)_j\delta_{ik} = I_{kj}\,(\Omega)_j$$

or

$$\mathbf{L} = I\boldsymbol{\Omega}$$

So this seems to have a pretty clean Dirac meaning. Here I am thinking of a particle which has some angular velocity vector $\boldsymbol{\Omega}$ and some inertia tensor I.

In the pure passive view with static frame S and S' there is only one vector **L** and only one **Ω.** You could study the components of $\mathbf{L} = I\boldsymbol{\Omega}$ in either frame. To do this study, you would write

$(L)_i = [I\Omega]_i = I_{ij}\Omega_j$ Frame S // $\mathbf{L} = I\boldsymbol{\Omega}$

$(L)'_i = [I\Omega]'_i = I'_{ij}\Omega'_j$ Frame S' // $(\mathbf{L})' = I'\boldsymbol{\Omega}'$

and this would have a Dirac form that makes sense. I think $I' = RIR^{-1}$ if $\mathbf{e}_n = R\mathbf{e}'_n$.

Note added 2.26.17: above I am just getting warmed up to address the rigid body equations mystery!
ok to here
Question: I seem to have no problem with the meaning of I' = RIR^{-1} in the passive view. Why do I have such a problem with the rank-1 rule **v'** = R**v** ? Since there exists no vector **v'** in the passive picture, what are you trying to say? I guess it is that idea that it is (**v**)' = R**v** which matches I' = RIR^{-1} . In Dirac,

$\langle \mathbf{e}_i | \mathbf{v} \rangle = v_i \quad = \langle R\mathbf{e}'_n | \mathbf{v} \rangle \quad = ???$

My answer to this question is this is as follows:

I' = RIR^{-1}	$\langle \mathbf{e}'_m | \mathcal{I} | \mathbf{e}'_n \rangle = \langle \mathbf{e}'_m | \mathbf{e}_i \rangle\langle \mathbf{e}_i | \mathcal{I} | \mathbf{e}_j \rangle\langle \mathbf{e}_j | \mathbf{e}'_n \rangle = R_{mi} I_{ij} R_{nj}$
(**v**)' = R**v**	// this is just a shorthand notation for what is below, has no Dirac not.
$(v)'_i = [R\mathbf{v}]_i = R_{ij} v_j$	$\langle \mathbf{e}'_i | \mathbf{v} \rangle = \langle \mathbf{e}_i | R\mathbf{v} \rangle = \langle \mathbf{e}_i | R | \mathbf{e}_j \rangle\langle \mathbf{e}_j | \mathbf{v} \rangle = R_{ij} v_j$

You could wedge this into Dirac by saying $\langle \mathbf{e}_i | (\mathbf{v})' \rangle = \langle \mathbf{e}'_i | \mathbf{v} \rangle$ so then $|(\mathbf{v})' \rangle = |R\mathbf{v}\rangle$. Not sure this is worth doing. Perhaps it is just fine.
ok to here

Question: In tensor doc and wedge doc I say $V'^n = R^n_m V^m$. Does this mean that these docs are all in the active view only? Or does this equation really mean $(V)'^n = R^n_m V^m$ and there is only one **V** ?

Answer 2.26.17: We cannot use Active View if **V'** already has some other meaning like v' does in frames doc. We stick then to the Passive View and the equation is (**V**)' = R**V** or $(V)'^n = R^n_m V^m$. I deal with this question at the end of Question 1 v3.
ok to here
Question: How do you relate x-space and x'-space of tensor doc to Frame S and Frame S' of frames doc?

Answer 2.26.17: This is fully addressed in Question 1 v3 with a slightly unexpected answer, see there.
ok to here
Question: How do $\mathbf{e}_n$ and $\mathbf{e}'_n$ in frames doc relate to basis vectors in tensor doc?

I have avoided facing up to these questions in order to get frames doc done. In tensor doc I have

$V'^a = R^a_b V^b$	rank 1 tensor
$M'^{ab} = R^a_{a'} R^b_{b'} M^{a'b'}$	rank 2 tensor

Things are on a uniform footing in each rank. But I seem to have a non-uniform footing in frames doc,.

Question: In frames doc, the idea of **V'** = R**V** works only if **b** = 0. Is there some restriction implied in tensor doc analogous to **b** = 0 without which you would not have $V'^a = R^a_b V^b$?

Answer 2.26.17: This is fully addressed in Question 1 v3, see there.
ok to here
Each of these questions could be a week of work to answer. (well, not really it turns out).

Question: All these questions are being motivated by my not understanding $\mathbf{L} = I\boldsymbol{\omega}$ in terms of frame doc. If L is Frame S, how can I be diagonal? This equation has the same confusion as $\mathbf{V}' = R\mathbf{V}$.

pending

Question: In frames doc Section 1 I treat R as if it were a tensor. But in tensor doc I make a big point about R NOT being a tensor, having one foot in each world. How does this all resolve?

Answer 2.26.17: Concerning the "one foot in each world" comment, I already have this in frames doc

$$R_{nm} = \langle \mathbf{e}'_n | \mathbf{e}_m \rangle = \langle R^{-1}\mathbf{e}_n | \mathbf{e}_m \rangle = \langle R^{T}\mathbf{e}_n | \mathbf{e}_m \rangle = \langle \mathbf{e}_n \,|\, \mathcal{R} \,|\, \mathbf{e}_m \rangle$$

and there you explicitly see one foot in each world. The R matrix is the matrix element of the unity operator in a mixed basis! I have just added a nice comment to this effect and have showed how one could use a mixed tensor for any tensor but notation is messy.

So I think in light if these comments, it is completely OK to treat $\mathcal{R}$ just as any other $\mathcal{T}$ in the Dirac space.

ok to here, only one "nasty question" remains unanswered: (see Question 2 docs)

Question Set (repeat). We have $\mathbf{L} = I\,\boldsymbol{\omega}$.

Is this equation "in a frame"? Is this the Frame S equation and there is a Frame S' equation that reads $\mathbf{L}' = I'\,\boldsymbol{\omega}'$ like $\mathbf{F} = m\mathbf{a}$ and $\mathbf{F}' = m\mathbf{a}'$?

For the case that Frame S' is the body frame, it seems that $\mathbf{L}' = 0$ for the rigid body or any particle of it, since the body is at rest in this body frame. Then $\mathbf{L}' = I'\,\boldsymbol{\omega}'$ would read $0 = I'\,\boldsymbol{\omega}'$. What does *that* mean? In one sense $\boldsymbol{\omega}' = 0$ in Frame S' since nothing is moving, but in another sense, Frame S' is moving relative to Frame S at some $\boldsymbol{\omega}'$ and perhaps $\boldsymbol{\omega}' = -\boldsymbol{\omega}$.

If $\mathbf{L} = I\,\boldsymbol{\omega}$ is a Frame S equation with Frame S being "the table on which the top is spinning" then how can you say I is diagonal? I think of the principle axes being the body Frame S' axes, maybe that is wrong.

Maybe $\mathbf{L} = I\,\boldsymbol{\omega}$ is *not* an equation that lives within a frame?