Question 3 PhL 2.26.17

Question 3: If you have some point **r** in space, how to you compute its components in all the Euler Angle frames of reference.

We are looking at Section G.5.

You want to compute for example

$$\mathbf{r} \bullet \hat{\boldsymbol{\zeta}}' = r_{\zeta'}$$

You know that

$$\hat{\boldsymbol{\zeta}}' = R_z(\varphi)R_x(\theta)\hat{\mathbf{z}} = R_z(\varphi)R_x(\theta)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \text{picks off the right column of matrix}$$

The matrix $R_z(\varphi)R_x(\theta)$ is the *transpose* of what is shown below in Maple, so

$$= \begin{pmatrix} \sin\theta\sin\varphi \\ -\sin\theta\cos\varphi \\ \cos\theta \end{pmatrix} = \sin\theta\sin\varphi\ \hat{\mathbf{x}} - \sin\theta\cos\varphi\ \hat{\mathbf{y}} + \cos\theta\ \hat{\mathbf{z}}$$

Alternatively use

$$\text{(b)} \qquad \begin{pmatrix} \hat{\boldsymbol{\xi}}' \\ \hat{\boldsymbol{\eta}}' \\ \hat{\boldsymbol{\zeta}}' \end{pmatrix} = R_x(-\theta)\, R_z(-\varphi) \begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{z}} \end{pmatrix} \qquad \text{// using (4)}$$

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evalm(Rx(-theta) &* Rz(-phi));
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$$\begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi)\cos(\theta) & \cos(\phi)\cos(\theta) & \sin(\theta) \\ \sin(\theta)\sin(\phi) & -\sin(\theta)\cos(\phi) & \cos(\theta) \end{bmatrix}$$

Then

$$\hat{\boldsymbol{\zeta}}' = \sin\theta\sin\varphi\ \hat{\mathbf{x}} - \sin\theta\cos\varphi\ \hat{\mathbf{y}} + \cos\theta\ \hat{\mathbf{z}}$$

So now consider

$$r_{\zeta'} = \mathbf{r} \bullet \hat{\boldsymbol{\zeta}}' = x\sin\theta\sin\varphi - y\sin\theta\cos\varphi + z\cos\theta$$

and this is the component sought after. For a vector V,

$$(\mathbf{V})_{\zeta'} = \mathbf{V} \bullet \hat{\zeta}' = \sin\theta\sin\varphi(V)_{\mathbf{x}} - \sin\theta\cos\varphi\,(V)_{\mathbf{y}} + \cos\theta(V)_{\mathbf{y}}$$

Now the rows of the above matrix are the $\begin{pmatrix} \hat{\xi}' \\ \hat{\eta}' \\ \hat{\zeta}' \end{pmatrix}$ so you get all three at once.