How did I change reference frames in sun earth doc? PhL 2.28.17

This is a due diligence test that luckily passed OK.

I just corrected a bad frame change in frames doc Section 8.8 regarding moon earth.

I am now concerned that I might have made this same error elsewhere in my docs. In particular, I am worried about sun earth doc (astronomy doc) where I shift from north pole to local location on the earth. How did I do that frame change in that doc ??? (very nervous right now. ...)

Here is one set of equations

The black equatorial circle is associated with Frame S with x,y,z axes, while the ecliptic red circle is associated with Frame S' with x',y',z' axes. Staring at the figure one sees that

$$\hat{\mathbf{z}}' = R_x(\theta_{tmax})\,\hat{\mathbf{z}} \qquad \text{and} \qquad \hat{\mathbf{x}}' = \hat{\mathbf{x}} \;\Rightarrow\; x' = x$$
$$\hat{\mathbf{y}}' = R_x(\theta_{tmax})\,\hat{\mathbf{y}} \tag{6.1}$$

where $\theta_{tmax} = 23°$. According to the rule that vectors rotate backwards from the unit vectors (see Appendix B), we then have in 2D that

$$\mathbf{r} = R_x(\theta_{tmax})\mathbf{r}' \quad \Rightarrow \quad \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} \cos\theta_{tmax} & -\sin\theta_{tmax} \\ \sin\theta_{tmax} & \cos\theta_{tmax} \end{pmatrix} \begin{pmatrix} y' \\ z' \end{pmatrix} \quad \Rightarrow$$

Notice the quote "according to the rule that vectors rotate backwards from the unit vectors". So at least I was aware of this fact at the time, so the last equation is then compatible with the unit vector equations above it. (whew!). I make reference to Appendix B which is my sun-earth take on active versus passive. I will read it right now. // It is just fine! It does not need notation like (V)' because in that doc I don't have vectors like v' which have other definitions. All OK at least at this point in sun earth.

At this point in sun-earth I am using Frame S = celestial sphere and Frame S' = tilted ecliptic as shown in Fig 6.1.

The next Big Deal is Section 7 where I have Frame S = celestial sphere and Frame S' = local system on surface of the earth. So how do I accomplish this transformation?

I first do Cartesian to Cartesian in Section 7 (c) based on this equation

$$\hat{\mathbf{x}} = \cos\varphi\sin\theta\,\hat{\mathbf{r}} + \cos\varphi\cos\theta\,\hat{\boldsymbol{\theta}} - \sin\varphi\,\hat{\boldsymbol{\varphi}}$$
$$\hat{\mathbf{y}} = \sin\varphi\sin\theta\,\hat{\mathbf{r}} + \sin\varphi\cos\theta\,\hat{\boldsymbol{\theta}} + \cos\varphi\,\hat{\boldsymbol{\varphi}}$$
$$\hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}\ . \tag{7.10}$$

which I got from a sun-earth appendix. I sure hope this agrees with frames doc on the same thing:

$$\hat{\mathbf{x}} = \sin\theta\cos\varphi\,\hat{\mathbf{r}} + \cos\theta\cos\varphi\,\hat{\boldsymbol{\theta}} - \sin\varphi\,\hat{\boldsymbol{\varphi}}$$
$$\hat{\mathbf{y}} = \sin\theta\sin\varphi\,\hat{\mathbf{r}} + \cos\theta\sin\varphi\,\hat{\boldsymbol{\theta}} + \cos\varphi\,\hat{\boldsymbol{\varphi}}$$
$$\hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}\ . \tag{E.2.7}$$

OK that is a very good thing! They agree. Look now at sun earth app A where this is derived. Ouch, I quote it from another of my docs, which is Ref [6] of sun earth which is frames doc!!! But I probably have changed eq nums? I refer to this as Appendix A (A.13.c) of frames doc. YES! That is still correct, I have not changed it! I have this in both App A and App E of frames doc.

I am now reading near (7.10) of sun earth doc. The vector **b** is earth center to local frame origin on surface of the earth in Fig 7.1, same as in frames doc. I am asking about the sun position in Frame S'.

In (7.1) I have

$$\hat{\mathbf{r}} = \mathbf{e}'_3 \qquad \hat{\boldsymbol{\theta}} = -\,\mathbf{e}'_2 \qquad \hat{\boldsymbol{\varphi}} = \mathbf{e}'_1 \ , \tag{7.1}$$

so these are how the Frame S basis vectors like $\hat{\mathbf{r}}$ are related to the Frame S' ones like $\mathbf{e}'_3$ in Fig 7.1.
I then install these into the triple of equations shown above as E.2.7 to get

$$\begin{aligned}
\hat{\mathbf{x}} &= \cos\varphi_b \sin\theta_1\, \mathbf{e}'_3 - \cos\varphi_b \cos\theta_1\, \mathbf{e}'_2 - \sin\varphi_b\, \mathbf{e}'_1 \\
\hat{\mathbf{y}} &= \sin\varphi_b \sin\theta_1\, \mathbf{e}'_3 - \sin\varphi_b \cos\theta_1\, \mathbf{e}'_2 + \cos\varphi_b\, \mathbf{e}'_1 \\
\hat{\mathbf{z}} &= \cos\theta_1\, \mathbf{e}'_3 + \sin\theta_1\, \mathbf{e}'_2 \quad .
\end{aligned} \tag{7.11}$$

And then I write

$$\begin{aligned}
\mathbf{r}' \approx \mathbf{r} &= x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}} \\
&= x\,(\cos\varphi_b \sin\theta_1\, \mathbf{e}'_3 - \cos\varphi_b \cos\theta_1\, \mathbf{e}'_2 - \sin\varphi_b\, \mathbf{e}'_1) \\
&+ y\,(\sin\varphi_b \sin\theta_1\, \mathbf{e}'_3 - \sin\varphi_b \cos\theta_1\, \mathbf{e}'_2 + \cos\varphi_b\, \mathbf{e}'_1) \\
&+ z\,(\cos\theta_1\, \mathbf{e}'_3 + \sin\theta_1\, \mathbf{e}'_2) \\
&= [-x \sin\varphi_b + y \cos\varphi_b]\mathbf{e}'_1 \\
&+ [-x \cos\varphi_b \cos\theta_1 - y \sin\varphi_b \cos\theta_1 + z \sin\theta_1]\mathbf{e}'_2 \\
&+ [\ x \cos\varphi_b \sin\theta_1 + y \sin\varphi_b \sin\theta_1 + z \cos\theta_1]\mathbf{e}'_3 \\
&= x'\,\mathbf{e}'_1 + y'\,\mathbf{e}'_2 + z'\,\mathbf{e}'_3
\end{aligned} \tag{7.12}$$

so that the Cartesian sun location coordinates in Frame S' are given by

$$\begin{aligned}
x' &= -x \sin\varphi_b + y \cos\varphi_b \\
y' &= -x \cos\varphi_b \cos\theta_1 - y \sin\varphi_b \cos\theta_1 + z \sin\theta_1 \\
z' &= \ x \cos\varphi_b \sin\theta_1 + y \sin\varphi_b \sin\theta_1 + z \cos\theta_1 \ .
\end{aligned} \tag{7.13}$$

This is "basis vector method" and the result looks good to me. I now have local Cartesian coordinates of the location of the sun. Here (x,y,z) is r in Frame S which is the center of earth aligned frame. I know that the sun's path in this Frame S is given by

$$\begin{aligned}
x &= R \cos\theta_t \cos\varphi \\
y &= R \cos\theta_t \sin\varphi
\end{aligned}$$

$$z = R \sin\theta_t \qquad (7.6)$$

and I then get

$$x' = R \cos\theta_t \sin(\varphi - \varphi_b)$$
$$y' = R [\sin\theta_t \sin\theta_1 - \cos\theta_t \cos\theta_1 \cos(\varphi - \varphi_b)]$$
$$z' = R [\sin\theta_t \cos\theta_1 + \cos\theta_t \sin\theta_1 \cos(\varphi - \varphi_b)] \qquad (7.14)$$

So these are the sun coordinates in local earth Frame S'. So this concludes my Cartesian → Cartesian transformation, it looks OK.

Then in Section 7 (d) I convert to θ' and φ' as Frame S' spherical coordinates!

$$x' = r'\sin\theta'\cos\varphi' \qquad y' = r'\sin\theta'\sin\varphi' \qquad z' = r'\cos\theta'$$

$$\cos\theta' = [\sin\theta_t \cos\theta_1 + \cos\theta_t \sin\theta_1 \cos(\varphi - \varphi_b)]$$
$$\sin\theta' = \rho'/r' = \sqrt{\cos^2\theta_t \sin^2(\varphi - \varphi_b) + \cos^2\gamma} \qquad (7.27)$$

$$\cos\varphi' = x'/\rho' = [\cos\theta_t \sin(\varphi - \varphi_b)] / \sqrt{\cos^2\theta_t \sin^2(\varphi - \varphi_b) + \cos^2\gamma}$$
$$\sin\varphi' = y'/\rho' = [\sin\theta_t \sin\theta_1 - \cos\theta_t \cos\theta_1 \cos(\varphi - \varphi_b)] / \sqrt{\cos^2\theta_t \sin^2(\varphi - \varphi_b) + \cos^2\gamma}$$
$$\tan\varphi' = y'/x' = [\sin\theta_t \sin\theta_1 - \cos\theta_t \cos\theta_1 \cos(\varphi - \varphi_b)] / [\cos\theta_t \sin(\varphi - \varphi_b)] \qquad (7.28)$$

Therefore, we may write

$$\theta' = \cos^{-1}[\sin\theta_t \cos\theta_1 + \cos\theta_t \sin\theta_1 \cos(\varphi - \varphi_b)]$$
$$\varphi' = \tan^{-1} ([\sin\theta_t \sin\theta_1 - \cos\theta_t \cos\theta_1 \cos(\varphi - \varphi_b)] / [\cos\theta_t \sin(\varphi - \varphi_b)]) . \qquad (7.29)$$

OK enough. I think I can rest easy that I did not do a major screw-up in sun earth on this subject. As noted, I was aware of the back-rotation of basis vectors and forward of regulars. Also, my plot results all came out reasonable so I don't think I could have made such a major error!