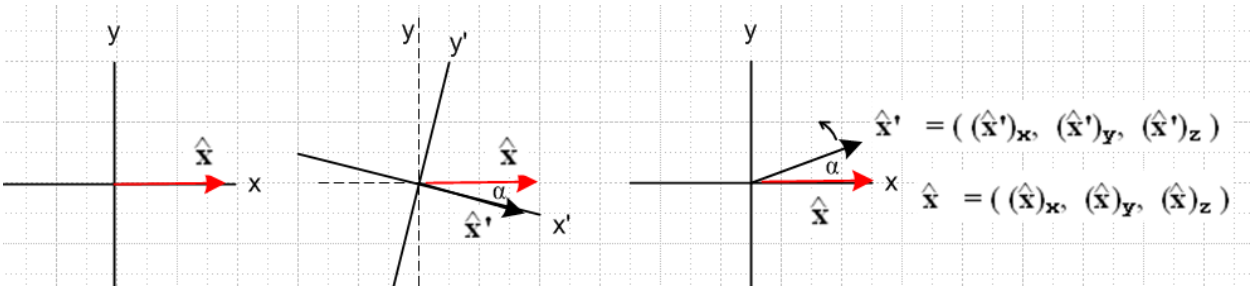
Paradox 1 of 2_23_17 v2. PhL 2.23.17

Consider these pictures



a Frame S b Frame S passive c Frame S active

(1) In picture b, "Frame S passive", the vector $\hat{\mathbf{x}}'$ points **southeast** and we think $\mathbf{e}'_i = R_z(-\alpha)\mathbf{e}_i$ and we say that the basis vectors are "back-rotated". In particular, $\hat{\mathbf{x}}' = R_z(-\alpha)\hat{\mathbf{x}}$ **.** This is the "passive view Picture". If you rotate year head to the right and look at b, you are in Frame S'. The Frame S components of the equation $\hat{\mathbf{x}}' = R_z(-\alpha)\hat{\mathbf{x}}$ are $(\hat{\mathbf{x}})'_i = [R_z(-\alpha)]_{ij}(\hat{\mathbf{x}})_j$.

(2) In picture c, "Frame S active", we write $\hat{\mathbf{x}}' = R_z(\alpha)\hat{\mathbf{x}}$ as we would write for any vector **V**, and vector $\hat{\mathbf{x}}'$ points **northeast**. This is the "active view Picture". Components are $(\hat{\mathbf{x}}')_i = [R_z(\alpha)]_{ij}(\hat{\mathbf{x}})_j$

All three drawings are in Frame S which we associate with our aligned piece of paper.

You obviously cannot have "both situations at once". In the first b situation $\hat{\mathbf{x}}'$ in Frame S points southeast, while in the second c picture $\hat{\mathbf{x}}'$ in Frame S points northeast.

So what do I say next? Here is one option

1. In frames doc, we want to take the passive view because: If we have a vector **V** in Frame S, we want to know: What are its **V**'s components in Frame S'?

In the above example, if we have a vector $\hat{\mathbf{x}}$ in Frame S, we get its components in Frame S' using paragraph (1), not paragraph (2). Paragraph (1) says $(\hat{\mathbf{x}})'_i = [R_z(-\alpha)]_{ij}(\hat{\mathbf{x}})_j$ which gives the back-rotated vector components and this points to the southeast.

2. Question: Adopting this frames doc passive view,
   (a) is there any role for the equation $\hat{\mathbf{x}}' = R_z(\alpha)\hat{\mathbf{x}}$?
   (b) is there any role for an equation $\mathbf{V}' = R_z(\alpha)\mathbf{V}$?

For item (b), we can certainly *define* $\mathbf{V}' = R_z(\alpha)\mathbf{V}$ as some new vector $\mathbf{V}'$ in Frame S.
But we cannot *define* $\hat{\mathbf{x}}' = R_z(\alpha)\hat{\mathbf{x}}$ because $\hat{\mathbf{x}}'$ is already defined as $\hat{\mathbf{x}}' = R_z(-\alpha)\hat{\mathbf{x}}$.

We might define $\mathbf{V'} = R_{\mathbf{z}}(\alpha)\mathbf{V}$ for some general vector **V**, but we cannot apply this to the position vector,. We cannot say $\mathbf{r'} = R_{\mathbf{z}}(\alpha)\mathbf{r}$ because for $\mathbf{r} = \hat{\mathbf{x}}$ we get the wrong answer.