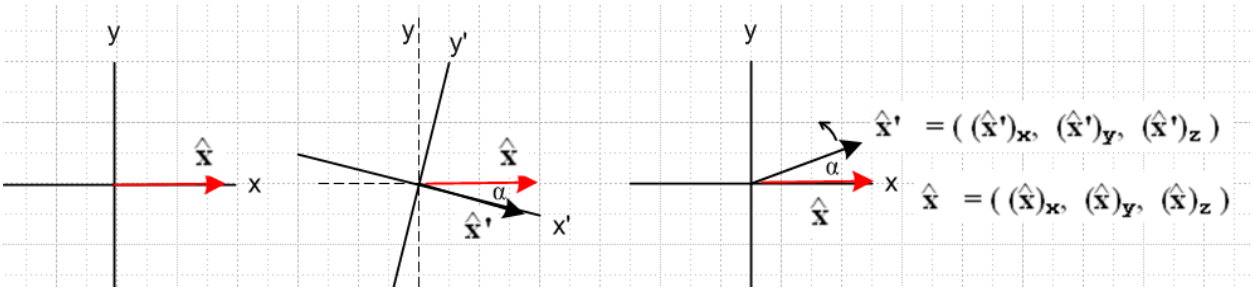
Paradox 1 of 2_23_17 v3. PhL 2.23.17

Statement of Paradox and Resolution are Below

Consider these pictures



a Frame S b Frame S passive c Frame S active

(1) In picture b, "Frame S passive", the vector $\hat{\mathbf{x}}'$ points **southeast** and we think $\mathbf{e}'_i = R_z(-\alpha)\mathbf{e}_i$ and we say that the basis vectors are "back-rotated". In particular, $\hat{\mathbf{x}}' = R_z(-\alpha)\hat{\mathbf{x}}$ **.** This is the "passive view Picture". If you rotate year head to the right and look at b, you are in Frame S'. The Frame S components of the equation $\hat{\mathbf{x}}' = R_z(-\alpha)\hat{\mathbf{x}}$ are $(\hat{\mathbf{x}}')_i = [R_z(-\alpha)]_{ij}(\hat{\mathbf{x}})_j$. Inverting, $(\hat{\mathbf{x}})_i = [R_z(\alpha)]_{ij}(\hat{\mathbf{x}}')_j$

(2) In picture c, "Frame S active", we write $\hat{\mathbf{x}}' = R_z(\alpha)\hat{\mathbf{x}}$ as we would write for any vector **V**, and vector $\hat{\mathbf{x}}'$ points **northeast**. This is the "active view Picture". Components are $(\hat{\mathbf{x}}')_i = [R_z(\alpha)]_{ij}(\hat{\mathbf{x}})_j$

All three drawings are in Frame S which we associate with our aligned piece of paper.

You obviously cannot have "both situations at once". In the first b situation $\hat{\mathbf{x}}'$ in Frame S points southeast, while in the second c picture $\hat{\mathbf{x}}'$ in Frame S points northeast.

Realization A: When I write equations like $\mathbf{V}' = R\mathbf{V}$, I have in mind that **V** is some vector associated with an Apparatus. It could be the position of a point in the Apparatus, so I would then write $\mathbf{r}' = R\mathbf{r}$. It could be a velocity or acceleration. so $\mathbf{v}' = R\mathbf{v}$ or $\mathbf{a}' = R\mathbf{a}$ for $\mathbf{b}= 0$ I think.

In Experiment A (or whatever), Apparatus stays fixed and the basis vectors back-rotate. Here is the key idea:

Fact: The vector $\hat{\mathbf{x}}$ is a vector, yes, but it is not a point in the Apparatus, it is related to the coordinate system of Frame S. As you back rotate such that $\hat{\mathbf{x}}' = R_z(-\alpha)\hat{\mathbf{x}}$, a vector **r** describing a point in your Apparatus does not change. It stays put in Frame S. It neither forward nor back rotates.

Suppose in Frame S you have $\mathbf{r} = \hat{\mathbf{x}}$ describing a particular point in your Apparatus. In Frame S' this apparatus point appears at $\mathbf{r}' = R_z(\alpha)\,\mathbf{r}$ and you then have $\mathbf{r}' = \cos\alpha\,\hat{\mathbf{x}}' + \sin\alpha\,\hat{\mathbf{y}}'$. This goes with the red arrow in b above.

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Statement of Paradox

Let's try to state and then resolve our **Paradox** which is this:

1. On the one hand, I say that basis vectors are back-rotated so $\mathbf{e}'_n = R^{-1}\mathbf{e}_n$. ($\hat{\mathbf{x}}' = R^{-1}\hat{\mathbf{x}}$)

In Frame S components this says $(\hat{\mathbf{x}}')_i = (R^{-1})_{ij}(\hat{\mathbf{x}})_j$.

2. On the other hand, the vector $\hat{\mathbf{x}}$ seems like a normal vector so it should actively rotate $\hat{\mathbf{x}}' = R\hat{\mathbf{x}}$.

In Frame S components, $(\hat{\mathbf{x}}')_i = R_{ij}(\hat{\mathbf{x}})_j$.

3. How can these both be true?

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Resolution Attempt 1: When you talk about "normal vectors" and their rule $\mathbf{V}' = R\mathbf{V}$, you are talking about a vector which is "part of" your Apparatus under study. In Experiment A where the Apparatus stays fixed and the basis vectors are back-rotated, the vector $\mathbf{V}$ *stays put* in Frame S and its components in Frame S' is what you mean by the equation $\mathbf{V}' = R\mathbf{V}$. If $\mathbf{r}$ is a point on the Apparatus, then $\mathbf{r}$ would stay put in this experiment. In Frame S' you would have $\mathbf{r}' = R\mathbf{r}$ as the new coordinates of that point in Frame S. So the rule $\mathbf{V}' = R\mathbf{V}$ applies for such a point, and you have $\mathbf{r}' = R\mathbf{r}$. Suppose a point in the Apparatus is located at $\mathbf{r} = \hat{\mathbf{x}}$. In Frame S' this same apparatus point is located at $\mathbf{r}' = R\hat{\mathbf{x}}$. Yes, $\hat{\mathbf{x}}' = R^{-1}\hat{\mathbf{x}}$. So you conclude that $\mathbf{r}' \neq \hat{\mathbf{x}}'$.

Now examine the sentences in the Paradox. Item 1 is correct, nothing wrong, $\hat{\mathbf{x}}' = R^{-1}\hat{\mathbf{x}}$.

Item 2 says "the vector $\hat{\mathbf{x}}$ seems like a normal vector so it should actively rotate $\hat{\mathbf{x}}' = R\hat{\mathbf{x}}$ ".

This is where mud is thrown onto the windshield. The phrase "normal vector" is not defined anywhere, first of all. Fuzzy thinking. What I mean by "normal vector" is a vector what is somehow "part of" the Apparatus. In a sense a normal vector is one which is "glued to" the Apparatus. This statement works for points of the Apparatus. The velocity of a point in the apparatus is "with the Apparatus".

So my first problem is how to describe what I mean by an "apparatus vector". It is any vector that is "associated with" the apparatus. The coordinate system is a second entity, it is not associated with the apparatus. So

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Here is the Paradox Resolution

1. **Apparatus Vectors** are associated with an Apparatus being observed. Examples include points **r** in the Apparatus, or velocities **v** of such points, or their accelerations **a** or angular momenta **L** or electric field **E**

at some point. If the Apparatus is forward rotated in space by R, all these apparatus vectors actively rotate according to the rule $\mathbf{V}' = R\mathbf{V}$.

2. **Basis Vectors** are associated with a coordinate system or a Frame of reference of an Observer. These might be written $\mathbf{e}_i$ or $\hat{\mathbf{x}}$ for Frame S, and other names for other Frames. If the Frame is backward rotated in space by R^{-1}, all these apparatus vectors actively rotate according to the rule $\mathbf{e}'_i = R^{-1}\mathbf{e}_i$.

3. In the **Passive View Experiment**, an Apparatus stays fixed in Frame S, and the Basis Vectors are back-rotated $\mathbf{e}'_i = R^{-1}\mathbf{e}_i$. The new coordinates of an apparatus vector are given by $\mathbf{V}' = R\mathbf{V}$.

4. In the **Active View Experiment**, an Apparatus is rotated by R, and the Basis Vectors $\mathbf{e}_i$ stay put. The new coordinates of an apparatus vector are given by $\mathbf{V}' = R\mathbf{V}$.

Fact: No vector can be both a Basis Vector and an Apparatus Vector. These are mutually exclusive classes of vectors.

It might be that an apparatus point in Frame S is located at $\mathbf{r} = \hat{\mathbf{x}}$.

In the Passive View Experiment, that apparatus vector $\mathbf{r}$ stays put. while the basis vector $\hat{\mathbf{x}}$ is back-rotated into $\hat{\mathbf{x}}' = R^{-1}\hat{\mathbf{x}}$. The coordinates of the apparatus vector $\mathbf{r}$ in Frame S' are given by $\mathbf{r}' = R\mathbf{r} = R\hat{\mathbf{x}}$. There is no equation anywhere which says $\hat{\mathbf{x}}' = R\hat{\mathbf{x}}$ because $\hat{\mathbf{x}}$ is not an Apparatus vector. We end up with $\mathbf{r}' = R\hat{\mathbf{x}} \neq \hat{\mathbf{x}}'$.

In the Active View Experiment, that apparatus vector $\mathbf{r}$ rotates into $\mathbf{r}' = R\mathbf{r}$ which is some new location in Frame S. The Basis vectors like $\hat{\mathbf{x}}$ stay put. There are no vectors named $\hat{\mathbf{x}}'$ and consequently there are no equations like $\hat{\mathbf{x}}' = R^{-1}\hat{\mathbf{x}}$ or $\hat{\mathbf{x}}' = R\hat{\mathbf{x}}$.

We now return to that statement 2:

Item 2 says "the vector $\hat{\mathbf{x}}$ seems like a normal vector so it should actively rotate $\hat{\mathbf{x}}' = R\hat{\mathbf{x}}$ ".

We now see that "normal vector" means "Apparatus Vector" and in fact $\hat{\mathbf{x}}$ is NOT in this class of vectors so the conclusion $\hat{\mathbf{x}}' = R\hat{\mathbf{x}}$ is incorrect.

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OK, I have written a section in Section 1.2 on all this stuff.

The next issue is this: What to do about Fig 1 equations like $\mathbf{r} = \mathbf{b} + \mathbf{r}'$ where you don't have $\mathbf{r}' = R\mathbf{r}$. What kind of "vectors" are these guys? Are these a third class of vectors?

Maybe I have to generalize my Active/Passive discussion so it applies more to Fig 1, and not just to Special Case #4. This is very confusing. I will try, while looking at the Fig.

In Fig 1 "where is the Apparatus? " It is not shown. We have Observers in the two Frames and each of them sees things.

Suppose I glue an Apparatus into Frame S. It has Apparatus Vectors, one of which is r to some point in the Apparatus. I really only care about the Passive View. The Apparatus stays put. WE have two things going on here, rotation and translation. I never made that very clear in frames doc.

Suppose **V** is an apparatus vector in Frame S. If Frame S' had the same origin, we could say that for any apparatus vector, we have **V**' = R**V** (passive) as seen in Frame S'. But this does not apply for **V** = **r**,**v** or a in Fig 1