Paradox 1 of 2_23_17. PhL 2.23.17

1. On the one hand, I say that basis vectors are back-rotated so $\mathbf{e'}_n = R^{-1}\mathbf{e}_n$. ($\hat{\mathbf{x}}' = R^{-1}\hat{\mathbf{x}}$)

In Frame S components this says $(\hat{\mathbf{x}}')_i = (R^{-1})_{ij}(\hat{\mathbf{x}})_j$.

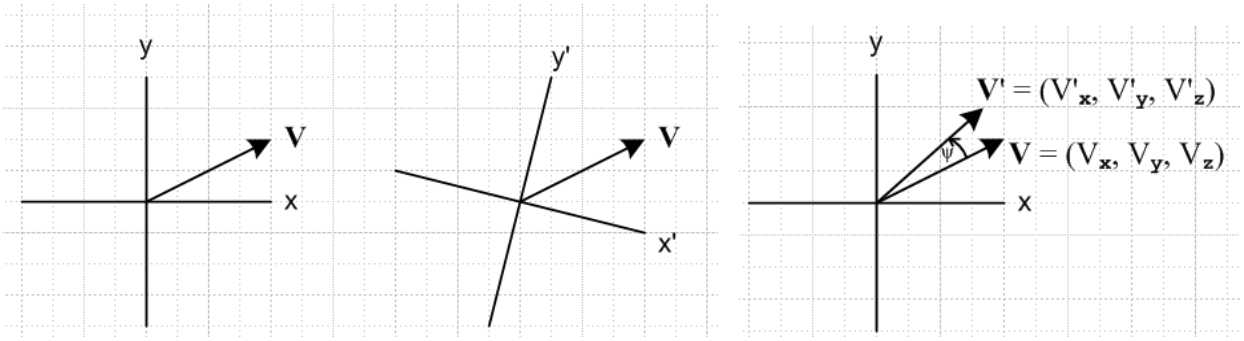
2. On the other hand, the vector $\hat{\mathbf{x}}$ seems like a normal vector so you should actively rotate $\hat{\mathbf{x}}' = R\hat{\mathbf{x}}$.

In Frame S components, $(\hat{\mathbf{x}}')_i = R_{ij}(\hat{\mathbf{x}})_j$. $\hat{\mathbf{x}}$ $\hat{\mathbf{x}}'$

3. How can these both be true? My notation must not be very good if such a simple paradox can exist within its confines.

Where are my most recent active/passive pix? OK it is "Interpreting Goldstein's Euler Angle Picture" doc and here is what I say there:

My usual picture is this



a Frame S b Frame S c Frame S active

The axes in b are right-hand-rule *back-rotated* by some angle positive α around 15 degrees.
The coordinates (components) of vector **V** are different in Frame S versus Frame S' as you can see.
In the **passive** view, I can get the Frame S' coordinates from the Frame S ones by this rule: **V'** = R**V**. I would write in my frames doc notation,

$(V)'_i = R_{ij}(\alpha)\,(V)_i$ (1) $\mathbf{V}' = R_z(\alpha)\mathbf{V}$ and $\mathbf{e}'_i = R_z(-\alpha)\mathbf{e}_i$ in Frame S

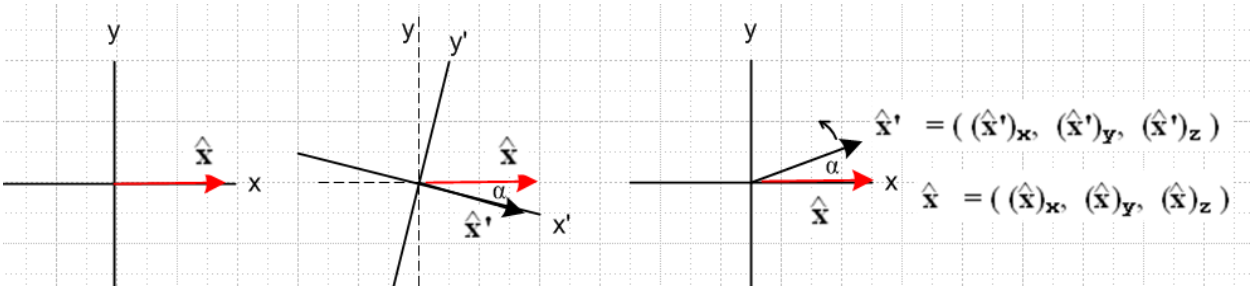
In the **active** view all within Frame S I would again write **V'** = R**V** but I would express this as

$(V')_i = R_{ij}(\alpha)\,(V)_i$ (2) $R = R_z(\alpha)$

In frames doc, it is item (1) that you normally want to do: express Frame S' components in terms of Frame S components, and maybe vice versa. You think "passive".

Reading it now, it all seems very reasonable for the generic vector **V**.

But what happens if I set **V** = $\hat{\mathbf{x}}$??? Let's draw the exact pictures corresponding to those above:



a Frame S b Frame S passive c Frame S active

Now look at item (1) above:

$(\hat{\mathbf{x}})'_i = R_{ij}(\alpha)\,(\hat{\mathbf{x}})_i$ (1)

ok

Frame S active	Frame S passive
$\hat{\mathbf{x}}' = R_z(\alpha)\hat{\mathbf{x}}$	and $\mathbf{e}'_i = R_z(-\alpha)\mathbf{e}_i$ in Frame S
	$\hat{\mathbf{x}}' = R_z(-\alpha)\,\hat{\mathbf{x}}$ in Frame S
	ok

In picture b, "Frame S passive", the vector $\hat{\mathbf{x}}'$ points **southeast** and we think $\mathbf{e}'_i = R_z(-\alpha)\mathbf{e}_i$ and we say that the basis vectors are "back-rotated". We have $(\hat{\mathbf{x}})'_i = R_{ij}(\alpha)\,(\hat{\mathbf{x}})_i$ as the Frame S' components of the vector $\hat{\mathbf{x}}$ **.** This is the "passive view Picture". If you rotate year head and look at b, you are in Frame S'.

In picture c, "Frame S active", the vector $\hat{\mathbf{x}}'$ points **northeast**. This is the "active view Picture".

Both these drawings are "in Frame S".

You obviously cannot have "both situations at once". In the first b situation $\hat{\mathbf{x}}'$ in Frame S points southeast, while in the second c picture $\hat{\mathbf{x}}'$ in Frame S points northeast.

It seems to be an either/or situation, but both situations are in Frame S. The paper is Frame S.

Look again at item (1) above,

The passive problem solved		Frame S active		Frame S passive
$(\hat{\mathbf{x}})'_i = R_{ij}(\alpha)\,(\hat{\mathbf{x}})_i$	(1)	$\hat{\mathbf{x}}' = R_z(\alpha)\hat{\mathbf{x}}$	or	$\mathbf{e}'_i = R_z(-\alpha)\mathbf{e}_i$ in Frame S

The equation $(\hat{\mathbf{x}})'_i = R_{ij}(\alpha)\,(\hat{\mathbf{x}})_i$ is the solution to the canonical problem Frames doc is addressing: "What are the coordinates of $\hat{\mathbf{x}}$ as observed from Frame S' ?" The statement $\hat{\mathbf{x}}' = R_z(\alpha)\hat{\mathbf{x}}$ is simply not true in this passive picture.

So how can I write $\boldsymbol{\omega}' = R\boldsymbol{\omega}$ or in general how can I be writing $\mathbf{a}' = R\mathbf{a}$ in Section 1? These are inherently active Picture equations. In the passive picture the vectors $\boldsymbol{\omega}'$ and $\mathbf{a}'$ don't even exist.

So what happens to all of tensor doc where I always write $\mathbf{V}' = R\mathbf{V}$? I say that $\mathbf{V}$ is a vector in x-space and then $\mathbf{V}'$ is the corresponding vector in x'-space. I am getting worried now.

$$V'^{i}(\mathbf{x}') = R^{i}{}_{j} V^{i}(\mathbf{x})$$

In tensor doc I expand vector $\mathbf{V}$ on two different bases:

$$\mathbf{V} = V_1\,\mathbf{u}_1 + V_2\,\mathbf{u}_2 + \ldots = \Sigma_n V_n\,\mathbf{u}_n \quad \text{where} \quad \mathbf{U}_n \bullet \mathbf{V} = V_n \quad \mathbf{U}_n = g_{ni}\,\mathbf{u}_i$$

$$\mathbf{V} = V'_1\,\mathbf{e}_1 + V'_2\,\mathbf{e}_2 + \ldots = \Sigma_n V'_n\,\mathbf{e}_n \quad \text{where} \quad \mathbf{E}_n \bullet \mathbf{V} = V'_n \quad \mathbf{E}_n = g'_{ni}\,\mathbf{e}_i$$

This seems to be a Passive Picture statement. If I say the $\mathbf{u}_i$ define Frame S and the $\mathbf{e}_i$ define Frame S', then the V'_i are the components of vector $\mathbf{V}$ in Frame S' and that is the canonical question solved. Note that both Frame S and Frame S' then exist within x-space.

Am I confusing "coordinates" (like $\mathbf{x}$ and $\mathbf{x}'$) with "Frames" (like S and S') ?

How does one reconcile the notion of Active and Passive Pictures with tensor doc?

I better stay in frames doc for now, don't want to metastasize my paradox into a huge unmanageable mess which is spinning out of control . I will come back *later* to tensor doc. Let's get stable first in the frames doc world. And right now things are not stable there.

I suspect there are big problems in Section 1.2 or 1.3 where I write $\mathbf{a}' = R\mathbf{a}$ which are Active Picture statements. I have not even discussed Pictures there. I am not ready to even think about Euler angles because basic Section 1 stuff is very unstable.

So *once again*, I have to go back to Section 1 and try to get it stabilized. All else drops by the wayside.

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$\hat{\mathbf{x}} = ((\hat{\mathbf{x}})_{\mathbf{x}}, (\hat{\mathbf{x}})_{\mathbf{y}}, (\hat{\mathbf{x}})_{\mathbf{z}})$

$\hat{\mathbf{x}}' = ((\hat{\mathbf{x}}')_{\mathbf{x}}, (\hat{\mathbf{x}}')_{\mathbf{y}}, (\hat{\mathbf{x}}')_{\mathbf{z}})$