New Frames doc edit log 2016

This edit log only goes through 12/10/16. Last item below was to change eq num format globally.

The original frames doc was published in 2012, see its edit log. This log starts in 2016 to track a major overhaul did mainly in Nov-Dec of 2016, but some material earlier in 2016. One very major change was a complete renumbering of all sections and equations to conform to my newer system where it is easier to find things. Figures and equations are in the same system, sections are all like N.M and so on.

Update 3/5/16.

Some guy from Poland put in a Maple request on this doc, so thought this would be a good time to do a review. I guess this "proofing frames" doc is what I now call an edit log.doc, so I will put notes here. I am reading the PDF and will add errata as they are encountered to the errata file. // Took a lot of hours.

Review

Interesting that the opening of Ch1 deals exactly with my issue in wedge doc. In the review below I am NOT checking all equations and equation refs. It is just a review to see what is going on in this doc, and whether I should release it to Researchgate.

Overview OK

Chapter 1 p 7

(a) had doubts about 1.2 right side, but it is correct.
(b) OK
(c) OK
(d) OK, meaning of two vectors being equal
(e) OK, so far I like my doc
(f) OK
(g) OK, I am still liking it
(h) OK
(i) OK
(j) OK

This is a pretty complicated collection of "facts", but we see the 8 accelerations.

Chapter 2 p 22

only one section, subject is "the G rule: $\partial_S \mathbf{a} = \partial_{S'} \mathbf{a} + \boldsymbol{\omega} \times \mathbf{a}$
I wonder if my presentation is the wordiest in the world?

Chapter 3 p 26

only one section, talks about Observer in Frame S' measuring various things he can measure.

Chapter 4 p 28

(a) OK, description of Fig 1.4 in some detail
(b) OK, Fig 4.1 is viewed from a different eye position
(c) seems OK, about vector **b**
(d) OK, special case where rotation axis contains Frame S origin
(e) OK, this time ω axis passes through Frame S' origin. I am liking it all so far.
(f) OK, I love it: picture of turntable and of the Rotor I rode with Karen in days of olde.
(g) OK, the earth and turntable are both Special Case #1 of earlier.
(h) OK, the flying camera platform as inverse problem

Chapter 5 p 35

very short, just states the general problem we are trying to solve.

Chapter 6 p 36

opening text, OK
(a) through (e), OK, just stating lots of equations
(f) Comment 1: be careful with prime location!
2: $\mathbf{r}' \neq \mathrm{R}\mathbf{r}$ $\mathbf{v}' \neq \mathrm{R}\mathbf{v}$ since non inertial frames.
3: **b** does not need concern about prime location

Chapter 7 p 39

(a) OK, relation between $\mathbf{a}_{S}$ and **a**' and the "Coriolis factor of 2"

$$\mathbf{a}'_S = \mathbf{a}' + \dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') \quad . \tag{7.4}$$

(b) OK, relates **a** to **a**'

$$\underset{S}{\mathbf{a}} = \underset{S'}{\mathbf{a}'} + \underset{\text{Euler}}{\dot{\boldsymbol{\omega}} \times \mathbf{r}'} + \underset{\text{Coriolis}}{2\,\boldsymbol{\omega} \times \mathbf{v}'} + \underset{\text{centripetal}}{\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}')} + \underset{\text{frame}}{\ddot{\mathbf{b}}_S} \tag{7.6a}$$

and a longer version is stated as well.

(c) OK. another relation of a's
(f) OK, summary of the previous 3 sections

(d) Acceleration Summary

$$\mathbf{a} = \mathbf{a}' + \dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') + \ddot{\mathbf{b}}_S \tag{7.6a}$$

$$\mathbf{a}'_S = \mathbf{a}' + \dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') \tag{7.4}$$

$$\mathbf{a}_{S'} = \mathbf{a}' + \ddot{\mathbf{b}}_{S'} \tag{7.7}$$

(e) OK, similar results for vector b

$$\ddot{\mathbf{b}}_S = \ddot{\mathbf{b}}_{S'} + \dot{\boldsymbol{\omega}} \times \mathbf{b} + 2\boldsymbol{\omega} \times \dot{\mathbf{b}}_{S'} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{b}) \ . \qquad (7.11)$$

Maybe this Chapter 7 is the one I called "dry" above. Yes, both Ch 6 and Ch 7 are dry as dust. Chapter are called Sections in this paper, by the way.

Note Added. Calculation regarding $\dot{\omega}$ on page 51 (now 52) bottom is wrong. Also I give no reference for the seasonal variation in ω for the earth. I found some day here

https://books.google.com/books?id=_6My7JR6dRgC&pg=PA133&lpg=PA133&dq=earth+rotation+%22seasonal+variations%22&source=bl&ots=NJhhvbNrkm&sig=bUMTZjRr3_LW5MJcqgmHzKrTo3Y&hl=en&sa=X&ved=0ahUKEwjqwI6rgqrLAhXktYMKHebrAl4Q6AEIVTAJ#v=onepage&q=seasonal&f=false

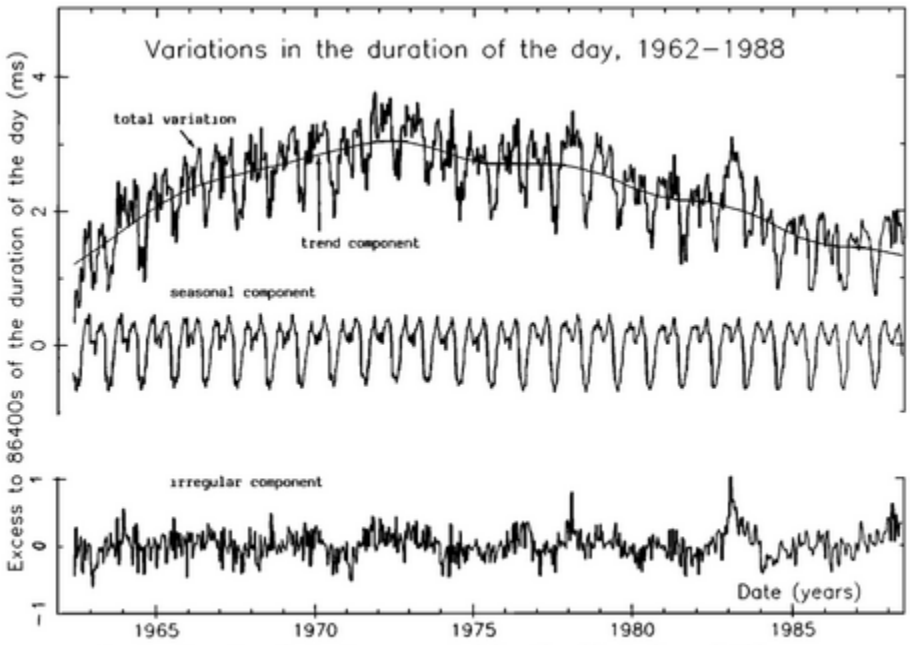


Fig. 1. Duration of the day, corrected for the effect of zonal tides. Total variation and decomposition in trend, seasonal and irregular components.

and the regular wave is the "seasonal component" which is indeed about 1 msec/day peak to peak and maybe then 0.5 msec RMS, so I am in the right ballpark with my claim.

Here is the repaired calculation:

$dT/T_{day} \sim 0.5$ msec$/T_{day}$

$\omega = 2\pi/T \qquad \dot{\omega} = d\omega/dt = -2\pi T^{-2} dT/dt = 2\pi(dT/dt)/T^2 \ (sec^{-1})$

Then $\dot{\omega} = 2\pi * (0.5 \text{ msec}/T_{day}) * (1/T_{day})^2 = \pi\ (1 \text{ msec})/T_{day}^3 = \text{sec}^{-2}$

$= \pi\ (.001)/(86400)^3 \sim 3 \times 10^{-3} / [10^5]^3 = 3 \times 10^{-18}$\

```
day := 24*60*60;
```

$$day := 86400$$

```
evalf(Pi* 0.001 / day^3);
```

$$.4870893628\ 10^{-17}$$

So the correct result is $\dot{\omega} \sim 10^{-17}$ sec^{-2} and not the 10^{-8} sec^{-2} which I quote. Another error is that it is the long vector r that is involved, not the short vector r'. Here then is my replacement text:

Let T = 24*60*60 = 86400 ~ 10^5 sec be the nominal period of a day. The day has a seasonal variation in its duration of roughly (see e.g. https://www.iers.org/IERS/EN/Science/EarthRotation/LODplot.html)

$dT/dt \sim 0.5$ msec/day ~ 10^{-8} // rms

with smaller short term variations. From this we compute a rough value for $\dot{\omega}$,

$|\dot{\omega}| = |d(2\pi/T)/dt| = 2\pi T^{-2}\ (dT/dt) \sim 6 * 10^{-10} * 10^{-8} \sim 10^{-17}$ sec^{-2} .

For activities on the surface of the earth, $r \approx R_E \sim 10^7$ m so

$\dot{\omega} \times r \sim 10^{-17} * 10^7 \sim 10^{-10} \sim 10^{-9}$ g

Thus in (8.12) we neglect the Euler term to get

$$\mathbf{F'}_{eff} = (m\mathbf{g}_0 + \text{possible other real forces}) - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - 2m\ \boldsymbol{\omega} \times \mathbf{v}' \quad . \qquad (8.13)$$

Comment: As shown later in Section 8 (g), the tidal force is ~ 10^{-7} g.

Bug? In Section 8 g I claim that Fig 8.11 is Special Case #2 with ω = 0 and two sets of axes always aligned. I don't understand that. Well due to the gimbals, frame S' is fixed wrt the stars in orientation. In

figure 4.5 I show rotation axis ω passing through the frame S' origin. I guess i am allowed to put that axis anywhere. In the earth case, then yes, ω = 0 since Frame S' is not rotating. The main thing is Ω .

This is a long chapter with lots of meat in it

Chapter 8 p 43

(a) OK, find expression for fictitious forces F_{fict}, do special case (not good for Earth)
(b) OK, I "interpret" the various fictitious forces, this is a difficult section to read today
(c)
OK: Arm-waving Coriolis interpretation, not bad.
OK: superposition of Coriolis and Centrifugal force. Only the first "drifts right".
if you ignore centrifugal, Coriolis would give circular motion!
(d) OK Special Case #1 simplify, then applies to earth surface problems.
(e) OK (found numerical error, but it does not affect conclusions, fix in errata doc and above)
(f) OK, tension in a tethered satellite pair, is tidal force etc
(g)
OK to end of digression on page 61
OK to end of page 64, nice picture and discussion of tides IMHO
OK to end of section (g) on page 68

Chapter 9 p 69

OK, compares notation to Marion's books and Taylor.

Chapter 10 p 71 comparing to Goldstein and GPS

(a) OK
(b) OK
(c) OK on a hidden Gold approximation.

Chapter 11 p 74

(a) OK, force torque and angular momentum stuff
(b) OK, shows L difference when referenced to points c and c'.
(c) OK, analogous discussion of "fictitious torques" in a rotating frame situation.
(d) OK, very fancy integral versions of things. (fluids etc)
(e) OK, fluid integral version of fictitious forces
all stuff verified in Lai equations thank goodness!
(f) OK, I give derivations of various Reynold's transport stuff.

Chapter 12 Forward Problem p 86

(a) OK, summarize "forward problem" equations in "non-swap notation"
(b) OK, summarize "forward problem" equations in "swap notation"

Chapter 13 Inverse Problem p 90

(a) OK, summarize "inverse problem" equations in "non-swap notation" : brute force
(b) OK, summarize "inverse problem" equations in "non-swap notation" : Swap Rules Method

(c) OK, summarize "inverse problem" equations in "non-swap notation" , more stuff than (a)
(d) OK, summarize "inverse problem" equations in "swap notation"
(e) OK, why the Swap Rules Method works

Chapter 14 Curvilinear versions of S and S' p 98

OK, we provide curvilinear coordinates ξ_i and ξ'_i for frames S and S', so four systems.

You might have ξ_i = sphericals and ξ'_i = toroidals

I then show how a key equations appears in all four systems. Fancy stuff!

A good topic for me to address.

Chapter 15 Ant on Turntable Problems p 101

(a) OK, basis ant kinematics in S and S' frames, details about various basis vectors,
This section has all setup work needed for the three ant problems.
(b) OK, ant crawls to origin of S'
(c) OK, ant spirals while crawling to origin in S'
(d) OK, the and flyover problem, but plots don't agree with Marion's numbers
(e) OK, the 4-projectiles problem with more plots

Appendix A: Rotation matrices p 134

OK

Appendix B: tensor doc notation comparison p 138

OK

Appendix C: G Rule for tensors p 141

OK, I like it, I hope it is right!

Appendix D: Foucault p 147

OK, opening material finds the small angle assumption solution
(a) setting up the spherical coordinate systems (kinematics)
(b) OK on qualitative solution.
(c) OK on three scalar equations to describe the spherical pendulum
(d) the full problem for a non-rotating earth
(e) the true Foucault Pendulum on the rotating earth.

References.

OK

So I have reread this entire paper today. I had forgotten how chock full of details and examples it is. Only 160 pages, one of my shorter docs. No one has ever emailed me until today, so that is why I took a pass and collected errata for a new release before Researchgate.

The guy from Poland asks for my Maple code, so I will next want to create a Maple index.

Update 11.17.16

I am reading this paper again as part of my Goldstein Ch 4 review. I think it is a good paper, and lots of errata are piling up, so I will probably do a cycle, but for now I am just reading

Overview	OK
Summary	skipped
Chapter 1	OK
Chapter 2	OK

Update 11.18.16

Continuing my reading (no equation checks on this pass).

Chapter 3 very short and clear, shows how rotating S' frame observer measures things.

Chapter 4

(a) OK detailed explanation of fig 4.1
(b) OK explanation of fig 4.2 where green circle is in plane of paper
(c) OK, seems mostly preparatory for later
(d) Special Case #1: rotation axis through Frame S origin, OK
(e) Special Case #2: rotation axis through Frame S' origin, OK
(f) turntable kinematics, excellent and OK (special case #1 example)
(g) the earth, another Special Case 1 example, just fine. OK
(h) the camera platform as Special Case 2 example

Chapter 5

OK, one paragraph stating our Problem.

Chapter 6 Determination of velocities

opening text OK
(a) OK on first of the four velocities.
(b) OK on the second
(c) OK on third // I don't do the 4th velocity for some reason
(d) velocity summary OK
(e) velocities for the special cases
(f) comments
1. OK
2. OK
3. OK

Chapter 7 Determination of accelerations

(a) OK, getting one of the 8 accelerations
(b) OK, lots of tedious algebra!!! Getting a second acceleration
(c) OK, getting a third acceleration
(d) OK, summary of these three accelerations
(e) OK, getting results for b double dot

Chapter 8. Fictitious Forces

(a) OK, development of these fictitious forces
(b) OK, a tough section, interpret Euler and Cor forces
(c) OK, another tough section. Maybe summaries at start will help.
(d) OK, fict forces in Special Case 1
(e) surface of the earth problems, g_0 vs g, seems OK
(f) tethered satellites discussion, OK
(g) tides on the earth a very LONG section, it does get support from other books OK

Pause for doing some of the Summary

1 - 10: OK

Chapter 9. Compare to Marion and Taylor OK

Chapter 10: Compare to Goldstein Very painful but probably OK.

Chapter 11

(a) introduction, seems OK, I talk about a bar and two chucks
(b) lots of messy algebra and a typo which is now on the errata list, OK
(c) if have no verification that what I say here is valid, but will call it OK
(d) fictitious torques in the Lai world, very messy so good to add, guess OK
(e) fictitious forces in the Lai world, and a translation table OK
(f) another Lai contact section, Reynolds Transport. Just trying to "tie in" to Lai. OK
end of Chapter 11

Comment: This is a brutal monograph! Hard to imagine anyone could read it through. It is a collection of what I have learned about these subjects. The subject is intrinsically difficult, I just don't think there is a Simple Simon way to do all this stuff. I have lots of examples,

Chapter 12. Forward summary OK

Chapter 13: The inverse problem and Swap Rules OK

Chapter 14: Curvilinear coordinates. OK, at least I face up to this question. And
I give a reasonably systematic answer with some examples.

Near (14.10), why are some of the ω symbols italicized, seems wrong and confusing? But ω is a vector like any other, and it has Cartesian components $(\omega)_i$ and curvilinear ones $(\omega)_i$ just the way I write things for the vector V, so I think this is all correct.

The ant problems are amazingly complicated for being such simple problems. The point is to exercise my general formulae given in Sections 12 and 13 which would apply to ANY problem.

Chapter 15. This is ALL the ant problems. I think they are all very good, I hope there are not too many algebra errors. Someone ELSE really has to check this stuff. So Ch 15 OK.

Appendices. Just reading through them. I really wonder if the G rule for tensors is backed up by any other author?

OK at 7:45 I finally finished this very long review. I think I will hold off on doing a Frames update until I am further along in my Goldstein review. The main thing is the G rule.

Update 11.26.16

By this time I have reviewed active/passive/covariance and A,B,C and Frames and all that stuff, and I am ready to start into updating frames doc.

I just rewrote Appendix C from scratch, it makes no mention of tensor doc or Appendix B, and I think it is very much better and clearer than the old Appendix C. I have installed the new one into frames v1 doc.

Now what to do with Appendix B? Should I just get rid of it? I really see no reason to keep it.

(1) I am trying to reduce connections to tensor doc which is not really "passive" oriented
(2) nobody cares about A/B notation.

OK, out it goes. I always have old copies in the earlier releases. So

Step 1: Delete the appendix B from v1 frames doc. DONE

Step 2: Rename all App C to be App B. DONE

Step 3: Rename all App D stuff to be App C. DONE

Step 4: Adjust the heading names for the new App B and App C.

I will leave tensor analysis in as a reference even though I might not refer to it any more.

Update 11.27.16

I went through a very long list of errata and made the fixes in v1. This included a long pass to add periods after equations that need periods (maybe 200). There are some errata left still that require more work and I will go after them next. I gathered them all together in the errata list at the bottom.

As of 3 PM I have worked through some of the more moderate errata, but some big ones are left. I think I am going to have to do another reading pass to get back into the details after my long digression on my "confusions with rotations" which lasted several days.

Another Frame Doc Proofing and Review

Overview -- good

Summary -- good, long, a few edits were done.

1(a) I just rewrote part of this section to make it clearer.

1(b) OK, did many fine edits

1(c) Much improvement, and added Active/Passive section. Did a lot of work here.

1(d) I maintain Example numbers always increasing.

1(e) very good, I think reader OK on matrix expo.

1 (f) very good, I checked the cone solution, I keep adding edits.

1(g) again, very clear

1(h) things are OK down to the start of angular momentum. And that is OK too, very good
no mysteries here.

1(i) OK, d/dt on scalars.

1(j) I better derive these 8 equations, at least on scratch. // Ouch, I have run aground regarding a claim about rotation matrices that I don't know how to prove! But then I realizes this whole section does not really do what I want. I want to show non-commute in all cases but the natural cases.

Update 11.28.16

I rewrote Section 1 (j) and it is immensely better now. I am finally done with Section 1. There is a huge amount of meat and potatoes in this section, I think it is excellent and sets the stage quite well. Added more facts to Section 1 (j) and eq nums are changed in that section. Proved the comm theorem in full.

So I now continue with my review

2. Added Marion books and Taylor and fully checked all references! I completely redid the derivation here of the G Rule, and removed some useless stuff, and it is much better.

3. Describes the observer at rest in Frame S', not much content here, a rest for the reader. All OK. This Section 3 introduces no new equation numbers.

4. This is a 7 page chapter, so I need to take is slowly, doing a close reading of everything and full eq num checks and so on.

4(a). OK, this is just a description emphasizing the generality of Figure 4.1.

4(b). OK, draws and describes Fig 4.2 where circular motion is in the plane of paper.

4(c) OK, just a few comments about motions of the b vector.

4(d) This is Special Case #1 where **b** does cone motion in Frame S.

4(e) This is special case #2, everything seems OK to me.

4(f) Excellent setup for the turntable with mention of Karen's Rotor.

4(g) I just capitalized Earth, Moon and Sun everywhere in this doc. This section is OK and clear.

4 (h) Very short, it is the camera platform as a special case 2 example.

So much for Chapter 4. I think it is ready for Prime Time.

5. Very short, states the Goal of the Next Two Sections.

6 opening	OK	Velocities
6 (a)	OK, not clear why the result is useful	
b (b)	OK, again still hazy where this is headed.	
6 (c)	OK, fixed a reference, still hazy	
6 (d)	OK summary. I have checked every detail and EN ref so far!	
6 (e)	the special cases, added new eq nums here, so we careful with later refs!	
6 (f)	cleaned up the 3rd comment, end on nice page boundary.	
7 opening	OK, cleaned up, parallel to 6	Accelerations
7(a)	OK, did every detail, it is good on showing Coriolis factor of 2	
7(b)	OK, full check on all	
7(c)	OK, very short	
7(d)	summary, gave it a new number	
7(e)	OK, new eq nums, back repair same.	
8(a)	OK, no changes , all checked	Fictitious Forces
8(b)	I don't like this section :	

The comments of Section 8 for intuition always go back to **r'** being the "long vector" on a merry go round or other. I think I veer off course in this section 8 (b).

I have rewritten Section 8 (b) in a separate file. I need to fix up words like "centrifugal" to get it cleaner, but then I will just install this replacement section into frames doc v1. I am going to dump the weird tilted cone picture which is impossible to understand and adds nothing. Figures in Section 8 are then going to get renumbered unless I can come up with a Figure to add. Enough for today at 7:30 PM. Progress was good, but many miles still to go.

Update 11.29.16

I have installed my new Section 8 (b) and I think it is much better. So I can now continue the review from yesterday with Section 8 (c)

8(c) This section needs no major fixes, I centered the pictures
arm waving section OK
comments OK
superposition idea OK

8(d) seems OK, point is to derive (8.12) for Special Case #1 problems (I guess).

8(e) Euler small for earth; g and g_0 in non-swap then swap notation. All OK

8(f) Bogged down right at the start, I don't know what "center of gravity" means for N objects each having some different gravity. This ate the rest of the day.

Update 11.30.16

At 2 PM I am done with "center of gravity" and ready to resume on Section 8 (f).

8(f) tether is done. I had to add equations here, so Section 8(g) will have to have its eq nums all altered and then I have to back-check xrefs. I have much more confidence now in the tether application of the rotating frames equations. I think the factor of 3 is correct.
I keep re-reading 8f on the tether, making small edits on each pass.

8(g) STOP.

What now does center of gravity say about the earth-moon drawing Fig 8.10? I would say that each has a certain center of gravity relative to the other (with the other treated as a point mass at its center).

8(g) RESUME, I will add a comment that R_{cog} exists but we are not using it.

The basic picture	OK
Frame S and Frame S'	OK
How the tidal model fits in	OK
One fictitious force in Frame S'	OK
Digression: The relation between r12 and Ω	OK

STOP. Sign error found below (8.31?) It causes me to disagree with Taylor, need to study.
Stopping at 8 PM, enough for today.

Update 12.6.16

I have now finished a first draft of 18-page Appendix D on the subject of the center of gravity and the tether. Not surprisingly, trying to answer questions raises more and new questions. So right now here are some of those:

1. I argue that $\mathbf{N}^{(rcog)} = 0$ and $\mathbf{N}^{(0)} = \mathbf{r}_{cog} \times \mathbf{F}$ as part of the definition of $\mathbf{r}_{cog}$. The other part is the distance rule which sets magnitude r_{cog}. Question: does this apply only in an inertial frame, or does it apply also in a rotating frame?

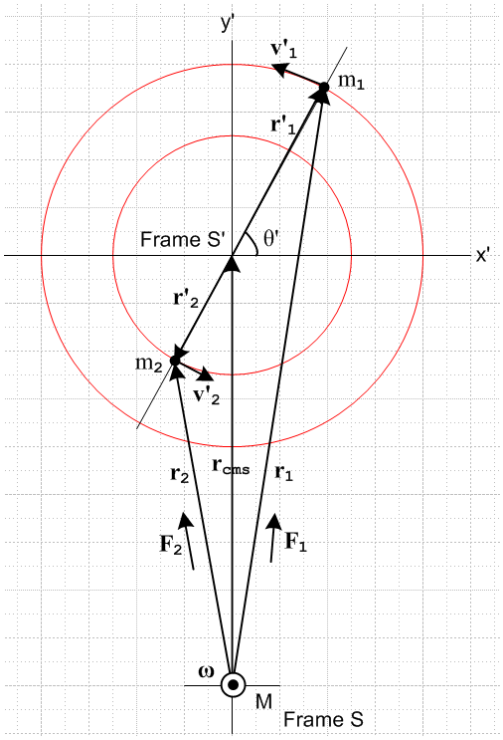
2. I compute the torque about tether center point ("origin") and I show "restoring force". What does this torque have to do with the torques of item 1 above? And what about the fact that this "origin" is not the center of gravity or of mass?

3. Does it make sense to say anything about a freeze-frame of the tether since you know it must be in motion? Am I apply statics to a case where there is motion and so statics is wrong?

I will go off in a new doc " App D questions" and try to resolve these questions and see what yet new questions arise from the study of these questions. Sisyphus lives!

Concession: It is Dec 6, I leave town Dec 20, there is no way I can get a new frames doc release done in that time and have it be good. So I hereby give up on this goal and will pursue multiple interests once again.

OK, did other things but then had a session here. I am trying to apply my completely general frames of reference equations to the tether problem even though I probably did that already in the main text. I then want to get an equation of motion for the two masses at some arbitrary angle. It now seems to me much better to put the CMS at the center of your circles picture to allow different masses. I convinced myself that the cog moves only very slightly relative to the cms and adds a motion which does not matter much in solving problems. So I drew a whole new CMS based tether picture.,



Since this whole thing is in a non-inertial frame (it orbits the earth), I have to use the full frames doc equations including fictitious forces and fictitious torques. I tried a long shot on that but made mistakes which I can try to fix tomorrow. Notice the simple gravity torques are:

$$\mathbf{N}_1^{(0)} = \mathbf{r}_1 \times \mathbf{F}_1 = 0$$
$$\mathbf{N}_2^{(0)} = \mathbf{r}_2 \times \mathbf{F}_2 = 0$$

whereas these to not vanish,

$$\mathbf{N}_1^{(0')} = \mathbf{r'}_1 \times \mathbf{F}_1$$
$$\mathbf{N}_2^{(0')} = \mathbf{r'}_2 \times \mathbf{F}_2$$

Update 12.8.16 through 12.10.16

I have decided to back up and update all eq nums to my standard format before trying to carry out an update of App D. The platform needs to be more solid than it is, with pieces sliding around. This will happen in the v2 doc I just made, in case I decide to go back.

1. Notation, important role of the Prime Symbol, and other Preliminaries

1.1 The basis vectors e_n and e'_n and two ways in which they are related a

old	new	
1.1	1.1.1	
1.2	1.1.2	
1.3	1.1.3	
1.4	1.1.4	done

1.2 Expansions of a vector and use of primes and parentheses b

old	new	
1.5	1.2.1	
1.6	1.2.2	
1.7	1.2.3	
1.8	1.2.4	done

1.3 Special case where a'_i is unambiguous c

old	new	
1.9	1.3.1	
1.10	1.3.2	
1.11	1.3.3	
1.12	1.3.4	
1.13	1.3.5	
1.14	1.3.6	done

1.4 When are Two Vectors Equal? d

Fig 1.1 1.4.1 done

1.5 The Small Rotation of a vector about an axis e

old	new	
1.15	1.5.1	
1.16	1.5.2	
1.17	1.5.3	
1.18	1.5.4	
1.19	1.5.5	
1.20	1.5.6	
1.21	1.5.7	
1.22	1.5.8	
Fig 1.2	1.5.9	
	1.5.10 new	done

1.6 The time rate of change of a rotating vector f

old	new	
1.23	1.6.1	
Fig 1.3	1.6.2	
1.24	1.6.3	done

1.7 Rate of change of the basis vectors g

old	new	
1.25	1.7.1	
1.26	1.7.2	
1.27	1.7.3	
1.28	1.7.4	done

1.8 Notations for the many time derivatives of vectors r, r', b and L h

old	
1.29	1.8.1
1.30	1.8.2
1.31	1.8.3
1.32	1.8.4
none	1.8.5
1.33	1.8.6
none	1.8.7
none	1.8.8
1.34	1.8.9
none	1.8.10

Fig 1.4	1.8.11	
none	1.8.12	
1.35	1.8.13	
none	1.8.14	
none	1.8.15	
none	1.8.16	
1.36	1.8.17	done

1.9 Notations for the many time derivatives of vectors r, r', b and L **i**

old	new	
1.37	1.9.1	done

1.10 Notations for the many time derivatives of vectors r, r', b and L

old	new	
1.38	1.10.1	
1.39	1.10.2	
1.40	1.10.3	
1.41	1.10.4	
1.42	1.10.5	done

2. The G Rule for arbitrary vector a and its derivation

No need to renumber this chapter since only one section!
But have to fix all refs. done

3. The Apparatus and its Observer at Rest in Frame S'

This Section has no numbered equations, but I fixed the refs.

4. The Relationship between the Two Frames S and S'

4.1 Explanation of Fig 4.1: Frame S in the plane of paper **a**

old	new	
Fig 4.1	(4.1.1)	done

4.2 Explanation of Fig 4.2: Vector ω pointing directly out of paper **b**

old	new	
Fig 4.2	(4.2.1)	done

4.3 Comments on $\dot{\mathbf{b}}_S$ and $\dot{\mathbf{b}}_{S'}$ **c**

old	new

(4.1) (4.3.1) done

4.4 Special Case #1 : ω axis through Frame S origin d

old	new	
Fig 4.3	4.4.1	
4.2	4.4.2	
4.3	4.4.3	
Fig 4.4	4.4.4	done

4.5 Special Case #2 : ω axis through Frame S' origin e

old	new	
Fig 4.5	4.5.1	
4.4	4.5.2	done

4.6 The Turntable f

old	new	
Fig 4.6	4.6.1	
Fig 4.7	4.6.2	
Fig 4.8	4.6.3	done

4.7 The Earth g

old	new	
Fig 4.9	(4.7.1)	done

4.8 The Flying Camera Platform
done

5. The Goal of the next two sections
done

6. Determination of velocities

For this Section I leave equation numbers as there were since so few of them.

6.1 Velocity $v_{S'}$ a
6.2 Velocity $v \equiv v_S$ b
6.3 Velocity v'_S c
6.4 Velocity Summary d
6.5 Velocities for Special Cases e
6.6 Comments f

But go through on xrefs! done

7. Determination of accelerations

Same idea, Leave eq nums as is, but replace letters with numbers in titles

7.1 Acceleration a'_S	**a**	**done**
7.2 Acceleration $a \equiv a_S$	**b**	**done**
7.3 Acceleration $a_{S'}$	**c**	**done**
7.4 Acceleration Summary	**d**	**done**
7.5 Relation between $\ddot{b}_S$ and $\ddot{b}_{S'}$	**e**	**done**

These last two sections are have heavy internal xref and I would not want to renumber them. There are only about 13 equations in each Section.

8. The Fictitious Forces

8.1 Development of the Fictitious Forces a

old	new	
none	8.1.1	
8.1	8.1.2	
8.2	8.1.3	
8.3	8.1.4	
8.4a	8.1.5	
8.4b	8.1.6	
8.5	8.1.7	
8.6	8.1.8	
8.2 app	8.1.9	
8.4a app	8.1.10	
8.4b app	8.1.11	
none	8.1.12	
8.6 app	8.1.13	done

8.2 Interpretation of the Centrifugal and Euler Fictitious Forces b

old	new	
Fig 8.1	8.2.1	
8.7	8.2.2	
8.8	8.2.3	
Fig 8.2	8.2.4	
8.9	8.2.5	
8.10	8.2.6	
8.11	8.2.7	
Fig 8.3	8.2.8	done

8.3 Interpretations of the Coriolis Fictitious Force c

Fig 8.5	8.3.1	
none	8.3.2	
Fig 8.6	8.3.3	
none	8.3.4	
none code	8.3.5	done

8.4 Special Case #1 Problems d

8.12	8.4.1	done

8.5 Problems on the surface of the Earth e

none	8.5.1	
none	8.5.2	
none	8.5.3	
none	8.5.4	
8.13	8.5.5	
8.14	8.5.6	
8.15	8.5.7	
8.16	8.5.8	
none	8.5.9	
8.13S	$8.5.5_{\mathbf{s}}$	
8.14S	$8.5.6_{\mathbf{s}}$	
8.15S	$8.5.7_{\mathbf{s}}$	
8.16S	$8.5.8_{\mathbf{s}}$	done

8.6 Tethered satellites and Tidal Forces

8.17	8.6.1	
8.18	8.6.2	
8.19	8.6.3	
8.20	8.6.4	
8.21	8.6.5	
Fig 8.7	8.6.6	
8.22	8.6.7	
8.23	none	
8.24	8.6.8	
none	8.6.9	
8.25	8.6.10	
Fig 8.8	8.6.11	
none	8.6.12	
8.26	8.6.13	
8.27	8.6.14	done

8.7 Tides on the Earth g

none	8.7.1
Fig 8.10	8.7.2
8.24	8.7.3
8.25	8.7.4
8.26	8.7.5
Fig 8.11	8.7.6
8.27	8.7.7
none	8.7.8
none	8.7.9
8.28	8.7.10
8.29	8.7.11
none	8.7.12
8.30	8.7.13
Fig 8.12	8.7.14
8.31	8.7.15
8.32	8.7.16
8.33	8.7.17
8.34	8.7.18
8.35	8.7.19
8.36	8.7.20
8.37	8.7.21
8.38	8.7.22
8.39	8.7.23
8.40	8.7.24
Fig 8.13	8.7.25
none code	8.7.26
8.41	8.7.27
8.42	8.7.28
8.43	8.7.29
Fig 8.14	8.7.30
Fig none	8.7.31
8.44	8.7.32
8.45	8.7.33
8.46	8.7.34
8.47	8.7.35
8.48	8.7.36
none	8.7.37
none	8.7.38 done!

9. Comparison with Marion (1970), Thornton & Marion (2003) and Taylor (2005)

No eq num changes here but do all xref! done

10. Comparison with Goldstein (1950) and Goldstein, Poole and Safko (2001)

No eq num changes here but do all xref! done

At this point I just pasted in replacements for sections 11.2 and 11.3 from "section 11b rewrite v1".
Now I continue:

11. Angular Momentum and Fictitious Torques; the Reynolds Transport Theorem

11.1 Introduction a

none	11.1.1	
none	11.1.2	
none	11.1.3	
Fig 11.1	11.1.4	
none	11.1.5	
none	11.1.6	
Fig 11.2	11.1.7	done

11.2 Expression of $\mathbf{L}^{(c)}$ and $\dot{\mathbf{L}}^{(c)}$ in terms of Frame S' objects b

This section was a complete rewrite in new eq nums, so I have no translation table.
Here is a partial translation table

But do the xrefs: done

11.3 Fictitious Torques and Newton's Rotational Law in a non-inertial frame c

This section was a complete rewrite in new eq nums, so I have no translation table.
But do the xrefs: done

old	new
11.2	11.2.14
11.3	11.2.14
11.4	11.2.15
11.5	11.2.16
11.6	11.3.2
11.8	11.3.5, 11.3.4
11.11	11.3.8
11.12	

11.4 Application: Fictitious Torques in Fluid Dynamics d

11.16	11.4.1
11.17	11.4.2
11.18	11.4.3

11.19	11.4.4	
11.20	11.4.5	
11.21	11.4.6	
none	11.4.7	done

11.5 Application: Fictitious Forces in Fluid Dynamics e

11.22	11.5.1	
11.23	11.5.2	
11.24	11.5.3	
11.25	11.5.4	
none	11.5.5	done

11.6 Comments on the Reynolds Transport Theorem f

none	11.6.1	
11.26	11.6.2	
11.27	11.6.3	
11.28	11.6.4	
11.29	11.6.5	
11.30	11.6.6	
none	11.6.7	
11.31	11.6.8	
11.32	11.6.9	
none	none	done

12. Summary of the Forward Problem Solution

old	new	
Fig12.1	12.1	
Fig12.2	12.1s	done

13. The Inverse Problem

13.1 Brute Force Method

13.1	13.1.1	
13.2a	13.1.2	
13.2b	13.1.3	
13.3a	13.1.4	
13.3b	13.1.5	
13.3c	13.1.6	done

13.2 Swap Rules Method

13.6	13.2.1

none 13.2.2 done

13.3 Summary of the Inverse Problem Equations (non-swap notation)

Fig 13.1 13.3.1
13.7 13.3.2

Pause: Why is my inverse problem incomplete compared to the forward problem equations?

For example, in Section 12.1 I give Forward Problem results with three distinct sections.
Definitions and Equations
Fictitious Forces (Section 8)
Fictitious Torques (Section 11)
All this stuff is repeated in Section 12.2 in swap notation.

In Section 13.1 I define the inverse problem, then in 13.2 I demonstrate the brute force method, and 13.2 I show how you get the same results from using Swap Rules.

Then in Section 13.3 I summarize the Inverse Problem equations, but I only have the first of those three sections!

Pause Pause: Stack Push! I no longer buy my argument as to why the Swap Rules which applied to a set of valid equations produces a new set of valid equations. I tried to show this graphically in Section 13, but I just don't believe my own argument. Although things end up generally the same in the final picture as in the initial picture, distances are different, things are not completely the same.

OK, I rewrote this section and I guess I am more or less convinced that the argument is OK. I installed the rewrite, and now Stack Pop.

Restabilize the Summary Sections. They have spun out of control with stale data.

Restabilize Section 12.1 : redid again, adding new eq nums!

Restabilize Section 12.2 : obtain this directly from 12.1 results
first part now OK, prior to fict forces
fict force part OK
torque OK
Done with 12.2, added new eq numbs overall.

Restabilize 13.3. OK. Now have (a) through (g)

13.3 has destabilized again. Stop answering the phone !!! 6 calls from MRL each one breaks my intense concentration in getting this fine detail right.

high noon on 12.10.16. I think the four summaries are now stable.

So let's now back up and try to continue our eq num alterations.

14. Rotating Frames in Curvilinear Coordinates

14.3 14.4
etc

Check xrefs. done

15. Ant on Turntable Problems

Kinematics common to all Ant Problems a //decided to have no number

Fig 15.1 15.1
15.1 15.2
15.2 15.3
15.3 15.4
15.4 15.5
15.2a 15.6
15.4a 15.7
15.5 15.8
15.6 15.9
15.7 15.10
15.8 15.11
15.9 15.12

15.1 Problem 1: Ant crawls at constant speed V to the Origin of Frame S' b

Fig 15.2 15.1.1
15.10 15.1.2
15.11 15.1.3
15.12 15.1.4
15.13 15.1.5
15.14 15.1.6
15.15 15.1.7
15.16 15.1.8
15.17 15.1.9
15.18 15.1.10
15.19 15.1.11
15.20 15.1.12
15.21 15.1.13
15.22 15.1.14
15.23 15.1.15
15.24 15.1.16
15.25 15.1.17
15.26 15.1.18
15.27 15.1.19
Fig 15.3 15.1.20

Fig 15.4	15.1.21
Fig 15.5	15.1.22
Fig 15.6	15.1.23

Now do all the references. done

15.2 Problem 2: Ant spirals in at constant V and Ω to the Origin of Frame S' c

15.29a	15.2.1
15.30a	15.2.2
15.31a	15.2.3
15.29b	15.2.4
15.30b	15.2.5
15.31b	15.2.6
15.32	15.2.7
15.33	15.2.8
15.34	15.2.9
15.34a	15.2.10
15.35	15.2.11
15.36	15.2.12
Fig 15.7	15.2.13
Fig 15.8	15.2.14
Fig 15.9	15.2.15

Now do refs: done

15.3 Problem 3: Inverse Problem: Ant flies in Frame S at constant velocity V d

15.37	15.3.1
15.38	15.3.2
15.39	15.3.3
15.40	15.3.4
15.41	15.3.5
15.42	15.3.6
15.43	15.3.7
15.44	15.3.8
Fig 15.10	15.3.9
Fig 15.11	15.3.10
Fig 15.12	15.3.11
Fig 15.13	15.3.12
Fig 15.14	15.3.13
Fig 15.15	15.3.14
Fig 15.16	15.3.15
Fig 15.17	15.3.16
Fig 15.18	15.3.17

refs: done

15.4 Problem 4: The Projectile Problem of Section 8.3

15.45	15.4.1
15.46	15.4.2
15.47	15.4.3
15.48	15.4.4
15.49	15.4.5
none	15.4.6
none	15.4.7
none	15.4.8
15.50	15.4.9
15.51	15.4.10
none	15.4.11
none	15.4.12
none	15.4.13

refs: done!

Appendix A: Derivation of R(ξ) for Spherical Coordinates

Making this change

A.1	A.1
none	A.2
A.2	A.3
A,n	A.n+1

Scan for refs: done

Appendix B: The G Rule for a Tensor of Rank n

Scan for refs: done

Appendix C: The Foucault Pendulum

none	C.1
none	C.2

C.1 Drawings, Notation, and Coordinates a

Fig C.1	C.1.1
C.1	C.1.2
Fig C.2	C.1.3
C.2	C.1.4

refs: done

C.2 Qualitative Solution b

Fig C.3 C.2.1

scan ref: done

C.3 The Equations of Motion for a Foucault Pendulum c

C.3 C.2.1
C.4 C.2.2
C.5 C.2.3
C.6 C.2.4
C.7 C.2.5
C.8 C.2.6
C.9 C.2.7
C.10 C.2.8
C.11 C.2.9

refs: done

C.4 The Spherical Pendulum d

C.12 C.4.1
C.13 C.4.2
C.14 C.4.3
C.15 C.4.4
C.16 C.4.5
C.17 C.4.6
C.18 C.4.7
C.19 C.4.8
C.20 C.4.9
C.21 C.4.10
none C.4.11
none C.4.12
C.22 C.4.13
C.23 C.4.14
Fig C.4 C.4.15
C.24 C.4.16
C.25 C.4.17

refs: done

C.5 The Foucault Pendulum e

C.26	C.5.1
none	C.5.2
none	C.5.3
C.27	C.5.4
C.28	C.5.5
C.29	C.5.6
C.30	C.5.7
none	C.5.8
none	C.5.9

refs: done

This concludes my incredibly painful and lengthy equation number update of frames doc. So now it is more maintainable.

Update 12.11.16

1. Created a new frames doc folder and moved all my 2016 work on frames doc into that folder.

2. Updated the Maple index in this new folder to the new numbering scheme.

2. Updated the Visio index in this new folder to the new numbering scheme. Took a while.

I think now I would like to rewrite Appendix D on center of gravity and related topics. I made a new v1 version of this appendix (it has never been installed).

Section D.1 This section is just fine, no changes needed (though I did tiny edits).

Section D.2. I want to redo this using a new picture of the tether. M_E ω_0

Update 12.13.16

I have now written New Appendix D sections D1 through D.5, and am working on D.6 about torque.

I just realized that my first angular momentum discussion is "hidden" in Section 1 and I need to make it more visible. I will move it to be the last section in Section 1 and this will involve some equation number changes

old	new	
1.8.11	1.9.1	Making this a new separate section 1.9
1.8.12	1.9.2	
1.8.13	1.9.3	
1.8.14	1.9.4	
1.8.15	1.9.5	
1.8.16	1.9.6	

1.8.17 1.9.7

1.9.1 1.10.1

1.10.1 1.11.1
1.10.2 1.11.2
1.10.3 1.11.3
1.10.4 1.11.4
1.10.5 1.11.5

Be careful with this update!!! Do it in stages.

Step 1: update the last 5 equation numbers globally in frames doc using a search DONE

Step 2: repeat for the second group above, globally. DONE

Step 3: Do the first group one at a time, each one global. DONE

Now at least the reader can FIND my opening comments on angular momentum.

Update 12.15.16

Today for the first time I was able to write down two ODE's for most-general dumbbell motion, that has been a goal for a long while now. They are very complicated, but I was able to obtain the small oscillation frequency that agrees with the tether PDF. Not sure what to do with this, and there are other sources to look at.

Update 12.31.16

Back from Cod, and Black PC seems to have recovered. I will start over with a new Appendix F v5 which has no primes for the satellite frame. I made that decision at the Cod.

Appendix F v5
- F.1 editing complete
- F.2 editing complete
- F.3 editing complete except for last series approx thing
- F.4 editing complete
- F.4 editing is underway, I have two equations of motion.

Update 1.2.17

I reread F.1 - F.3 above, and think I should now go back and update Appendix D to my new notation, But first I am going to attempt Appendix F v6 where I use $r_1 = r$ interchangeably. Well, I am working my way through Ap F v6 with still better notation.

Update 1.3.17

I am proofing App F v6 yet another time, still making edits. Done with F1 and F2. Done also with F3. I will pause here and now go back and rewrite Appendix D in my new notation. This will be New Appendix D v4.doc.

D1 done
D2 done
D3 done
D4 done
D5 done

So this App D v4 is now upgraded to be compatible with the notation of App F v6. I now return to this App F to add a piece to the end of F.3: done.

I will start next session at the beginning of F.4 which will just be a review.

Update 1.10.17

Much has been done and today is a bit of a breakthrough in that I obtained Cartesian equations of motion which run well in Maple. I need to ponder a bit how to arrange this long appendix.

Update 1.11.17

(1) wrote up Maple notes on dsolve inside my user guide
(2) rewrote Section F.7 using the 2nd order ODE's instead of the 1st order ones.
(3) wrote Section F.8 on derivation of the Cartesian equations

Update 1.12.17

I have tried to convert the Cartesian EOM to Angular EOM which is the simple direction to do it! Everything seems to be right except the "constant term" does not match. This is the term without the derivative of anything. It seems that my two earlier results have an extra constant term that should not be present, and the same error was made in both the torque and the force analyses! The term in question is this one:

$$\ddot{\theta} + \sin\theta\cos\theta(3\omega^2 - \dot{\varphi}^2) + \omega\sin\theta\cos\varphi\,(\omega\cos\theta\cos\varphi - 2\sin\theta\,\dot{\varphi}) = 0 \qquad // \hat{\varphi}$$

So my later Cartesian analysis is saying the red stuff should not be there! I now start at (F.5.7) and trace this backwards in red to see where this red term comes from. I trace this back in Section F.4 and it looks like it is supposed to be there.

I trace it backwards as well in Section F.6 and it looks solid there as well!!

So we have the Puzzler of the day. I think it should be there and I somehow am losing it in my Cartesian EOM derivation and subsequent conversion to angular EOMs.

Update 1.14.17

I have now finished Sections F.7,8,9,10 and things look good.

Immediate problem: the out of plane libration looks different depending on how I enter the A factor, so will look into that a bit. Before doing this, I am going to assemble Appendix F into one piece, it is too confusing in separate chunks. Assembly complete.

I wondered why mws v5 and v6 were making different plots, then I found an error in my entry of A for v5 and now it won't run at all, so the different plots problem goes away!!

So I have no remaining "mysteries" except I want to get tension T plotted somehow.

The problem plotting T: I have $T = - m_1 r_1 [\ddot{x}/x + \omega^2(1 - 3(z/b))]$ and this blows up when x(t) crosses zero. but tension should never blow up!

Bug: when I change odeplot number of points, the length of the orbit changes! Well, I think I have this one taken care of.

So Appendix F is finally in a full first cut, it has 10 sections.

Update 1.15.17

I did a successful 3D plot of an orbit.

Next I try to plot the tension T and I get a significant discrepancy between the numerical solution and an earlier far approximation expression. The scale is correct, but the amplitude variation is not correct.

So how exactly does the "far approximation" get into my work? I use it along the way and I end up with no equations containing b. So I cannot set b large relative to r_1 anywhere because b does not appear!

If I make the motion very small with x = y = .1, then the T's agree better. The problem is that the angular T expression is just the tidal force, it is the result you get with no velocity at all of m_1. So the difference between the two estimates is due to velocity and its effects like centrifugal!!

Let's redo the angular version:

$$\hat{\mathbf{r}}_1: \ [r_1/b](- \dot{\theta}^2 - \dot{\varphi}^2 \sin^2\theta) = - (\omega^2 b^3/r'_1{}^3)(\cos\theta + [r_1/b]) - T/(m_1 b) + \omega^2 \cos\theta - \omega^2 [r_1/b](\sin^2\theta\cos^2\varphi - 1)$$
$$- \ 2\omega[r_1/b] (\dot{\theta} \sin\varphi + \dot{\varphi} \sin\theta \cos\theta\cos\varphi) \quad \text{correct} \qquad (F.6.27)$$

Assuming small velocities we drop the left side and the last term to get

$$\hat{\mathbf{r}}_1: \qquad 0 \approx - (\omega^2 b^3/r'_1{}^3)(\cos\theta + [r_1/b]) - T/(m_1 b) + \omega^2 \cos\theta - \omega^2 [r_1/b](\sin^2\theta\cos^2\varphi - 1) \qquad (F.6.28)$$

From (F.6.25) one has

$$(b/r'_1)^3 \approx 1 - 3(r_1/b)\cos\theta$$

so (F.6.28) becomes

$$0 \approx -(\omega^2[1 - 3(r_1/b)\cos\theta])(\cos\theta + [r_1/b]) - T/(m_1 b) + \omega^2\cos\theta - \omega^2[r_1/b](\sin^2\theta\cos^2\varphi - 1)$$

$$= -\omega^2\cos\theta + 3\,\omega^2(r_1/b)\cos^2\theta - \omega^2[r_1/b] - T/(m_1 b) + \omega^2\cos\theta - \omega^2[r_1/b](\sin^2\theta\cos^2\varphi - 1)$$

$$= 3\,\omega^2(r_1/b)\cos^2\theta - \omega^2[r_1/b] - T/(m_1 b) - \omega^2[r_1/b](\sin^2\theta\cos^2\varphi - 1)$$

Then

$$T/(m_1 b) = 3\,\omega^2(r_1/b)\cos^2\theta - \omega^2[r_1/b](\sin^2\theta\cos^2\varphi - 1) - \omega^2[r_1/b]$$

$$= 3\,\omega^2(r_1/b)\cos^2\theta - \omega^2[r_1/b](\sin^2\theta\cos^2\varphi)$$

$$= 3\,\omega^2(r_1/b)[\cos^2\theta - \sin^2\theta\cos^2\varphi] \qquad \text{// wrong!}$$

$$T = 3m_1\,\omega^2 r_1[\cos^2\theta - \sin^2\theta\cos^2\varphi]$$

$$= (3m_1\,\omega^2/r_1)[r_1^2\cos^2\theta - r_1^2\sin^2\theta\cos^2\varphi]$$

$$= (3m_1\,\omega^2/r_1)[z^2 - x^2]$$

But when I plot this, I seem to get a totally wrong result in scale!! // Fixed

I have reworked T in the angular world to get

$$T = m_1 r_1[\,\omega^2(3\cos^2\theta - \sin^2\theta\cos^2\varphi) + \dot{\theta}^2 + \dot{\varphi}^2\sin^2\theta - 2\omega(\dot{\theta}\sin\varphi + \dot{\varphi}\sin\theta\cos\theta\cos\varphi)\,] . \qquad \text{(F.6.27)}$$

In the Cartesian world I know that

$$T = -m_1 r_1[\,\ddot{x}/x + \omega^2\,]$$

Thus I should be able to show that

$$\omega^2(3\cos^2\theta - \sin^2\theta\cos^2\varphi) + \dot{\theta}^2 + \dot{\varphi}^2\sin^2\theta - 2\omega(\dot{\theta}\sin\varphi + \dot{\varphi}\sin\theta\cos\theta\cos\varphi) + \ddot{x}/x + \omega^2 = 0$$

or

$$-\ddot{x}/x = \omega^2(3\cos^2\theta - \sin^2\theta\cos^2\varphi) + \dot{\theta}^2 + \dot{\varphi}^2\sin^2\theta - 2\omega(\dot{\theta}\sin\varphi + \dot{\varphi}\sin\theta\cos\theta\cos\varphi) + \omega^2$$

$$-\ddot{x}/x = \omega^2(3\cos^2\theta - \sin^2\theta\cos^2\varphi + 1) + \dot{\theta}^2 + \dot{\varphi}^2 \sin^2\theta - 2\omega(\dot{\theta}\sin\varphi + \dot{\varphi}\sin\theta\cos\theta\cos\varphi)$$

or

$$-\ddot{x} = x[\omega^2(3\cos^2\theta - \sin^2\theta\cos^2\varphi + 1) + \dot{\theta}^2 + \dot{\varphi}^2 \sin^2\theta - 2\omega(\dot{\theta}\sin\varphi + \dot{\varphi}\sin\theta\cos\theta\cos\varphi)]$$

This certainly seems far-fetched but let's try it in Maple

$$f1 := -\ddot{x}$$

$$f2 := x[\omega^2(3\cos^2\theta - \sin^2\theta\cos^2\varphi + 1) + \dot{\theta}^2 + \dot{\varphi}^2 \sin^2\theta - 2\omega(\dot{\theta}\sin\varphi + \dot{\varphi}\sin\theta\cos\theta\cos\varphi)]$$

OK I have confirmation that the Cartesian equation gives the angular one for T:

Update 1.16.17

I did a little more at the end of App F, cleaned up its TOC, and am ready to proof the monster. All by itself it is 46 pages long!!! I will update the little subsection overviews right after I proof each section.

But before doing this, I feel the need to review certain other sections of the paper.

$\theta \quad \hat{\boldsymbol{\theta}} \quad \hat{\boldsymbol{\rho}} \quad \rho \quad \hat{\boldsymbol{\varphi}} \quad \varphi$

Section 8

8.1 Development of the Fictitious Forces **OK**
8.2 Interpretation of the Centrifugal and Euler Fictitious Forces **OK**

Did some work here, added to App E. This is a tough section but I want to keep it.

8.3 skip
8.4 skip
8.5 OK, about g and g_0
8.6 Tethered satellites and Tidal Forces **OK**

8.7 Tides on the Earth

This section is a mess and I have put it in a separate doc for a rewrite. It wrongly states stuff about the earth's rotation axis versus the plane of the moon's orbit. And it is horribly unclear. The more I red this section. the more wrong it seems! The whole presentation stinks. Maybe I will get happy again after more work on this, but more work is needed.

I am confused by Fig 1 of frames doc. Could the origin of Frame S' be rotating but the axes are not rotating, or are the axes of Frame S' soldered to the **b** vector?

Update 1.19.17

Have continued on the tide stuff, decided to trash the last section on phase locked earth, this section is already way too long and that last section added little. Confidence level is pretty high.

I now want to proof the new Tides section one section at time. There are over 50 equation numbers so I don't want to screw up at the start.

8.7 Special Case #3 OK

8.7 Tides on the Earth

The basic picture	OK	1 through 7
Force Equations	OK	8 thru 13
The relation between r_{12} and Ω	OK	14,15,16
Tidal Force at an arbitrary point on the Earth	OK	17 thru 20
Tidal force in Cartesian coordinates	OK	21 thru 28
Tidal force in Polar coordinates	OK	29 thru 32
Equation of the water surface	OK	33 thru 42
Tidal patterns for an arbitrary rotation axis of the Earth	OK	43-50
Example 1: ($\hat{\mathbf{z}}' = \hat{\mathbf{y}}$)	OK	51 thru 54
Example 2: (tipping $\hat{\mathbf{z}}'$ toward the $\hat{\mathbf{x}}$ direction)	OK	55, 56
Tidal patterns for the real Earth (but still a water world)	OK	57 thru 62

All done! I think this is a greatly improved section over my last frames release. It ends up having no bearing the 2 or 3 question.

Maybe now I should actually read Taylor and Butikov. Not sure of the status of the satellite stuff now, I digressed to get the tides under control!

Update 1.20.17

I made the numbers more accurate, and added more caveats, and I did due diligence by looking at both the papers of Taylor and Butikov. I am now going to install these sections into frames doc v2;

I will now update the summary as well. Done, no change made.

Status: I wanted to get this tide stuff nailed down before updating the satellite section of frames doc. I thought there might be some connection, but now I don't really think so.

12:15 starting a reading of the main doc 8.6 on tethers.

Status at 7:05 PM. I think Section 8.6 (tether) and 8.8 (Earth tide) are finally stable. I finally addressed the issue of the 2 versus the 3 and I no longer think wiki is wrong. You have to say what system you are talking about! So this has been a long time coming. What remains is a ton of proofing (I hope that is all!).

Update 1.21.17

Well I was bothered at night by the fact that I could not make the potential version of the tidal surface work. Taylor does it, Butakov does it, why couldn't I do it? I did major battle all day on this, always pursuing wrong threads. Tried to integrate the force, ugly. Realized that "by inspection" really works. Used my Cartesian tidal force and came up with the same answer only after much painful detail and two more versions of the potential doc. I added this as a "footnote" at the end of Section 8. And I added a comment that I think the water volume is not handled properly in the usual presentations.

I think now it is REALLY done!

Update 1.22.17

Did some due diligence by scanning the tethers in space pdf and other web docs. Did not find anything I did wrong, added comment 5 on real tethers.

Argg! I see I now have two appendices on spherical coordinates A and E. I will have to combine them and repair all references?? Well, App A is really just about a specific topic.

Make a list of current references to Appendix A equations:

No specific equation references in all of the main text!
Appendix C
above (C.1.4) ref to (A.14c) // OK as is
below (C.2.5) ref to (A.16) // fixed up
no others

Make a list of current references to Appendix A or App. A:
Below (14.5) // OK as is

So I can perhaps just remove duplicate stuff from Appendix A.

Done. I removed some stuff from App A and put it at the end of App E, and I fixed up all references to this material which were only those shown in red above.

Next task: Review Appendix C on the spherical pendulum, maybe add Maple simulation.

Appendix C Review: Foucault Pendulum

opening text: OK
C.1 Drawings, Notation, Coordinates and Basis Vectors OK, lots of work here
C.2 Qualitative Solution OK, very short
C.3 The Equations of Motion for a Foucault Pendulum OK
C.4 The Spherical Pendulum
The equations of motion OK
L_z as constant of the motion OK
E as constant of the motion OK
Closed form solution to the spherical pendulum problem (outline) OK
The nature of the general solution OK
The spherical pendulum for small θ is still complicated unless h is very small OK
The Conical Motion solution OK
C.5 The Foucault Pendulum OK

So maybe I add C.6 with a simple Maple simulation, similar to what I did for the dumbbell satellite. I could start with my dumbbell Maple file, probably things are very similar.

I did some work, have that same problem with $\theta = 0$, so I really want to redo the pendulum in Cartesian coordinates from scratch, and check θ with that, and do the simulation there! I will do it tomorrow!

Update 1.24.17

I have added Sections C.6 and C.7 and C.8 in parallel to the dumbbell stuff in Appendix F. Am now starting to do some Foucault simulations! I would like to do the Pantheon!! The thing is deflecting to the left instead of the right, something is wrong! Oh, $\omega < 0$!! All fixed.

Many hours later: I think I have finished Appendix C. Reviewing latter sections now

C.6 Cartesian Equations of Motion for the Foucault Pendulum ok
C.7 Verification of the Cartesian equations of motion and string tension ok
C.8 Numerical solutions of the equations of motion (Cartesian Coordinates) ok

It looks good, finally. I think Appendix C is now "up to snuff". I installed the above sections so now all of Appendix C is in the main doc. Did a lot today with Maple, had usual problems but all is well.

Update 1.25.17

Created a new Maple index for the entire frames doc with today's date in title.

Now, let's start by proofing each Appendix including eq refs, and then updating its summary.

Appendix A: Derivation of R(ξ) for Spherical Coordinates

This is a short appendix and I read everything and did a few clarifying edits. It has a very narrow purpose which is to find a certain matrix appearing in the text. I also reviewed how App A is referenced and did small edits there. Now go do the summary. DONE.

Appendix B: The G Rule for a Tensor of Rank n

ok thru (B.4)

STOP. I want to change the way I do references to NOT use numbers. DONE. Resume App B:

ok thru (B.15)
ok thru (B.21)
ok thru (B.26)

OK, this is an excellent appendix, right to the point, derives all the cases. I checked all xrefs and fixed a few. Summary:

Appendix C: The Foucault Pendulum

Opening section:

Needed a lot of work and was in fact wrong as stated! I tied it in with later work in the section and got the rotation sense understood. Took rest of morning for this tiny piece of work!

C.1 Drawings, Notation, Coordinates and Basis Vectors

OK very longwinded but I want no confusion about how the variables are defined and what the pictures look like. Fussbudget.

C.2 Qualitative Solution

A short section, restates perhaps the obvious already noted in the small angle opening section, but OK.

C.3 Equations of Motion for the Foucault Pendulum (Spherical Coordinates)

All equations were numbered C.2.x instead of C.3.x. Fixed this locally but then have to worry about external references! Otherwise all is OK.

C.4 The Spherical Pendulum

I am sidetracked. I want to see someone's formula for the precession rate of a basic spherical pendulum and I cannot find anything useful. Even for small angles. This problem must have some elementary solution. Will look more tomorrow.

Update 1.26.17

Got the Airy precession figured out, a topic with very rare appearances in the literature! Added it, ready to continue proofing. I will read quickly through the start of C, then slow down at C.4.

Opening OK
C.1 OK
C.2 OK
C.3 OK

C.4 The Spherical Pendulum

I am removing this text which is pure garbage, now that I know more about the precession.

_____

Since h is an arbitrary constant of the motion, it seems pretty clear that the angular rotation rate in the φ direction is "decorrelated" from that in the θ direction. If, for example, one randomly launches a spherical pendulum in a modest thin elliptical orbit (small h), that orbit precesses fairly quickly about the vertical axis because $\dot{\phi}$ and $\dot{\theta}$ are not related in any simple way. A perturbation analysis shows that such a thin elliptical orbit processes in the direction of the elliptical motion (try it with a string and weight). This precession of course has nothing to do with the rotation of the Earth since the spherical pendulum being discussed has nothing to do with the rotating Earth, it is just a mass on a string in a uniform g field.

The lack of correlation between $\dot{\phi}$ and $\dot{\theta}$ exists whether or not the angle θ is small during the motion, so even for small angle motion there is precession of the orbit.

_____-

Update 1.27.17

Will start again with C.4 in review. Reviewed from the start of C quickly to just after the animation links in C.4.

C.4 The Spherical Pendulum **OK**

C.5 The Foucault Pendulum

I need a reference for my claimed simple pendulum solution with the sn function. I found a paper which claims this

$$\sin(\theta/2) = \sin\frac{\theta_0}{2}\, sn\left[K\left(\sin^2\frac{\theta_0}{2}\right) - \omega_0 t\,;\, \sin^2\frac{\theta_0}{2}\right] \tag{30}$$

which is more complicated than my solution which is

$$\sin(\theta/2) = \sin(\theta_0/2)\ \mathrm{sn}(\Omega t;\ k=\sin(\theta_0/2)) \quad . \qquad \Omega = \sqrt{g/\ell} \tag{C.5.2}$$

So where did MY solution come from. I have a plane pendulum doc on this.

OK, I got this sn stuff all cleaned up. BUT

Major malfunction: In Section C.5 I claim that since Foucault does only a small deflection, that $\dot{\varphi}$ is small and can be neglected in equations. This is wrong! As soon as there is any tiny Coriolis deflection at all, one has φ going around full 2π on every swing! that is the way spherical coordinates work. So in fact $\dot{\varphi}$ is on the same order as $\dot{\theta}$ and there is no logic in neglecting $\dot{\varphi}$ terms as I propose here. This needs a major rewrite.

Can I bail it out with a different justification of the assumptions? I will try a rewrite here:

Justification of assumptions

In the second of equations (C.2.9) we neglected both $\dot{\varphi}$ terms. Was this justified?

$$| \sin\theta\cos\theta\, \dot{\varphi}^2 | \approx \dot{\varphi}^2 \approx \Omega^2$$

$$|- 2\omega \,[(-\cos\beta\cos\theta + \sin\beta\cos\varphi\sin\theta)\sin\theta\, \dot{\varphi}]\, | \sim \omega\Omega$$

$$| (g/\ell) \sin\theta | \sim \Omega^2$$

So you could neglect the Ωω term and then you get the plane pendulum.

since $\omega^2 << \Omega^2$. For a typical Foucault pendulum

$\Omega = 2\pi/T_{\Omega}$	$\omega = 2\pi/T_{day}$		
$T_{day}^2 >> T_{\Omega}^2$		?	
$(24*3600)^2 >> (16.4)^2$		?	// Pantheon
$7.46 \times 10^9 >> 269$	.	yes	

Well maybe the second equation can be bailed out and it then describes the plane pendulum and that is my logic flow.

What about the third equation which is this

$$2\cos\theta\, \dot{\theta}\, \dot{\varphi} + \sin\theta\, \ddot{\varphi} = - 2\omega \,[(-\cos\beta\cos\theta + \sin\beta\cos\varphi\sin\theta)\, \dot{\theta}\,]$$

$$\Omega^2 \qquad \Omega^2 \qquad \omega\,\Omega$$

If you neglect the right side, you are back to the spherical pendulum.

Update 1.29.17

I have just finished Section C.5 and it is VERY much improved in many ways. I like it! Readers will like it. I spent a lot of time getting C.4 and C.5 updated.

I would like now to review Sections C.6-8 to make sure things are compatible. Also, want to look again at the Appendix F angular Maple work, maybe "gear" will improve it somehow. Of course for the pendulum I never tried to run the simulation right through the origin $\theta = 0$ which is where you get the problem.

Section C.6 is OK, I got rid of r as a duplicate version of $\boldsymbol{\ell}$ and I just use $\boldsymbol{\ell}$.

Section C.7 is OK, have some $\boldsymbol{\ell} = r$ here, constantly fixing up eq nums.

Section C.8 is now OK, got rid of the 58 second Earth and replaced it with a Merry Go Round pendulum at the North pole which has the exact same plots.

I am now ready to integrate C.4 and C.5 into the main document. DONE.

I need now to update the Maple and Visio stuff again *for* Appendix C and clean the desktop. DONE.

I will now read through Appendix D and if OK I will install it.

D.1 OK
D.2 OK
D.3 OK
D.4 OK
D.5 OK

All refs exist in Refs (only Symon). Appendix D is ready to install!

Appendix E.
- E.1 OK
- E.2 OK
- E.3 OK
- E.4 OK
- E.5 OK
- E.6 OK I revamped this, it is much better (affine connection)

Appendix E is ready to install: DONE. But then I realized I have similar stuff in E.4 and E.6 and also that I use the tensor doc developmental notation, so had to massage things a bit.

Updated the summaries for App C,D and E. Appendix F is next, will start tomorrow on that. It is a monster I know.

Update 1.30.17

I spent about an two hours trying to see why there is not some way to do the pendulum without using ODE's since I did this in the projectiles problem. The deal is this: if you KNOW a trajectory in an inertial

frame, you can easily convert it to a trajectory in some other frame. But for the pendulum I don't know the trajectory in ANY frame, so just doing a "little equations" conversion does not work.

Ready now for Appendix F which is a whopping 48 pages long!

Appendix F review

defer the summary section

F.1 Kinematics of the satellite in rotating Frame S

opening text OK
Description of the Figure OK
Naming of coordinates OK
Some Basic Kinematic Facts OK
Summary: OK
eq num OK

F.2 Angular momentum of the satellite and its time derivative in Frame S

OK
eq num OK

F.3 The torque on the Dumbbell Satellite in Frame S'

opening section OK
Far Approximation OK
eq num

F.4 The fictitious torque on the satellite in Frame S

opening and main text OK
How might one interpret this simple result? OK
eq num OK

F.5 Equations of Motion for the satellite in Frame S (Spherical Coordinates)

main text no sections OK
eq num OK

F.6 Force analysis of the satellite in Frame S (Spherical Coordinates)

opening text comment OK
Obtain the three equations of motion OK
Verify the angular equations of motion OK
Obtaining the tension in the stick (or tether) OK
Tidal Force OK
eq num (F.6.22) is missing!!!! I will have to make some changes here.
repair done.

F.7 Numerical solutions of the equations of motion (Spherical Coordinates)

I have done a lot of plot work here to make it better, and started using "gear" method.
opening text OK

all subsections OK
eq nums OK

I will continue tomorrow. There are still 3 long sections to go, is now 8 PM.

Update 1.31.17

Let's do a fast review of F up to this point,

Pause to create a total and new Visio index including everything! I changed ω_0 to ω in F.1.1.

Restart review at start of F.

Fig F.3.11 make same $\omega_0 \rightarrow \omega$ change in sat page 8. Also fix axes labels, done.

F.8 Force analysis of the satellite in Frame S (Cartesian Coordinates)
text all OK.
eq nums OK

F.9 Verification of the Cartesian equations of motion and stick tension
added new opening comment
text OK
eq nums OK

F.10 Numerical solutions of the equations of motion (Cartesian Coordinates)
text all OK
eq num OK

DONE!!!

I just realized that I really can make nice x,y,z plots using the θ,φ numerical stuff, so I added a section at the end of Section C.5 to show an example of this, also plotting the tension. This is a very good thing to add.

Still have to do the App F summaries! Done.
Installing Appendix F. Done, and we are 286 pages. Last release was 161 pages.
Not sure what comes next. The opening summary has to be updated of course, and maybe the overview as well.

Updated the App F mini summary, and added such a summary to App C as well. Then reviewed and updated all the summaries at the very beginning.

Is it possible that I am done???????

FULL REVIEW OF THE MAIN TEXT

(this main text alone took 3 full days to review, there were small changes made).

Update 2.1.17

Each edit pass I find stuff wrong and stuff to be added, meaning the doc is not stable. So I guess it is time for another pass. Also some red text remains here and there.

TOC: Fills 3 pages, total page count is 287.

Overview: added a super-summary opening paragraph which can be an abstract as well. Read the entire Overview, I think it is OK. Now takes 3.3 pages

1. Notation, important role of the Prime Symbol, and other Preliminaries
opening text OK
Proof of (1.1.2): OK
Time dependence of the R_{ij} OK
1.2 Expansions of a vector and use of primes and parentheses OK

so far everything seems very clear to me on this reading, details and examples

1.3 Special case where a'_i is unambiguous
opening text OK
Example 2: Consider equation (1.1.1) , // a good simple example, OK
Active and Passive OK
Example 3: OK
Footnote: did a repair here, now OK

again, all seems clear to me on this reading -- so far, so good

1.4 When are two vectors equal?
OK, and the two examples are OK. Example numbers just increment across subsections.

1.5 The small rotation of a vector about an axis
all OK. This is a mind-expanding document IMHO, in terms of content and those footnotes.

1.6 The time rate of change of a rotating vector
all OK. Such details are rarely presented in any text! Combining with Section 1.4.
Each of the 6 comments is useful I think.

1.7 Rate of change of the basis vectors
all OK, stresses that you MUST indicate a frame for d/dt on a vector.

1.8 Notations for the many time derivatives of vectors r, r', b and L
this section is more difficult that previous ones, notion of cross and natural properties.
but I think it is all OK.
I take great pains to make my notation clear, other authors don't always do this!

1.9 Angular momentum
Mentions reference points, and does natural time derivatives, nothing more here. All OK

1.10 No frame label is needed for d/dt of a scalar function
OK, and scalar comment is also OK

1.11 When do operations d/dt and taking a component "commute" ?
I checked all eq num in the summary in italics.
the proof is a bit ugly but that is how you do it.

eq num check for all of Section 1:
pagination generally OK but will do that later.

Section 1 is I think OK. I will not read it again!

2. The G Rule for arbitrary vector a and its derivation
very very good! I like it, short, but interesting comments. Previews, examples. all OK
eq num check = OK

3. The Apparatus and its Observer at Rest in Frame S'
I just ran aground on the reference points for the L vectors here!!! Spent some time on this, added a little cross product theorem to Appendix A, then came back here and think it is better now regarding how the Observer measures angular momentum L and $\partial_{S'}$L. Very short section, is now OK.
eq num = OK since no equations!

4. The Relationship between the Two Frames S and S'
4.1 Explanation of Fig (4.1.1): Frame S in the plane of paper
OK, this is a winded description of the meaning of Fig 1.1, I think it is worth while
4.2 Explanation of Fig (4.2.1) : Vector ω pointing directly out of paper
OK, just describes an alternate picture where Frame S' origin rotates in the plane of paper.
4.3 Comments on $\dot{\mathbf{b}}_S$ and $\dot{\mathbf{b}}_{S'}$
OK, not much content but at least the author is facing these strange objects.
4.4 Special Case #1 : ω axis through Frame S origin
OK, trying to stay general by allowing $\hat{\boldsymbol{\omega}}$ to still move in time
4.5 Special Case #2 : ω axis through Frame S' origin
OK ω passes through Frame S' origin. Very short.
4.6 The Turntable
OK, very clean and short, has the rotor picture
4.7 The Earth
OK, shown in non-swap, but warns that Appendix C is in Swap
4.8 The Flying Camera Platform
OK, only a paragraph.
eq nums all OK

5. The Goal of the next two sections
OK short and sweet, I did add a small extension comment to this a while ago
no eq nums at all

6. Determination of velocities
6.1 Velocity $v_{S'}$
6.2 Velocity $v \equiv v_S$
6.3 Velocity v'_S
6.4 Velocity Summary
I carefully reviewed the above 4 sections with a fine tooth comb not long ago, so OK now.
6.5 Velocities for Special Cases
6.6 Comments
Comments all are OK and I am trying to drill into the reader earlier concepts and claims with examples.
eq nums OK

7. Determination of accelerations
7.1 Acceleration a'_S OK and explains the Coriolis factor of 2
7.2 Acceleration $a \equiv a_S$
7.3 Acceleration $a_{S'}$
7.4 Acceleration Summary
7.5 Relation between $\ddot{\mathbf{b}}_S$ and $\ddot{\mathbf{b}}_{S'}$
this section is too painful to proof. I know I did a solid proofing not too long ago, so accepting as OK.
eq nums OK

8. The Fictitious Forces
8.1 Development of the Fictitious Forces
seems just fine, not sure what those approx equations are doing to do for me. OK
eq nums OK

STOP. I think something is going to be wrong with my discussion involving

$$\mathbf{F'}_{fict} = -m\ddot{\mathbf{b}}_S - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') - m\dot{\boldsymbol{\omega}} \times \mathbf{r}'$$

$$= -m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - m\dot{\boldsymbol{\omega}} \times \mathbf{r}$$

I claim people do this wrong with long and short vectors, and I note that those approx formulas don't have the frame contributions. I may be doing some bad comparisons with Marion et al, so things are going to have to slow down again now. I am just suspicious of what I have done. Maybe jump ahead a bit to the comparison sections. // Instead, I have started into a rewrite of Section 8.2.

Update 2.2.17

I finished my rewrite of Section 8.2 and installed it, it is very much better than the old one which I wrote not understanding the basic facts! I suspect this will have a bearing on my text comparisons in later sections. But for now I just continue the overall doc review.

8.3 Interpretations of the Coriolis Fictitious Force
Qualitative Arm-Waving Interpretation of the Coriolis Force
OK, no problems
Superposition Interpretation of the Coriolis Force
OK, very good, no problems.
eq nums = OK

8.4 Special Case #1 Problems
OK, a little repetitive but now Coriolis is included.

8.5 Problems on the surface of the Earth
OK, but now sure why I added the long "swap notation" version at the end.
eq num = OK

8.6 Tethered satellites and Tidal Forces
Question: I write T = $f(r_1, m_1) = - f(r_2.m_2)$, but with m1=m2 this says T = $f(r_1, m_1) = - f(r_1.m_1)$???
Wrong! Here r_1 and r_2 are long vectors! when m1=m2 you do not have r1=r2.
Section is A-OK
eq nums = OK

8.7 Special Case #3
ok, and I got my gimbal comment back in!
eq nums = OK

8.8 Tides on the Earth
The basic picture OK
Force Equations OK
The relation between r_{12} and Ω OK
Tidal Force at an arbitrary point on the Earth OK
Tidal force in Cartesian coordinates OK
2 versus 3 OK
Tidal force in polar coordinates OK
Equation of the water surface OK
Why is the equation shown that of an ellipse??? It is not an ellipse. I fixed this up.
Tidal patterns for an arbitrary rotation axis of the Earth OK
Tidal patterns for the rotating Earth (but still a water world in which water flows instantly) OK
Tides on the real Earth OK
Footnote: Equation of the Water Surface by the Potential Method OK
eq nums = OK

This really is a monster section on the tides! It is 23 pages long! I think it is pretty good in that it brings up a lot of topics. There are 73 equations, surely the most in any of my sections.

Chunk cut out of section 8 just now:

If in some problem the force $m\ddot{\mathbf{b}}_S$ can be neglected compared to all other forces, we can write

$$\mathbf{F} = m\mathbf{a} \approx m\mathbf{a}' + m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') + 2m\,\boldsymbol{\omega} \times \mathbf{v}' + m\dot{\boldsymbol{\omega}} \times \mathbf{r}' \qquad (8.1.3)_{approx} \qquad (8.1.9)$$

$$m\mathbf{a}' = \mathbf{F}'_{eff} \approx \mathbf{F} - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') - 2m\,\boldsymbol{\omega} \times \mathbf{v}' - m\dot{\boldsymbol{\omega}} \times \mathbf{r}' \qquad (8.1.5)_{approx} \qquad (8.1.10)$$

$$m\mathbf{a}' = \mathbf{F}'_{eff} \approx m\mathbf{a} - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') - 2m\,\boldsymbol{\omega} \times \mathbf{v}' - m\dot{\boldsymbol{\omega}} \times \mathbf{r}' \qquad (8.1.6)_{approx} \qquad (8.1.11)$$

$$\mathbf{F}'_{eff} = \mathbf{F} + \mathbf{F}'_{fict} \qquad (8.1.7) \qquad (8.1.12)$$

$$\mathbf{F}'_{fict} \approx \underbrace{-m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}')}_{\text{centrifugal}} \underbrace{- 2m\,\boldsymbol{\omega} \times \mathbf{v}'}_{\text{Coriolis}} \underbrace{- m\dot{\boldsymbol{\omega}} \times \mathbf{r}'}_{\text{Euler}} . \qquad (8.1.8)_{approx} \qquad (8.1.13)$$

As we shall see in Section 8.4, this list of approximate equations is not too relevant for "Earth problems", but we write the above list mainly for our comparison with Marion and Goldstein given below.

9. Comparison with Marion (1970), Thornton & Marion (2003) and Taylor (2005)
did a major overhaul here and just removed Taylor since he doesn't have much
all done, and it is better OK
eq num OK

10. Notation comparison with Goldstein (1950) and Goldstein, Poole and Safko (2001)
rewrote this as well, now OK
eq num OK

11. Angular Momentum and Fictitious Torques; the Reynolds Transport Theorem

11.1 Introduction
OK
eq nums OK

11.2 Expression of $\mathbf{L}^{(c)}$ and $\dot{\mathbf{L}}^{(c)}$ in terms of Frame S' objects
Huge detail here to leave a checking trail, but all seems OK
eq nums OK

11.3 Fictitious Torques and Newton's Rotational Law in a non-inertial frame
all clear, OK and eq num OK

11.4 Application: Fictitious Torques in Fluid Dynamics
This is a bunbuster section, but it seems right and has Lai verification. Brought down to earth by having a garden sprinkler sitting there. OK

11.5 Application: Fictitious Forces in Fluid Dynamics
A bit parallel to previous, with more Lai verifications. OK
eq nums OK

11.6 Comments on the Reynolds Transport Theorem
just a tour of Lai and tying in our stuff here. I guess I wanted to get this written down somewhere so I would have it if I ever need it later.
OK

I think I will pause here and review some of the opening section summaries! DONE, made some small changes. Tomorrow I will resume with Section 12, started at 4AM today and is now 6:30 PM.

Update 2.3.17

I had to repair Section 10 on Goldstein comparison. The two frames could not be co-sited at the Earth's surface because then neither would be an inertial frame. I moved them both to Earth center and all seems OK again. So I will now continue the long review.

12. Summary of the Forward Problem Solution
I checked EVERYTHING in full detail in this section, all is OK.

13. The Inverse Problem
opening text OK

13.1 Brute Force Method
OK, obtained those inverse equations

13.2 Swap Rules Method
I added the notation swap rules to show how they are different.
No proof yet of the Swap Rules.
OK and eq num OK

13.3 Summary of the Inverse Problem Equations (non-swap notation)
OK, very simple method, I checked each equation.

13.4 Summary of the Inverse Problem Equations (swap notation)
Checked every equation, all is OK.

13.5 Why the Swap Rules Work
Excellent, no changes needed, OK. and eq nums OK

So concludes Chapter 13 on "the inverse problem". This frames doc is a very good doc IMHO.

14. Rotating Frames in Curvilinear Coordinates
Excellent, easy to read, uses App A, uses my curvilinear notation trick, not confusing
OK! and eq nums OK

15. Ant on Turntable Problems

Kinematics common to all Ant Problems
OK, no issues.

What do we know about all the basis vectors?
Lots of technical detail here about basis vectors, all seems OK.

Relation between Frame S and Frame S'
OK

Motion of vector **b**
OK
eq num OK

15.1 Problem 1: Ant crawls at constant speed V to the Origin of Frame S'
This is a long section, taking a break before doing a review.

Ant's Motion in Frame S'.
plan to removed excess stuff in red
OK

Trajectory **r**(t) of the ant in Frame S
OK

Velocity **v**(t) of the ant in Frame S
OK

Acceleration **a**(t) of the ant in Frame S
OK

Selected Plots
OK

Some results to be used in the next problem.
OK

eq num = OK

This is a long section. I added a Chapter 15 opening text section.

15.2 Problem 2: Ant spirals in at constant V and Ω to the Origin of Frame S'

Ant's Motion in Frame S'
OK, straightforward stuff

Trajectory **r**(t) of the ant in Frame S
OK

Velocity **v**(t) of the ant in Frame S
OK

Acceleration **a**(t) of the ant in Frame S
OK

Trajectory Plots
OK, the orbital sander

eq nums = OK

15.3 Problem 3: Inverse Problem: Ant flies in Frame S at constant velocity V

Ant's Motion in Frame S
OK

Trajectory **r'**(t) of the ant in Frame S'
OK

Trajectory **r'**(t) of the ant in Frame S': Alternate Method
OK, anticipating M&T discrepancy so I do something twice to make sure of the answer

Velocity **v'**(t) of the ant in Frame S'
OK

Acceleration **a'**(t) of the ant in Frame S'
OK

Trajectory Plots
OK

Pause. Found that (8.3.13) does not exist. I will now make these changes

add new (8.3.4)
map existing (8.3.4) → (8.3.5)
map existing (8.3.5) → (8.3.6)
first change references to old 8.3.4 : none
first change references to old 8.3.5 : none
so I have nothing to fix.

15.4 Problem 4: The Projectile Problem of Section 8.3

The Hard Way
OK

The Four Projectiles
orange path came out brown for some reason.
I fixed this up.
OK all done
eq num = OK

All done now with all of the Main Text, only appendices are left!!! Time 4:30 PM.
[I thought I was done with Section 15, but I return to it heavily below]

Appendix A: Derivation of R(ξ) and Properties of Rotation Matrices
This went quickly, I had cleaned it up recently,
eq nums OK. All done.

Appendix B: The G Rule for a Tensor of Rank n
OK, and eq nums are OK as well. Been here recently.

Appendix E: Spherical Coordinates and Unit Vectors

E.1 Angle Conventions
OK and eg nums = OK (only one)

E.2 Matrix Approach
OK and eg nums = OK

E.3 The motion of a particle in spherical coordinates
OK and eg nums = OK

E.4 Curvilinear coordinates approach
OK and eg nums = OK

E.5 Polar Coordinates
OK and eg nums = OK

E.6 The Affine Connection
OK and eg nums = OK

I ran into no problems in App E, I have been very recently anyway.

I still have App C, D and F to deal with. Signing off at 8:45 PM.

Update 2.4.17

This AM I just read through Ch 15 's ant problems and made a few tiny cosmetic changes. When I got to the projectile problem, I fear something is wrong! Something just seems wrong with my original picture and I have to figure that out. Also, I don't understand my last two plots, they seem wrong intuitively. So will work on this a while today in the post-Judi time frame.

Update 2.5.17

I had some bad errors in Section 15.4 which only showed up at small velocities, I think things are fixed now. I am going now to go through ALL of Section 15 again.

15. Ant on Turntable Problems
opening text OK
Kinematics common to all Ant Problems

I did not get far. Just did a complete rewrite of this "common" section so let's start again

15. Ant on Turntable Problems
opening text OK
Kinematics common to all Ant Problems
OK and eq nums OK (but they are new!)

15.1 Problem 1: Ant crawls at constant speed V to the Origin of Frame S'

Ant's Motion in Frame S'.
OK, job here is simply to state **r'**, **v'** and **a'**

Trajectory **r**(t) of the ant in Frame S
OK, obtained **r**(t)

Velocity **v**(t) of the ant in Frame S
OK, obtained **v**(t)

Acceleration **a**(t) of the ant in Frame S
OK, obtained **a**(t)

Summary of the Solution to Problem 1
OK

Selected Plots
OK

Some results to be used in the next problem.
OK

15.2 Problem 2: Ant spirals in at constant V and Ω to the Origin of Frame S'
BEWARE, I THINK THERE ARE PROBLEMS HERE

Update 2.6.17

I ended up doing a very major rewrite (in place) of this section in order to make $R_{\mathbf{z}}(\Omega t+\varphi)$ appear in a natural manner. This required matrix notation and extreme care. I added a piece in Section 1 on this matrix business so I could call upon it here. I think the result is much better than it was.

I think finally that Section 15 is OK. It has been many days on this.

I think the opening section can be simplified now that I have added stuff to Section 1. Need to do this now.

Yet Another Review of Section 15.
opening OK

Kinematics common to all Ant Problems
opening text OK

What do we know about all the basis vectors?	OK
Writing the basis vector relations in matrix notation	OK
Relation between Frame S and Frame S'	OK
eq nums	OK

15.1 Problem 1: Ant crawls at constant speed V to the Origin of Frame S'

Ant's Motion in Frame S'.	OK
Trajectory **r**(t) of the ant in Frame S	OK
Velocity **v**(t) of the ant in Frame S	OK
Acceleration **a**(t) of the ant in Frame S	OK
Summary of the Solution to Problem 1	OK
Selected Plots	OK
Reader Exercise:	OK
Results expressed in matrix notation	OK

15.2 Problem 2: Ant spirals in at constant V and Ω to the Origin of Frame S'

Ant's Motion in Frame S'	OK
Trajectory **r**(t) of the ant in Frame S	OK
Velocity **v**(t) of the ant in Frame S	OK
Acceleration **a**(t) of the ant in Frame S	OK
Trajectory Plots	OK

I have a good overall impression of problem 2.

15.3 Problem 3: Inverse Problem: Ant flies in Frame S at constant velocity V

Ant's Motion in Frame S	OK
Trajectory **r'**(t) of the ant in Frame S'	OK
Trajectory **r'**(t) of the ant in Frame S': Alternate Method	OK
Velocity **v'**(t) of the ant in Frame S'	OK

Acceleration **a'**(t) of the ant in Frame S' OK
reference to the movie and two more plots OK
eq nums OK

15.4 Problem 4: The Projectile Problem of Section 8.3
opening text OK
The Hard Way OK
The Four Projectiles OK
eq nums
I have a good overall impression of problems 3 and 4

I think I can finally stop worrying about this Section 15. It had many problems of different types (clarity, math logic, actual errors in the projectile problem, general organization). I think it is a great chapter to demonstrate the main points of this document. It is 37 pages long, no one is going to find more detail on this topic anywhere in this world.

Check out the Section 15 overview: OK

STATUS: I have completed a full review of the entire document on this pass EXCEPT C,D and F. So these come next!

Update 2.7.17

Defer the overview for now****** /// DONE.

Nutshell Analysis of the Foucault Pendulum
OK, very good and concise summary of the essential fac.

C.1 Drawings, Notation, Coordinates and Basis Vectors
OK, no problems, eq nums OK

C.2 Qualitative Solution
OK, and worth having, the qual situation, and eq nums OK (only one).

C.3 Equations of Motion for the Foucault Pendulum (Spherical Coordinates)
OK and very good. Tiny edits only. eq nums OK. It reads like a story.

C.4 The Simple Pendulum
opening text is just fine

Exact Solution for the Simple Pendulum
Very excellent, completely readable, tiny edits. Will add the refs.
Had to tick up last few eq nums.

C.5 The Spherical and Foucault Pendulums
opening text gives index of this section, it will be long!

(a) The equations of motion for the Spherical Pendulum OK
(b) L_z as constant of the motion OK
(c) E as another constant of the motion OK
(d) Exact solution to the Spherical Pendulum (outline) OK
(e) The nature of the general solution for the Spherical Pendulum OK
(f) The Conical Motion solution of the Spherical Pendulum OK
(g) The thin ellipse scenario for the Spherical Pendulum OK
(h) The Intrinsic Airy Precession of the Spherical Pendulum OK
(i) The Foucault Mode of the Spherical Pendulum OK
(j) Interference between the Airy and Foucault precession OK
(k) The Foucault Pendulum at the Pantheon in Paris OK
(ℓ) General Numerical Orbits of the Spherical Pendulum OK

This is a killer good section on its topic!!!

C.6 Equations of Motion for the Foucault Pendulum (Cartesian Coordinates)
OK, obtained the x,y z equations of motion, first 10 equations of section

Small oscillation limit OK

All eq nums OK for this section.

C.7 Verification of the Cartesian equations of motion and string tension
main text OK, heavy use of Maple
Tension equation verification OK
eq nums all OK

C.8 Numerical solutions of the equations of motion (Cartesian Coordinates)
opening text OK
The Foucault Pendulum at the Pantheon in Paris (revisited) OK
Foucault Pendulum on a rotating platform OK
Desktop Spherical Pendulum OK
full eq num check = OK, had to fix the last two which have no refs OK

This concludes my review of all of Appendix C. I caught some good fixes, it is ready to ship.

What remains now are Appendix D and Appendix F.

Appendix D: Center of gravity and torque for a tethered satellite

D.1 Definition of Center of Gravity
totally clear, found no problems OK

D.2 Center of Gravity for a 2-mass Dumbbell Satellite
opening text OK

Center of Mass OK
Angles OK
Where is Center of Gravity of the Satellite? OK
This section establishes Fact 1 and Fact 2 about r_{cog} location.
eq nums = OK

D.3 Dumbbell Satellite Center of Gravity with the Far Approximation: Numerical Examples
opening text (more Facts are developed) OK

Numerical Examples very good, a few edits OK
eq check full = OK

D.4 Dumbbell Satellite Center of Gravity for equal masses and no approximation
all OK, a rather tech topic, no approx results, then limits for approx OK
eq nums =

D.5 Center of Gravity for a single-sphere satellite
OK, high school math on display and eq nums are OK

That concludes my review of App D. Only F remains! It is a monster.

Appendix F : The Dumbbell (Tethered) satellite as an example of rotating frame analysis

defer review of overview **********

F.1 Kinematics of the satellite in rotating Frame S
opening text OK
Description of the Figure
lots of works here related to the drawing, all OK
Naming of coordinates OK
Some Basic Kinematic Facts OK
eq nums = OK

F.2 Angular momentum of the satellite and its time derivative in Frame S
all OK, a very limited scope for each of these small sections OK
eq num = OK

F.3 The torque on the Dumbbell Satellite in Frame S'
opening text, found some small errors and fixed them.

Far Approximation OK
eq nums = OK

F.4 The fictitious torque on the satellite in Frame S
opening text OK
How might one interpret this simple result? OK

a painful section, eq nums = OK

F.5 Equations of Motion for the satellite in Frame S (Spherical Coordinates)
all OK, finally have the EOM in θ,φ after previous 4 sections of hard work! OK
eq num = OK

F.6 Force analysis of the satellite in Frame S (Spherical Coordinates)
opening text OK
Obtain the three equations of motion OK
Verify the angular equations of motion OK
Obtaining the tension in the stick (or tether) OK
Tidal Force OK
eq num = OK

F.7 Numerical solutions of the equations of motion (Spherical Coordinates)
OK, just a few basic plots
eq nums = OK

F.8 Force analysis of the satellite in Frame S (Cartesian Coordinates)
OK, and eq nums = OK . We have the x and y EOM.

F.9 Verification of the Cartesian equations of motion and stick tension
OK, and eq nums = OK. Duplicate of method used in App C.

F.10 Numerical solutions of the equations of motion (Cartesian Coordinates)
OK, and eq nums = OK. Not very exciting plots but shows how to do it.
An abrupt ending to the paper.

ALL DONE WITH THIS HUGE PROOFING PASS!! It took 7 days to do it, though of course this includes time needed for doing various repairs to problems discovered.

1. update all indices
2. clean up
3. reread overview stuff a last time
4. spelling checks
5. pagination
6. release!

Update 2.8.17

- Updated the Visio index for changes made in the last edit pass, hope I got everything.

- Updated the References section with new refs, discovered I had been erroneously been writing Stegun when correct name is Irene Segun. Added errata to three other docs on this.

- Verified all links in my Refs, they are all OK today Feb 8, 2017.

• Very and update all Section Headers! They were OK through App C. I fixed up the rest and all looks good.

• Spelling check global. Doc is too big, so do this 50 pages at a time and be careful!

start thru page 52: found and fixed two typos

page 53 thru page 101: found and fixed two typos

page 102 thru page 150: found and fixed two typos

page 150 thru page 200: no typos, one paren adjust

page 200 thru page 250: no typos, two adjusts

page 250 thru end: no typos

All done with the global spell check! Total of 6 corrections made.

BE VERY careful with all new edits in terms of spelling!!!

• figure centering: full pass, did small adjusts on many

• Review the entire overview section: done, made tiny changes.

Decision: spacing between subsections is on ad hoc basis, no general rule.
Same applies to starting new subsections on a new page.

PAGINATION

TOC OK

Overview/summary: done

Section 1: OK thru p 24 done
Some subsections begin in a new page, some do not, used min spacing between subsections.

Section 2: OK thru p 27 done

Section 3: done

Section 4: OK thru p 36 done

Section 5: done

Section 6: done

Section 7: done

Section 8: OK thru p 78 done

Section 9: done

v

Section 11: OK thru p 90 done

Section 12: done

Section 13: OK thru p 111 done

Section 14: done

Section 15: OK thru p 146 done

Appendix A: done

Appendix B: done

Appendix C: OK thru p 203 done

Appendix D: OK thru p 225 done

Appendix E: OK thru p 232 done

Appendix F: OK thru p 283 done

References: done

This concludes a FULL pagination of the entire frames doc, is now 6 PM Weds.

Cleaned up the new frames doc folder a bit, but did not read all the docs!

Let's now do a full eq num review looking for misplaced eq nums, thrown, missing, whatever!
I got up to F.4.4 and now have an airport run to do. // resumed and all done.
I see no missing eq nums in the right margin, but the PDF will no doubt throw some.

Needs decision on what to capitalize: Earth? Sun? Moon? Northern Hemisphere? Etc.

Update 2.9.17

Time now to take a look at the result.

Am wondering about the **general solution** to the spherical pendulum. I have the two EOM.

$$\ddot{\theta} - \sin\theta\cos\theta\,\dot{\varphi}^2 = -(g/\ell)\sin\theta \qquad \hat{\boldsymbol{\theta}}$$

$$2\cos\theta\,\dot{\theta}\,\dot{\varphi} + \sin\theta\,\ddot{\varphi} = 0 \quad . \qquad \hat{\boldsymbol{\varphi}} \qquad \text{(C.5.1)}$$

Then I insert second into first to get

$$\ddot{\theta} - h^2\cos\theta/\sin^3\theta + (g/\ell)\sin\theta = 0 \qquad \text{(C.5.7)}$$

and this is a fully decoupled equation in θ. I work things down to this point:

$$t(\theta) = \int_z^{z_0} dz' \frac{1}{\sqrt{(2g/\ell)\,[\,z'^3 - (\mathcal{E}\ell/g)\,z'^2 - z' + (\ell/g)\,(\mathcal{E} - h^2/2\,)\,]}} \qquad // \; z = \cos\theta,\; z_0 = \cos\theta_0$$

$$= \sqrt{\ell/(2g)}\int_z^{z_0} dx \frac{1}{\sqrt{(x-a)(x-b)(x-c)}} \qquad \text{(C.5.13)}$$

where a,b,c are the roots of the cubic equation $x^3 - (\mathcal{E}\ell/g)\,x^2 - x + (\ell/g)\,(\mathcal{E} - h^2/2\,) = 0$. The...

Dimensions $\ell/g = L/(LT^{-2}) = T^2$ is correct. The roots are functions of $\mathcal{E}$ (dim = T^{-2}) STOP!! Last term then has bad dimensions! What happened here? $(\mathcal{E}\ell/g)$ = OK since $T^{-2}T^2 = 1$. Oh, dim(h) = T^{-1} so all dimensions are OK. Roots are functions of dimensionless quantities $(\mathcal{E}\ell/g)$ and $(\ell h^2/g)$ where

$$E = m\,\ell^2\mathcal{E} \qquad \mathcal{E} = E/(m\ell^2) = M(L/T)^2/(ML^2) = 1/(T^2) = \text{correct again}$$

So you could define dimensionless $\alpha = (\mathcal{E}\ell/g)$ and $\beta = (\ell/g)\,(\mathcal{E} - h^2/2\,)$ as two constants of the motion which in effect replace E and L_z^2. But both of these are arbitrary with E having either sign. I say'

$$E = (1/2)mv^2 - mg\ell\cos\theta = (1/2)\,m\,\ell^2(\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2) - mg\ell\cos\theta \qquad \text{(C.5.8)}$$

what sign is this? For very large v it could be as large as you want. But v = 0 it is $-mg\ell$ since cosθ = 1 at the resting point. I might mention this either sign business. Done. so you have

$$x^3 - \alpha x^2 - x + \beta = 0$$
$$a1 \quad a2 \quad a3$$

I think there ARE formulas for the roots of a cubic. Looking at page 32 of Schaum (with my corrections) we have $a_1 = -\alpha$, $a_2 = -1$, $a_3 = \beta$. Then

$$Q = (-3-\alpha^2)/9 \qquad R = (9\alpha - 27\beta + 2\alpha^3)/54$$

$D = Q^3+R^2$ $S = [R+D]^{1/3}$ $T = [R-D]^{1/3}$

Here is how Maple evaluates these things

```
restart; unprotect('D'); unassign('D');
a1 := -alpha:  a2 := -1: a3 := beta:
Q := (3*a2-a1^2)/9;
```

$$Q := -\frac{1}{3} - \frac{1}{9}\alpha^2$$

```
R := (9*a1*a2-27*a3-2*a1^3)/54;
```

$$R := \frac{1}{6}\alpha - \frac{1}{2}\beta + \frac{1}{27}\alpha^3$$

```
D := Q^3+R^2:simplify(%);
```

$$-\frac{1}{27} - \frac{1}{108}\alpha^2 - \frac{1}{6}\alpha\beta + \frac{1}{4}\beta^2 - \frac{1}{27}\beta\alpha^3$$

```
D*108: expand(%);
```

$$-4 - \alpha^2 - 18\alpha\beta + 27\beta^2 - 4\beta\alpha^3$$

```
S := (R+D)^(1/3): simplify(%);
```

$$\frac{1}{6}(36\alpha - 108\beta + 8\alpha^3 - 8 - 2\alpha^2 - 36\alpha\beta + 54\beta^2 - 8\beta\alpha^3)^{\left(\frac{1}{3}\right)}$$

```
T := (R-D)^(1/3): simplify(%);
```

$$\frac{1}{6}(36\alpha - 108\beta + 8\alpha^3 + 8 + 2\alpha^2 + 36\alpha\beta - 54\beta^2 + 8\beta\alpha^3)^{\left(\frac{1}{3}\right)}$$

```
alpha := E*L/g;
```

$$\alpha := \frac{EL}{g}$$

```
beta := (L/g)*(E-h^2/2);
```

$$\beta := \frac{L\left(E - \frac{1}{2}h^2\right)}{g}$$

```
D;
```

$$\left(-\frac{1}{3} - \frac{1}{9}\frac{E^2L^2}{g^2}\right)^3 + \left(\frac{1}{6}\frac{EL}{g} - \frac{1}{2}\frac{L\left(E - \frac{1}{2}h^2\right)}{g} + \frac{1}{27}\frac{E^3L^3}{g^3}\right)^2$$

.

From the last D could be either sign, so have to consider the two solutions separately. Nothing factors, so in general it is a big mess, but you could write it down! You then find the roots x_1, x_2 and x_3 which appear as a,b,c in my formula. THEN you evaluate the messy Maple expression at both endpoints

```
Int(((x-a)*(x-b)*(x-c))^(-1/2),x);
```

$$\int \frac{1}{\sqrt{(x-a)(x-b)(x-c)}}\,dx$$

```
value(%);
```

$$2\frac{(-a+b)\sqrt{\frac{x-a}{-a+b}}\sqrt{\frac{x-c}{a-c}}\sqrt{\frac{x-b}{a-b}}\,\text{EllipticF}\left(\sqrt{\frac{x-a}{-a+b}}, \sqrt{\frac{a-b}{a-c}}\right)}{\sqrt{x^3 - x^2c - bx^2 + bxc - ax^2 + axc + abx - abc}}$$

and you get a result which is a function of $z = \cos\theta$ and $z_0 = \cos\theta_0$.

Question: How does this solution describe high speed over the top solutions? I don't think it does, so I must have assumed something somewhere. I assumed $\theta(0) = \theta_0$. How would you apply both boundary conditions to this problem?? It really needs two, not just one. But the first order ODE (C.5.9) is a first order ODE, so it can have only $\theta(0) = \theta_0$ as a BC.

$V_e(\theta)$ θ_{min} θ_{max} ε plotted for $\theta = 0$ to π

OK, I have added a few small extra text into this solution and think all is well. They were good comments to add! I have reviewed it and I think it is all OK.

Question: do my equations of motion appear somewhere in the literature? Maybe I should add a reference.

The spherical pendulum equations DO appear in wiki as Euler Lagrange equations, so I added a long comment on this above (C.4.1). I will now repaginate Appendix C after this point. I then added the Lagrangian derivation of the spher pend EOM and repaginated C again. Always getting better.

Again: does someone state the Foucault equations of motion? Nothing shows up in a quick search, I will not hunt this out.

R.E. Hunt, *Lecture Notes for the Mathematical Tripos* (Cambridge University, 2007), Chapter 7 on Rotating frames, see http://www.damtp.cam.ac.uk/user/reh10/lectures/ .

Removed the above ref since I no longer Hunt's G rule derivation. not referred to at all!

So what else do I want to look at before publishing? Nada.

How about periods after equations?

PDF First attempt on Alta. 109 bookmarks, piledriving, then fails. Second attempt is creating pages. Bookmarks all look good (no extra starting spaces, none missing. No blank pages. Browsed the entire pdf looking at typesetting, could find nothing wrong.

What about words like Earth being capitalized. Review this please! **********

Found a few lower case Earths and updated them. There are no lower case northern hemispheres. No southern either. Just updated two moon to Moon. There are no lower case sun. So I mostly did this once before, but I will now have to do a new PDF.

Release Feb 9, 2017

Now I want to review why this took so long!

Review of efforts which went into this release.

The existing release before today's is 24 Oct 2012 and it is 161 pages long. Here then are some of the major changes which I made in this last release push which started 3/5/16!!

- In 3/16 I did a general read-through. It was not yet on Researchgate. I sent Maple stuff to guy.
Then nothing happened until November.

- November 2016. While reviewing Goldstein again, I decided to take another look at frames doc. Read through it again, called it "brutal".

- 11.26.16 Updated the G-Rule appendix which was C at that time. App B at that time was an ugly thing comparing my frames doc notation to tensor doc notation.

- 11.27.16 I went through a very long list of errata that had been piling up. Added 200 equation periods. Rewrote section a (a). Lots of editing in Section 4.

- 11.29.16 Replaced Section 8 (b)

- 11.30.16 worried about location of center of gravity for the earth moon system. Main Taylor tide formula had a sign error.

- 12.6.16 Have now written a new center of gravity appendix, it is now D. This was a major effort because I really knew nothing about it. Had scare with the swap rules.

- 12.10.16 Updated the entire doc to my modern section numbering format, took 3 days.

- 12.11.16 Created the "new frames doc" folder. Update Maple and Visio indices to the new eq numbering.

- 12.13.16 Have rewritten App D on center of gravity, many confusions.

- 12.14.16 first write down of the angular ODE's for the dumbbell satellite.

Cod trip at this time

- 12.31.16 wrote Appendix F on the dumbbell satellite, it had 6 drafts!!

- 1.11.17 wrote Maple notes on dsolve.

- 1.15.17 first 3D plot

- 1.19.17 doing a full rewrite on the tides section in Section 8. Many days on this

- 1.22.17 study tethers a bit. Update Foucault Appendix C.

• 1.24.17 More App C updates so it is parallel to App F in method.

• 1.26.17 learn about the Airy precession, get it worked in.

• 1.27.17 added plane pendulum complete solution to App C.

• 2.1.17 start full review of the main text after working on Appendices for a long time. Cleaned up my critique of long and short vectors, another rewrite of Section 8.2.

• 2.1.17 Update Marion and Goldstein sections. Review ongoing, taking 3 days.

• 2.5.17 find errors in the 4-projectile problem solutions, fixed them.

• 2.6.17 major rewriting in ant problem 15.2 showing how angle sums appear.

• 2.8.17 doing pre-release work: spelling, etc. Pagination.

• 2.9.17 small additions on Lagrangian and general spherical pendulum solution.

Basically the I spent 11.26.16 through 2.9.17 on this release, less Cod which was perhaps 11 days, so Excel says that would be 75 days or 10.7 very long and tedious weeks!!!

So how did Goldstein cause me to do this update. I have some notes in Goldstein folder of 11.16 which bring up questions. // I have looked at them and I think I really do have to finish my review of Goldstein Chapter 5 on rigid body motion before I release this paper to Researchgate. I can release it to my little xmission location, however. I just think there may be a few things I will want to ADD when I do that Goldstein review.

I will do a PDF cycle and then an xmission release, coming right now. DONE! Made a release folder.

Update 2.11.17

I am wondering what happened to my claim that (d**ω**/dt) is the same in both frames. The reason of course is

$$(d\boldsymbol{\omega}/dt)_S = (d\boldsymbol{\omega}/dt)_{S'} + \boldsymbol{\omega} \times \boldsymbol{\omega} = (d\boldsymbol{\omega}/dt)_{S'}$$

I KNOW I had this stated somewhere in frames doc, but I cannot find it. Found it, in 2.6 as examples of the G rule, right where it should be.

Update 2.14.17

I have written a new adder appendix G and am now proofing it.

G.1 Generators and finite rotation matrices.
text OK, eq nums OK (redid them)

G.2 About the general rotation matrix
text OK, eq nums OK (redid them)

G.3 The CHB and Sandwich Formulas
text OK, eq nums OK (redid them a little maybe)

Thrown out chunk:

_____

The "states" having values j and m are written in "Dirac notation" as |jm> where all the |jm> span an infinite dimensional Hilbert space that is the direct sum of the spaces spanned by |jm> for each particular j. This same infinite space is also spanned by the coordinates |θ,φ> which are a different "basis". Then for example

$$\langle\theta,\varphi|jm\rangle = Y_{jm}(\theta,\varphi)$$

$$\langle\theta,\varphi|\mathcal{J}_3|jm\rangle = \langle\theta,\varphi \mid J_3\, jm\rangle = m\langle\theta,\varphi|\, jm\rangle = m\ Y_{jm}(\theta,\varphi)$$

$$\langle\theta,\varphi|\mathcal{J}_3|jm\rangle = \langle J_3\ \theta,\varphi \mid\ jm\rangle = -i\partial_\varphi\langle\theta,\varphi|\, jm\rangle = -i\partial_\varphi\, Y_{jm}(\theta,\varphi)$$

_____-

OK, I spent this entire day cleaning up all of Appendix G. But I now have a new bothersome question.

Question: I show these complicated results in frames doc

$$\mathbf{L}^{(c)} = \mathbf{L}'^{(c')} + m(\mathbf{r}'-\mathbf{c}') \times [\,(\boldsymbol{\omega} \times \mathbf{r}') + \dot{\mathbf{b}}_S] \tag{11.2.14}$$

$$\dot{\mathbf{L}}^{(c)} = \dot{\mathbf{L}}'^{(c')} + m(\mathbf{r}'-\mathbf{c}') \times [\,\dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\,\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') + \ddot{\mathbf{b}}_S]$$
$$- m(\dot{\mathbf{c}}' + \boldsymbol{\omega} \times \mathbf{c}' + \dot{\mathbf{b}}_S) \times (\mathbf{v}' + \boldsymbol{\omega} \times \mathbf{r}' + \dot{\mathbf{b}}_S) + m\,\dot{\mathbf{c}}' \times \mathbf{v}' \tag{11.2.15}$$

But how does this relate to the simple G-rule statement that

$$(d\mathbf{L}/dt)_S = (d\mathbf{L}/dt)_{S'} + \boldsymbol{\omega} \times \mathbf{L}$$

which appears in Goldstein p 158. This makes it appear that the fictitious torque is just $\boldsymbol{\omega} \times \mathbf{L}$ instead of the very complicated formula I have for such torque.

Update 2.15.17

I have addressed this question in "Challenge on angular momentum.doc", things are going OK.

I then rewrite the four sections like 12.2 and added a new Special Case #4 to them which is basically the entire Goldstein world. I think that is a great improvement, though needs review before installing.

I learned a bit about Goldstein rigid body motion stuff and how is notation ties in.

I think at some point I can derive the App G Goldstein equations from his picture.

signing off at 8 PM.

Update 2.16.17

Reviewed the four docs and will install them soon.

Now lets look at the Goldstein notation chapter : DONE, added Case #4 refs, fixed up two illogicals. Review Marion notation section as well : DONE.

I am now ready to do the graphical ω equations derivation.

Question: I want to decode the Euler angle stuff, but I am faced with this question.

How do you *justify* the idea that **e** = R **e'** for basis vectors, but **v'** = R**v** for "normal vectors"? This seems a contradiction. I do address this question already in some doc.

1. Tensor doc active area says this

In the "active view" of things, we can think of **V'** = R**V** as creating a new vector **V'** in x-space from the old vector **V** by rotating the vector **V** by R. In the "passive view", we think of the V'_i as the components of the original vector **V** projected onto the backwards-rotated basis vectors $\mathbf{u'}_n = R^{-1}\,\mathbf{u}_n$. To verify this relation between the basis vectors, we can write

$$V'_n = \mathbf{V} \bullet \mathbf{u'}_n \;\; = \mathbf{V} \bullet R^{-1}\mathbf{u}_n \;= R\mathbf{V} \bullet RR^{-1}\mathbf{u}_n = R\mathbf{V} \bullet \mathbf{u}_n \;= \mathbf{V'} \bullet \mathbf{u}_n = V'_n\,. \qquad \text{(K.5.26)}$$

So one can think *either* of **V** being rotated forward in x-space into **V'** where **V'** has x-space components V'_n , *or* one can think of the V'_n as the components of **V** one measures in frame that is backwards rotated by R^{-1} , that is, $\mathbf{u'}_n = R^{-1}\,\mathbf{u}_n$.

So here I would just have to say that basis vectors do not transform as normal vectors.

I already have a very good comment in Section 1.3 which pretty clearly states that basis vectors do not transform as normal vectors. So let's just accept that for now.

Update 2.18.17

I have done several days of battle with the Goldstein ω formula on page 134, pieces right now are flung far and wide. Today I an doubting my Gold chapter on correlation of notation, so let's take a look.

It is Goldstein Ch 4 that is of concern here. Outline: pp 93-142, about 50 long pages.

4.1 Coordinates of a rigid body. Page 94 pic he has no-prime = space and prime=body. Other stuff here is no interest to me.

4.2 Orthog transformation (ie, rotations) . Rotations, and he has my active rotation picture p 100.

4.3 Matrix stuff. Details of matrix algebra, All old hat.

4.4 Euler Angles. Here are his pictures with Frame S aligned with paper and Frame S' rotated. Page 109 shows three matrices but they are not given nice names like $R_z(\theta)$, just B or C. $\mathbf{x}' = A\mathbf{x}$. He says nothing about space or body here.

4.5 Cayley Klein. This looks like SU(2) to me. Very long, of zero interest today.

4.6 Euler's Theorem on rigid body. Not much of a theorem. Rotation matrices have real eigenvalues, they are roots of secular equation. Doing linear algebra here.

4.7. Small rotations. Idea of dr = r x dΩ, he is approaching the G rule.

4.8 Rate of change of a vector. G Rule! Uses terms space and body. Now we get to the page 134 formula, and he associates S' = body, S = space for the first time. So prime goes with the body. I think this agrees with that Euler picture if the body is a top, say. This is his too-short derivation of the page 134 formula.

4.9 Coriolis. No primes or noprimes here, all just subscript r and s for rotating and body. Only r has no subscript, and I think he is Special Case #4 so not needed. And the chapter ends!

OK, so very last section is what I base my comparison on! I compare things and I conclude that his rotating or body frame is my S, while space is my S'. This conflicts with his earlier Euler picture. Maybe I should rethink this comparison!!!

Did that, and I have updated the Goldstein Section. Now the Euler angle picture is compatible with my interpretation of his notation in that comparison section.

Scraps:

One preliminary test is that we can take the determinant of both sides to make sure we get the same thing. Below in ** we shall prove that $\det(e^{A}) = e^{tr(A)}$ in any number of dimensions, so our check is then

$\det[R \exp(-i\theta \mathbf{n}\bullet\mathbf{J}) R^{-1}] = \det[\exp(-i\theta \mathbf{n}'\bullet\mathbf{J})]$?

$\det[\exp(-i\theta \mathbf{n}\bullet\mathbf{J})] = \det[\exp(-i\theta \mathbf{n}'\bullet\mathbf{J})]$?

$\exp[tr(-i\theta \mathbf{n}\bullet\mathbf{J})] = \exp[tr(-i\theta \mathbf{n}'\bullet\mathbf{J})]$?

$\exp[-i\theta n_k (trJ_k)] = \exp[-i\theta n'_k (trJ_k)]$? // the J_i generators (G.1.2) are traceless

exp[0] = exp[0] ? 1 = 1 ? yes

Update 2.20.17

To provide better support for frames doc, I have decided on a heavy rewrite of Sections 1.1 and 1.2. I am now going to proof this, and it will require a massive adjustment of cross references document wide. I will first proof this new stuff, then the rest of Chapter 1, and THEN I will do the xref adjusts outside this chapter.

Proofing: It is not stable, on each proofing pass I make major additions and changes. Try again

Proofing:

Section 1.1 is just excellent now, all the key basic basis ideas are now included and presented in a logical order. And eq num OK

Section 1.2

opening text OK

Matrix Notation to show how components are related. OK

Dot Products OK

eq num OK

I will continue to accumulate the new section 1 into my new doc, and will now add and edit existing 1.3.

Section 1.3

opening text OK

now only two examples

rest is also OK

eq num OK that went fast.

Section 1.4 added and done as well.

Coming soon: all remote xrefs to equations of the form (1.1.*) and (1.2.*) need be updated * DONE.

Major action. I will now archive off the old sections 1.1 through 1.4.

I want next to conclude proofing the rest of Section 1 including internal Sec 1 ref adjusts.

Proof 1.5 OK

I browsed the rest of Section 1, and I will just include it in my general xref update coming next.

Big Xref Effort.

1. Scan for all (1.1. refs and update one by one! Nothing till section 1.11, big box there. // Next end of 4.1. Already up to section 14! Now in 15. // Done. This includes App A thru F. Did G v2 separately. All done, it was not as major for (1.1.

2. Scan for all (1.2 refse and update. Done main, now G: Nothing in G.

3. I made no changes in (1.3. so no need to scan.

OK all done with this Xref action. Was much easier than I was expecting. I guess I don't really use all that stuff I explain a lot in Section 1! It is mostly just "background" for my notation.

What's next? Next is to add the Euler graphic section to App G v2. This is in Euler angle writeup doc as G.4, I will proof it a bit in that doc before installing.

I did a pass on The Euler Angle method of computing ω in App G.5 now in a separate doc . When this is proofed a bit, I will conclude with the mega section on how to compute ω in a different manner which does not require any pictures. I hope to get both a Frame S and Frame S' version of **ω**. Signing off 10PM

Update 2.21.17

Spent time editing my new Section G.5 within "Euler angle writeup.doc", it gets better with each edit. I think it would be ready to install.

Somewhere, however, I want to say WHY these Euler angles are useful in a real world problem, and that means I want to do something like a top.

Meanwhile, while proofing G.5 I decided that my Basis Theorem is just not clean because the notation I write as Re'_n lacks meaning in a complicated environment. So I did YET ANOTHER rewrite of Section 1.1 and I now want to see if it has the power needed to make G.5 good.

Very Bad News. Although I spent another whole day trying to fine-tune my notation, when I apply it to concatenated transformations, things simply do not work! So at least another week is going to get burned trying to make this go. I am back to my Paradox but this time it is worse than before!

Update 2.22.17

I bailed it out and have a new cut for section 1.1 which supports everything I need. I will proof it now, and then do the same cross ref thing I did before.

Proofing Section 1.1

start until Notation Problem	text all OK, eq num all OK
The Notation Problem	text OK, no numbered eq
Dirac Notation and the Basis Theorem	text OK, eq OK thru 1.1.22
The Basis Theorem	OK eq thru 30
Alternate shorthand notations	OK eq thru 32
The matrices R and R'	OK thru 38
Time dependence of the R_{ij}	no eq
Dealing with Multiple Concatenated Transformations	OK and eq OK

All right! I hope Section 1.1 will stay put now. Next comes the Big Xref:

1. Scan entire doc for all (1.1. refs and update them, done
2. Same scan for App G v2. done
3. Same scan for Euler writeup done

I made no changes to section 1.2 and onward, so I think these are the only xref's I need worry about.

BUT, let's read through 1.2 to see if I replicated something. done. 1.2 is OK
Do 1.3 as well: OK

Next, want to nail down Section G.5 in the "writeup" and then get it installed.

Proof App G.5 in separate doc.
OK through QED (3). I added a paragraph of notation comment with ref to Section 1.1.
all done.

I am now going to install this G.5 into App G v2: done.

Next, I read old docs to make sure I have not done something wrong, and right off the bat I see that I have not clearly explained how to

Update 2.23.17

This AM I got unhappy again with Section 1, with a paradox involving having both $\hat{\mathbf{x}}' = R^{-1}\hat{\mathbf{x}}$ (basis vector) and $\hat{\mathbf{x}}' = R\hat{\mathbf{x}}$ (normal vector) at the same time. Thus Paradox has been lurking for a long time and recurred last night at bedtime. I have resolved this paradox by defining two classes of vectors which are Apparatus Vectors and Basis Vectors. This directly relates to the Active / Passive view business.

I do mention Active/Passive in Section 3.1 but I think it is too late there and the discussion is not quite right either.

I created a new paragraph or two in Section 1 critque about Actibve/Passive.Basis/Apparatos. But then I come fo Fig 1 and it all seems to fall apart.

At least I think I have resolved my little paradox with $\hat{\mathbf{x}}' = R^{-1}\hat{\mathbf{x}}$ and $\hat{\mathbf{x}}' = R\hat{\mathbf{x}}$ contradicting each other. And I have it written up. But connecting this discussion to Fig 1 is unclear. I started today at 4:45AM and it is now 4PM and I am just done for this day.

For the Cartesian spaces we are using for Frame S and Frame S', the tensor is very simple, but for a non-Cartesian space like that of $(x'_1,x'_2,x'_3) = (r,\theta,\varphi)$ of spherical coordinates, it is more complicated as shown in (E.6.8). In that case $\mathbf{e}_n \bullet \mathbf{e}_m = \delta_{nm}$ but $\mathbf{e}'_n \bullet \mathbf{e}'_m = \bar{g}'_{nm}$.

I am now going to cut out and omit a chunk of text which I think adds nothing to frames doc. It is in Section 1.3. The first part has been replaced, the last part is not useful to the reader. I don't think the idea of covariance is very useful at this point either so happy to remove that.

I have already shown (1.3.4) for a Special Case #4 Kinematic Vector, so I blue some out below.

We shall now examine the type of relationship between **a**' and **a** in which $(a')_i = (a)'_i$ and therefore we can use the notation a'_i without ambiguity. First, the components $(a)'_i$ and $(a)_i$ are related in the following simple manner, using (1.1.8),

$$(a)'_n = \mathbf{e}'_n \bullet \mathbf{a} = (\mathbf{e}'_n)_m (a)_m = R_{nm}(a)_m \,. \qquad \text{ok} \tag{1.3.1}$$

Now suppose we *define* a new vector **a**' in this way,

$$\mathbf{a}' \equiv R\mathbf{a} \quad . \qquad \text{active rotation of } \mathbf{a} \text{ gives a new vector } \mathbf{a}' \tag{1.3.2}$$

If a vector **a** transforms into **a**' according to (1.3.2), we say it is a "vector under rotations" which means it "transforms as a vector under rotations".

When written in Frame S components this says

$$(a')_n = R_{nm}(a)_m \quad . \tag{1.3.3}$$

Comparison of (1.3.1) and (1.3.3) shows that

$$(a)'_n = (a')_n \tag{1.3.4}$$

and therefore in this case we can use

$$a'_n \equiv (a)'_n = (a')_n \quad . \tag{1.3.5}$$

Thus, if the vectors **a** and **a**' are related by $\mathbf{a}' = R\mathbf{a}$ where R is the rotation appearing in $\mathbf{e}_n = R\,\mathbf{e}'_n$, then we can dispense with the parentheses as shown in (1.3.4). We still have $(a')'_i$ which requires parentheses.

Example 1: Consider the equation in the Basis Theorem (1.1.30),

$$\mathbf{e}'_n = R^{-1}\,\mathbf{e}_n \quad .$$

Since this is *not* of the form $\mathbf{a}' \equiv R\mathbf{a}$, we may not dispense with the parentheses. In fact from (1.1.8) we have

$$\begin{aligned} (\mathbf{e}_n)'_i &= R_{in} \\ (\mathbf{e}'_n)_i &= R_{ni} \end{aligned} \tag{1.3.6}$$

and these are not the same because rotation matrices are not symmetric. Unlike normal vectors for which one writes the transform $\mathbf{a}' = R\mathbf{a}$, the basis vectors are "back-rotated" as noted earlier so $\mathbf{e}'_n = (R^{-1})\mathbf{e}_n$.

Active and Passive

One can think of $\mathbf{a}' = R\mathbf{a}$ as an **active** rotation of vector **a** into another vector **a'** within Frame S. In this case, the components of **a'** are $(a')_i$. The alternative is to think of vector **a** as not moving at all in Frame S, but the basis vectors are back-rotated from $\mathbf{e}_n$ to $\mathbf{e}'_n$ taking us to Frame S'. In this back-rotated basis the components of **a** are $(a)'_i$. This is the **passive** view of a rotation and is in fact the view we take in most of this document because we want to *observe* activities in Frame S from Frame S' and vice versa. Each view has its usefulness and we have just shown that if $\mathbf{a}' = R\mathbf{a}$, then $(a)'_n = (a')_n$.

In the active view, the "apparatus" is rotated and the axes stay put, while in the passive view the apparatus stays put but the axes are back-rotated. There is a third view in which the apparatus and the axes are both rotated in the same direction, and this view is useful in the discussion of covariance of equations (e.g. Lucht *Tensor* ***). One can regard the three views as three "experiments" one might perform.

Example 2: Soon we shall be dealing with the Fig 1 equation $\mathbf{r}' = \mathbf{r} - \mathbf{b}$. Since this is *not* of the form $\mathbf{r}' \equiv R\mathbf{r}$, we may *not* dispense with the parentheses, and we expect that $(r')_i$ and $(r)'_i$ will be different.

Footnote: More generally, if R is the linearized version of some general transformation $\mathbf{x}' = \mathbf{F}(\mathbf{x})$ at a point **x**, so that $d\mathbf{x}' = R(\mathbf{x})\, d\mathbf{x}$, then (1.3.2) that $\mathbf{a}' = R\mathbf{a}$ says that **a** "transforms as a vector with respect to the underlying transformation **F** ". In general $R(\mathbf{x})$ is a combination of rotation and stretch and is a function of location. In our current document we deal only with $R(x) = R$ = a rotation that is the same at all points in space. It turns out however that the notation a'_i is unambiguous in the general case as well as we shall now show. Lucht *Tensor* uses a different notation for basis vectors, and to make the connection between our current document and *Tensor* one must take

$$\{\mathbf{e}_n, \mathbf{e}'_n\} \quad \rightarrow \quad \{\mathbf{u}_n, \mathbf{e}_n\} \qquad\qquad \text{Frame S} = \{\mathbf{e}_n\} \rightarrow \text{Frame S} = \{\mathbf{u}_n\}$$
$$\mathbf{e}_n = R\,\mathbf{e}'_n \quad \rightarrow \quad \mathbf{u}_n = R\mathbf{e}_i \qquad\qquad \text{Frame S'} = \{\mathbf{e}'_n\} \rightarrow \text{Frame S'} = \{\mathbf{e}_n\} \ .$$

Then in the language of *Tensor* where • is the "covariant dot product", one has (implied summations),

$$(a')_n = \mathbf{a}' \bullet \mathbf{u}_n = (R\mathbf{a}) \bullet (R\mathbf{e}_n) = (R\mathbf{a})_i (R\mathbf{e}_n)^i = R_i{}^j\ (a)_j\ R^i{}_k\ (\mathbf{e}_n)^k = (R_i{}^j R^i{}_k)\,(a)_j (\mathbf{e}_n)^k$$

$$= \delta^j{}_k (a)_j (\mathbf{e}_n)^k = (a)_j (\mathbf{e}_n)^j = \mathbf{a} \bullet \mathbf{e}_n = (a)'_n \text{ of this document.}$$

In this general case, within Frame S the $\mathbf{u}_n$ are still axis-aligned basis vectors but the $\mathbf{e}_n$ are generally not axis-aligned and are generally not unit vectors. For example, in spherical coordinates $\mathbf{e}_1 = \hat{\mathbf{r}}$, $\mathbf{e}_2 = r\,\hat{\boldsymbol{\theta}}$ and $\mathbf{e}_3 = r\sin\theta\,\hat{\boldsymbol{\varphi}}$ as shown in (E.6.8) below.

Basically Section 1.3 is completely gone, so I will have to jimmy things a bit.

OK, I am going to save off the old Sections 1.2 and 1,3 and replace them with completely new sections. Section 1.1 says as is with one added comment where it refers to **V**' = R**V**. All DONE!

I redid eq nums in Sections 1.2 and 1.3, I will have tomorrow to do an xref adjust on these guys,
****** DONE!

Today was yet another Sisyphus bailout of frames doc! It happens every day now, I am getting used to it. I hope I have immunized myself against the paradox which let do today's rewrites. I included the paradox resolution right in the doc where I can always find it any time. This was initialized when I tried to understand the meaning of Goldstein's Euler picture and something went very wrong with $\hat{\mathbf{x}}$ and $\hat{\mathbf{x}}'$ in some way I need to update tomorrow, hopefully having fixed that problem! With my luck, it won't be fixed! Manana! ********* FIXED!

Update 2.24.17

Clip removed from G.5 since R = R' not too helpful :

But in Frame S' coordinates, one has $(\hat{\boldsymbol{\xi}})'_i = [R'_z(\varphi)]_{ij}(\hat{\mathbf{x}})'_i$ where $R'_z(\varphi)$ is a different matrix (see the discussion in Section 1.1).

Task done: I have finished off Section G.5 about the Goldstein Euler angles. My two examples at the end finally work correctly after all the repairs made yesterday on Section 1, so there is a payoff. I moved the ω calculation to section G.6 since G.5 is so long.

Task done: finished new section G.6 on the computation of ω in the two bases. My intention is that G.7 will be a repeat of this computation using my other method.

XREFs:
did all (1.1.
did all (1.2.
did all (1.3.

OK, by 7:30 PM I have all of Appendix G in place (9 sections!) and only one section is in process, and that is G.8 about the rigid body motion equations of Goldstein. I put in the generalizations, I put in the second ω derivation, I had no major setbacks today, for the first time in many days. I have been reviewing docs and will continue with that tomorrow. It was a good day.

Update 2.26.17

I created a list of "hard questions" and I think answered them all satisfactorily, but it took much of today to do it. I have the rigid body think figured out finally. I have gotten more interested in the vector notation (**V**)' = R**V** so I am going to reread section 1.1 now (it has cooled for a few days). I have done a lot with the Active Passive stuff and want to see if new stuff should be added.

Section 1.1

ok up to Dirac Notation start
ok up to start of the Basis Theorem, made one change (**V**)' = R**V** with ref promised!
ok up to The matrices R and R'
ok up to Other Dirac Facts
ok up to concatenated
ok through the end

Well, this is not where the tricky stuff happens. Nothing much changed on this pass!
Tomorrow I will continue and read Section 1.2 which is more critical (I am tired).

Update 2.27.17

I went through Section 1.2 which is very brief and am now editing Section 1.3 . I think I can remove this text from the start,

In this discussion, Frame S and Frame S' have the same origin, which we shall later call **Special Case #4**. This is a more restricted situation than that illustrated in Fig 1 where **r** is a particle in an Apparatus but the Frames have different origins separated by vector **b**.

because "when is a vector a vector" allows the general case not just Special Case #4.

My rewrite of Section 1.3 is very severe and I now include the Active/Passive pictures. I think it now says all the right things. I have had to redo the eq nums of section 1.3 so xref needed ********** . // I have now gone through Section 1.3 about 4 times and think it is stable.

Section 1.4 I just read and seems OK, not much dependent on previous stuff.

Reading Section 1.5 now. I will cut this from the end of Section 1.5 since it appears later in G.9

Footnote: The equation $R(\boldsymbol{\varphi}) = \exp(-i\,\boldsymbol{\varphi} \bullet \mathbf{J})$ can be interpreted as a rotation in N dimensions with a set of three appropriate NxN generator matrices $(J_k)_{ij}$. Only when N = 3 is (1.5.2) valid or even meaningful. In group theoretic language, these NxN matrices J_k form an N-dimensional irreducible representation of the Lie Algebra so(N) which is $[J_a, J_b] = i\varepsilon_{abc}J_c$. When **J** is exponentiated as shown in (1.5.5), the rotations $R(\boldsymbol{\varphi})$ are then elements of the Lie Group known as SO(N). The meaning is Special (det = +1), Orthogonal (as in $R^TR = 1$), and N dimensions. The vector $\boldsymbol{\varphi}$ then has N components. For N = 2, the generators are $J_k = \sigma_k/2$ where σ_k are the so-called 2x2 Pauli matrices associated with "spin 1/2". See Appendix G.* **********

Section 1.6 is OK
Section 1.7 is OK
Section 1.8 is OK.
Section 1.9 I skip, OK on ang mom intro.
Section 1.10 is OK
Section 1.11 is OK.

Section 2 is just fine, I like it! Proof of the G rule is good and simple.

Section 3. Observer in Frame S' . Nothing conflicts to changes recently made in Section 1.1-3,

Section 4.

Section 4.1 is OK, no conflicts, added a Basis Theorem comment.
Section 4.2 is OK.
Section 4.3 is OK.
OK I think I have scanned enough to make sure my Section 1.1,2,3 changes don't affect any later comments.

Let's do the xref update noted above since I changed eq num in Section 1.3 (only).

Search for all ext 1.3. : DONE, only 1.3.5→1.3.10

Suspicious stuff near (8.8.44). I am talking about the Earth, and I move its rotation axis from $\hat{\mathbf{z}}$ to a new axis

$$\hat{\mathbf{z}}' = R_z(\varphi_1)\, R_y(\theta_1)\, \hat{\mathbf{z}} \equiv R_1\, \hat{\mathbf{z}} = \begin{pmatrix} \cos\theta_1\cos\varphi_1 & -\sin\theta_1 & \sin\theta_1\cos\varphi_1 \\ \cos\theta_1\sin\varphi_1 & \cos\varphi_1 & \sin\theta_1\sin\varphi_1 \\ -\sin\theta_1 & 0 & \cos\theta_1 \end{pmatrix} \hat{\mathbf{z}}$$

This is exactly in the mold of the Basis Theorem. What then happens to a Kinematic Vector? It must be

$$\mathbf{V}' = R_1^{-1}\mathbf{V}$$

This then must apply to r, so I should write

$$\mathbf{r}' = R_1^{-1}\mathbf{r}$$

So I have a screw-up in this section!!! ????

Am I defining a Frame S' here? I want to view the tides from Frame S' I think, so yes, I am in effect defining a Frame S'. This is JUST THE KIND OF THING I am trying to get right with all my work. Separate doc please, Question

7:30 PM. I am trying to rewrite a chunk of Section 8 regarding those tide plots. Something is wrong because I think plots for mirror image latitudes should simply be time shifted, but that is not what the plots are showing. Something is wrong.

I have $\cos\theta = \cos\theta_1 \sin\theta'_L - \sin\theta_1\cos\theta'_L\cos(\omega t)$ and

$$h(t) = a\,[\; 2\,(\cos\theta_1 \sin\theta'_L - \sin\theta_1\cos\theta'_L\cos(\omega t))^2 - 1\;]$$

If you negate θ'_L you negate only the first term of cosθ, and this gets compensated by a time shift which would negate the second term. Therefore the h(t) plots for +60 and -60 latitude should have the same shape but are just horizontally time shifted. But this is not what Maple shows!

Wrong, it is exactly what Maple is showing.

OK, I have a rewrite of this part of Section 8 all done. Tomorrow I will proof it then install it. I had things very wrong before, by the way, due to my saying that $\hat{\mathbf{z}}' = R_z(\varphi_1)R_x(\theta_1)\,\hat{\mathbf{z}} \Rightarrow \mathbf{r}' = R_z(\varphi_1)R_x(\theta_1)\,\mathbf{r}$. It was exactly to clean up this type of stupid error that I did the Section 1. 1 rewrite in recent days. I just stumbled onto this Section 8 error today because I wanted to check on the matrices there.

Update 2.28.17

I have proofed and improved the new Section 8 segment in its own doc, and will now save out the old segment and install the new one. Then I need to do some xref checks.

XREF: I want to examine xrefs to these equation numbers:

(8.8.44) through (8.8.56).

I checked each one by searching eg for (8.8.44 in case a and b. There are no xrefs at ALL to these equation numbers outside of that section I fixed.

OK what's next?? Oops, I need to replace fig 8.8.60! Done.

But then I realized that my green and blue cone pictures will mean nothing to the reader because I have not explained them. This resulted in yet another rewrite of the section starting at (8.8.58). // Did this in a separate doc, and will proof after a break and will then install. // Well, I finished this and installed.

I looked but could not find clean data on the rotation axis of the moon and its precession period. Hard to find anything on this subject. Lots or Russian selenocentric Cassini coordinate system fancy stuff. The reflectors on the moon gave people something to do in this regard (moon coordinate systems of high precision, doing astronomy from the moon surface etc).

OK, what's next? I need to clean up Appendix G.5 about Euler angles.

I have done this, and had a little surprise. Found easier way to transform vectors. Now all my linear combination formulas are just for academic interest. I show lots of things on both sides of the basis theorem. It was the examples at the end of this Section G.5 that caused my many digressions because I was not sure of the simple process of computing Frame S' components of a Kinematic vector. I did this for position vector **r**, and of course you can use that same formula for $\hat{\mathbf{x}}$ as a special case, even though $\hat{\mathbf{x}}$ is not a kinematic vector. My Moon footnote gives some application of Euler angles to the reader. I added the r arrow to the picture, to show that it applies to any vector V, not just to basis vectors as are drawn there.

The drawing basically shows a bunch a sets of axes or coordinate systems. I am the one who added the unit vector structure, and the Kinematic Transformation structure. I felt Euler angles was a complicated subject so I said a lot about them.

Next I would like to get the rigid body stuff written up before I forget it all, and then I need to review more of my "paradox docs" to make sure things are stable.

Update 3/1/17.

I want to **"scan" (not read)** the entire doc, looking for abuses of the (**V**') = R**V** idea or other basis theorem refs without ref to right place, other places where components are computed, etc etc. In this review I will skip sections 1,2,3 since they have been heavily reviewed and rewritten in last few days. I am looking for stale old stuff. Want to see if Special Case #4 is properly referred to and things not over-restricted.

1.4 ok
1.5 matrix stuff, but not in ref to frames, all-vector equations are OK
1.6 ok
1.7 ok
1.8 all vector equations and definition of natural, bolded word natural, key idea
1.9 L, all vector eq only
1.10 OK
1.11 lots of component notation properly done with primes. This involves Frame change from S to S', no transforms explicit. Uses $(\mathbf{e}_i)'_j = R_{ji}$ which is in facts box and is correct. This is all components so no problems or issues. All OK

2. G rule all OK.

3. Observer all OK, all vector equations

4. how frames S and S' are related
4.1 summary at end could include (**V**)' = R**V** but will not add.
4.2 redraw Fig 1, OK
4.3 about b, all vector eq. Could write (**V**)' = R**V** as $\mathbf{V}_{S'} = R\mathbf{V}_S$ but too heavy labeling.
4.4 Special Case #1 all vec eq
4.5 Special Case #2 all vec eq
4.6 turntable, describes how basis vectors are placed, rotor pic
4.7 the earth, describes how basis vectors are placed
4.8 flying camera OK

5. Goals ok

6. Velox all vec eq until 6.6
6.6 reminder can eval any eq in either frame OK, state $\mathbf{r}' \neq R\mathbf{r}$, $\mathbf{v}' \neq R\mathbf{v}$ very good! $\mathbf{b}' = R\mathbf{b}$, $\boldsymbol{\omega}' = R\boldsymbol{\omega}$ **ok**

7 Accel all vec eq. all OK

8. Fict force
8.1 fict force all vec eq

8.2 interp centrifugal and Euler, all vec eq
8.3 interp coriolis, all vec eq
8.4 SC #1 probs, all vec eq
8.5 surface of Earth probs, swap and no swap, all vec eq,OK
8.6 no xforms, all OK
8.7 Special Case #3, all vec eq, frypan
8.8 Tides. Lots of unit vec use.
(8.8.43) Here we have Basis Theorem type action first time since Section 1!! This was all rewritten yesterday and had errors! Pix bigger and clearer and both hemispheres.
All rest of sec 8 is OK

9. Marion: comparing vector equations only

10. Goldstein. I define him as Special Case #4 because he has **r** = **r'**. All OK here.

11. Ang mom and fict torques
11.1 torque basis, all OK
11.2 all vec eq, L and Ldot in diff frameds
11.3 cleaned up a blued comment and ref to inertia. All vec eq here. OK
11.4 fluid Lai stuff, all vec eq
11.5 more fluid, vec eq
11.6 Reynolds, way out there, no component anything here

12 summary forward problem
12.1 Summary Forward non-swap all vec eq with SC #4
12.2. Summary Forward swap all vec eq with SC #4

13 summary inverse problem
13.1 all vec eq
13.2 swap rules method, all vec eq
13.3 Summary Inverse non-swap all vec eq with SC #4
13.4 Summary Inverse swap all vec eq with SC #4
13.5 why swap rules work

14 Curvilinear coords, pay attention here!!! Basis Theorem referred to.

Hold the phone!! I write

$\hat{\boldsymbol{e}}_n = [R(\xi)\,]^{-1}{}_{nm}\,\mathbf{e}_m$ // compare to $\mathbf{e'}_n = R_{nm}\,\mathbf{e}_m$ lin comb OK

and // compare to $\mathbf{e'}_n = R_{nm}\,\mathbf{e}_m$ lin comb

$$\begin{pmatrix}\hat{\boldsymbol{e}}_1\\ \hat{\boldsymbol{e}}_2\\ \hat{\boldsymbol{e}}_3\end{pmatrix} = \begin{pmatrix}\hat{\mathbf{r}}\\ \hat{\boldsymbol{\theta}}\\ \hat{\boldsymbol{\varphi}}\end{pmatrix} = \begin{pmatrix}\cos\varphi\sin\theta & \sin\varphi\sin\theta & \cos\theta\\ \cos\varphi\cos\theta & \sin\varphi\cos\theta & -\sin\theta\\ -\sin\varphi & \cos\varphi & 0\end{pmatrix}\begin{pmatrix}\hat{\mathbf{x}}\\ \hat{\mathbf{y}}\\ \hat{\mathbf{z}}\end{pmatrix} = [R(\xi)\,]^{-1}\begin{pmatrix}\hat{\mathbf{x}}\\ \hat{\mathbf{y}}\\ \hat{\mathbf{z}}\end{pmatrix} = [R(\xi)\,]^{-1}\begin{pmatrix}\mathbf{e}_1\\ \mathbf{e}_2\\ \mathbf{e}_3\end{pmatrix} \qquad (14.6)$$

So far OK, but in Alt short not in (1.1.31) I write

$$\mathbf{e}_n = R\mathbf{e}'_n \quad \Leftrightarrow \quad (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3) = R\,(\mathbf{e}'_1, \mathbf{e}'_2, \mathbf{e}'_3) = \mathscr{R}\,(\mathbf{e}'_1, \mathbf{e}'_2, \mathbf{e}'_3) \tag{1.1.31}$$

$$\mathbf{e}'_n = R_{nm}\,\mathbf{e}_m \quad \Leftrightarrow \quad \begin{pmatrix} \mathbf{e}'_1 \\ \mathbf{e}'_2 \\ \mathbf{e}'_3 \end{pmatrix} = R \begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{pmatrix}. \tag{1.1.32}$$

I have a connection R = $[R(\xi)\,]^{-1}$ which seems very bad. Something is wrong here!!

Yes, it was another screw-up. I have rewritten Section 14 AND Appendix A (was not too hard) so that basically the change made was $[R(\xi)\,]^{-1} \leftrightarrow R(\xi)$ so we don't have a notation mismatch!

Now let's must scan for R(ξ) to see xrefs to it possibly from elsewhere in frames doc: // nothing in the main doc, check App G just to make sure. All OK,

All right, archive old Section 14 and old Appendix A and install the new!!! All done.

That was a good find! I now continue my scan:

14 Curvilinear coords, pay attention here!!! Did above repairs, all is good now, OK

15. Ant trails. Opening section. Pay Attention!!!!

Trouble in section spanning (15.5) through (15.9) only, rest of opening text section is OK. Here I am applying Section 14 ideas, but I have altered those ideas just now. // Repairs done and installed.

15.1 Primes on vector eq here are natural primes. OK
15.2 Notation like $(v')_x$ and $(a')_y$ for naturals seems OK
15.3 And flies over. This time see $(\mathbf{v}')'_1 = v'_x$, notation, OK
15.4 OK

Now keep scanning in appendices, I continue to look for $(\mathbf{V}') = R\mathbf{V}$ idea or basis theorem stuff.

Appendix A: just rewrite this, it is OK

Appendix B: See $\mathbf{A} = \Sigma_i A_i \mathbf{e}_i = \Sigma_i A'_i \mathbf{e}'_i$ type stuff, all OK. Really = $\Sigma_i(A)\,'_i\mathbf{e}'_i$ and added comment to the effect. I think all is OK here.

Appendix C: Foucault

App C opening text OK
C.1 OK
C.2 OK

C.3 vector equations then at end get components on $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ etc, OK
C.4 plane pend all OK
C.5 really no basis vector stuff anywhere here
C.6 all Frame S Cartesian components
C.7 all OK, verifications
C.8 all OK

My scanned for issues are far from the world of Appendix C!

Appendix D center of gravity all OK

Appendix E: all OK

Appendix F: all OK

Appendix G:
G.5 has basis theorem stuff.
all done

This concludes the scan. Very little of frames doc deals directly with the things I just scanned for, I just wanted to make sure nothing was out of date and I in fact did find things out of date and I have fixed them above.

NEXT, I want to finish reviewing the various "paradox docs" to make sure nothing has been left hanging.

Euler Angle Paradox1 doc. This one has been sitting around for a long time now.

Update 3/2/17.

OB 6 AM, first session was to try and break the ω calculation paradox, but it would not budge one iota after several hours.

It then occurred to me that R is not a tensor, and there is no object R', so I am saying misleading things in Section 1.1 on this topic, so maybe I should go fix that and forget about The ω Paradox for now. This then might have some bearing on the Paradox. As things stand, I have bad stuff in Section 1.1.

I think this problem is isolated to Section 1.1 only. The only other appearance of it is in G.7 where I have the paradox. how can I scan for other possible references to R' ?

Ouch! If I just search for R' it has infected Section 1.3 as well, and it is heavily involved in the triple and double concatenation work at the end of Section 1.1. It is also in G.7 only in G.

BUT, as I set out to <remove R' > I find that it simply *cannot be done*. Once you say $\mathcal{R}|\mathbf{e'}_n\rangle = |\mathbf{e}_n\rangle$ then your are hooked. Nobody can stop you from considering

$$R_{ij} = \langle \mathbf{e}_i \,|\, \mathcal{R} \,|\, \mathbf{e}_j \rangle$$

$$R'_{ij} = \langle \mathbf{e}'_i \,|\, \mathcal{R} \,|\, \mathbf{e}'_j \rangle \ . \qquad (1.1.34)$$

I am never saying "R is a tensor". but I am saying $\mathcal{R}$ is an operator in the Hilbert Space.

So I guess I will leave it in there.

It is now 3PM. I did some more work and got some promising results, but things are really as confused as ever. It will take me a solid week to clean this up, I can tell. And then I have a week probably on the rigid body stuff.

SO: I have to give up on getting this done before Tax Season. Probably that will get into the Blair trip and then into the Cod trip. I have therefore failed to bring Frames Doc under control in time for a release pre Cod. I guess I am not surprised. When you keep adding more stuff, it keeps adding more time. I am probably already 3-4 months on this frames doc "update" and it will probably consider another 3 months, so let's Face the Music. Time to start on Tax Season. Do a good backup!

Update 3/9/17.

It is 2PM on 3.9.17 and I am ready after a week's break to again attack the problem of the ω computation by my Method 2. I must admit, I don't really understand it.

Update 3/11/17.

I describe $(\mathbf{V})' = R\mathbf{V}$ as the passive view when the name $\mathbf{V}'$ is otherwise used up.

Question: How does this work with tensors?

Answer: The notation $(\mathbf{V})' = R\mathbf{V}$ means $(V)'_i = R_{ij}V_j$. In Dirac it says

$$\langle \mathbf{e}'_i | \mathbf{V} \rangle = \langle \mathbf{e}'_i | e_j \rangle\langle e_j | \mathbf{V} \rangle = R_{ij}V_j$$

For a rank-2 tensor you would say

$$(T)'_{ij} = \langle \mathbf{e}'_i | T | \mathbf{e}'_j \rangle = \langle \mathbf{e}'_i | e_n \rangle\langle e_n | T | e_m \rangle\langle e_m | \mathbf{e}'_j \rangle = R_{in}(T)_{nm}R_{jm} = [RTR^T]_{ij}$$

so in passive world you would write this as

$$(T)' = RTR^T$$

Question: Consider

$$A \equiv -[\,(\dot{\theta}\cos\psi + \dot{\varphi}\sin\theta\sin\psi)(iJ_1) + (\dot{\theta}\sin\psi - \dot{\varphi}\sin\theta\cos\psi)(iJ_2) + (\dot{\psi} + \dot{\varphi}\cos\theta)(iJ_3)\,]$$

Is this a rank-2 tensor? In paradox2 v2.doc I convinced myself that

$(J_i)'_{mn} = (J_i)_{mn}$ or $(J_i)' = (J_i)$

If J_i were a rank-2 tensor, we would have instead $(J_i)' = R(J_i)R^T \neq (J_i)$. This then argues that the object A above is in fact NOT a rank-2 tensor. Thus

$(A)' = RAR^T$ is not valid for object A

However, if the symbol (A') is not already used, we could define

$(A') \equiv RAR^T \neq A$

as some new object. We would do the same for J_i and write

$(J_i') \equiv RJ_iR^T$

Then we have these two *different* objects :

$$A \equiv -[(\dot{\theta}\cos\psi + \dot{\varphi}\sin\theta\sin\psi)(iJ_1) + (\dot{\theta}\sin\psi - \dot{\varphi}\sin\theta\cos\psi)(iJ_2) + (\dot{\psi} + \dot{\varphi}\cos\theta)(iJ_3)]$$

$$(A') \equiv -[(\dot{\theta}\cos\psi + \dot{\varphi}\sin\theta\sin\psi)(iJ'_1) + (\dot{\theta}\sin\psi - \dot{\varphi}\sin\theta\cos\psi)(iJ'_2) + (\dot{\psi} + \dot{\varphi}\cos\theta)(iJ'_3)]$$

and we also have this third object

$(A)' = A$

So I am now confused. Which "rule" defines a rank-2 tensor ?

Rule 1: $(A)' = RAR^T$

Rule 2: $(A') = RAR^T$

In the active world, (A') is a new tensor in Frame S.
In the passive world. (A)' is the same tensor A but the basis is back-rotated so it has different components.

Conclusion: My formalism of Section 1 is unable to make clear statements about rank-2 tensors, so I really need to find out what is wrong and fix it. Only then am I justified in doing things with A_{ij} and making statements about it.

I have made some progress here, see "Section 1 Formalism...".

I am now I think ready to take a new stab at Section G.7. Good luck! // At 2:30 PM I finished this stab at Section G.7. Amazingly I did not run into any problems. I think it is all hanging together.

Proof the new Section G.7 in separate doc, it is 12 pages.

opening text	ok
DETERMINATION OF THE FRAME S' COMPONENTS OF **ω**	
First expression for: $[(d\mathbf{e}_n/dt)_{S'}]'_i$	ok
Second expression for: $[(d\mathbf{e}_n/dt)_{S'}]'_i$	ok
Equate the two expressions for: $[(d\mathbf{e}_n/dt)_{S'}]'_i$	ok
Computation of A and Statement of Final Result	ok
DETERMINATION OF THE FRAME S COMPONENTS OF **ω**	
Plans A,B,C	ok
First expression for: $[(d\mathbf{e}_n/dt)_{S'}]_i$	ok
Second expression for: $[(d\mathbf{e}_n/dt)_{S'}]_i$	ok
Equate the two expressions for: $[(d\mathbf{e}_n/dt)_{S'}]_i$	ok
Computation of B and Statement of Final Result	ok
Review of this Section	ok

It passes muster!!! I just saved the old G.7 and installed this new G.7. Finally!

What remains is

1. Section G.8 on rigid body setup equations.
   Rough notes are located in Appendix G_8 rigid body in App G folder
   See also "Question 2 about L=Iw.doc" in the Hard Questions Folder

2. App H on now this relates to tensor doc.
   See Question 1 v3.doc, perhaps, and nearby docs.

Update 3/12/17

Wrote a first draft of Section G.8 on rigid body motion. It went pretty well and I arrived at the final equations, but there is much more to add. Think I resolved many "issues" along the way and it is all now written down.

Update 3/14/17

Section G.8 draft is improving, but no eq nums there yet. I first want to clean up my statements about dot products in the Passive View.

OK to (1.1.10) on the dot product topic.
OK to (1.1.23) where Comment previews the Passive View.
OK to (1.1.32)
OK to (1.1.39)
OK to the end of Section 1.1. I fixed a few things, it is all OK where dot products appeared.

I have just continued on.
OK thru (1.3.5)

OK through end of 1.3. The dot product stuff assumes Active where stated which I guess is OK.

done with this little due diligence review of Sections 1.1,2,3.

Am confused now about the free precession picture.

Update 3/16/17

I am going to change the definition of Ω so I then be inconsistent with Goldstein BUT consistent with:

- T&W page 449 where they have $$\Omega \equiv \frac{I_3 - I_1}{I_1}\omega_3 \qquad \textbf{(11.131)}$$

and where they also have $$\left.\begin{aligned}\omega_1(t) &= A\cos\Omega t\\ \omega_2(t) &= A\sin\Omega t\end{aligned}\right\} \qquad \textbf{(11.138)}$$ which is the more conventional form for circular motion where x gets the cosine.

- GPS page where they have $$\Omega = \frac{I_3 - I_1}{I_1}\omega_3. \qquad (5.49)$$

$$\omega_1 = A\cos\Omega t.$$

'or ω_2 can be found (5.48):

$$\omega_2 = A\sin\Omega t.$$

This is much better, I will do a global repair right now, starting in G.8 on torque free L-cone

Little problem. Consider

$\omega_1 = \omega\sin(\theta-\alpha)\sin[(L/I'_1)t\,] =$
$\omega_2 = -\omega\sin(\theta-\alpha)\cos[(L/I'_1)t\,]$
$\omega_3 = \omega\cos(\theta-\alpha)$.

Want cosine on top line. Use these facts

$\cos(\pi/2-\psi) = \sin\psi \quad \Rightarrow \quad \cos(\psi-\pi/2) = \sin\psi \quad \psi = (L/I'_1)t$

$\sin(\pi/2-\psi) = \cos\psi \quad \Rightarrow \quad \sin(\psi-\pi/2) = -\cos\psi$

Then $\sin[(L/I'_1)t\,] = \sin\psi = \cos(\psi-\pi/2) = \cos[(L/I'_1)t - \pi/2]$

$-\cos[(L/I'_1)t\,] = -\cos\psi = \sin(\psi-\pi/2) = \sin[(L/I'_1)t - \pi/2]$

Equations are then

$\omega_1 = \omega \sin(\theta-\alpha)\cos[(L/I'_1)t - \pi/2] =$
$\omega_2 = \omega \sin(\theta-\alpha)\sin[(L/I'_1)t - \pi/2]$
$\omega_3 = \omega \cos(\theta-\alpha)$.

Which is standard form

Status 8 PM. I think I should take Section G.8 out of App G and make it be Appendix H, then the TOC can be much better. There is too much going on here for a single G.8 subsection which is way off topic from App G title of rotation matrices and Euler angles. I will do that tomorrow. The Rigid body stuff is coming out pretty good I think.

Update 3/17/17

When I start an Appendix, I never know how much I am going to write. I think it is time for some reorg as folllows

Appendix G: Rotation Matrices and Related Theorems

Appendix H: The Euler Angles

Appendix I : Rigid Body Dynamics

I quickly formed the new Appendix G v3.doc. It was too long and too cluttered with multiple topics. Moved all the Euler angle details into a new container App H.

Let's now try to divide H into a few sections.

The old G.5.* is now H.1.* .
The old G.6.* is now H.4.* .
The old G.7.* is now H.5* .

Tentatively Appendix H is now constructed. Now start on Appendix I on rigid body

G.8.1 thru G.8.10 become I.1.1 through I.1.10

G.8.11 thru G.8.14 become I.2.1 thru I.2.4

G.8.15 becomes I.3.1

G.8.16 thru G.8.18 becomes I.4.1 through I.4.3

and beyond that I did not have any equation numbers yet!

OK, I have now roughed in the newly formed Appendices G,H,I. Lots or proofing needed and of course I is not finished yet.

Proof Appendix G:

G.1 all OK
G.2 all OK, added Hint regarding θ terms vanishing
G.3 all OK, I keep adjusting eq nums when I find off by 1 or whatever
G.4 snag. I think I need to write Tensor Doc Appendix J right now. I have the rough into somewhere. I keep getting confused in my statements regarding Tensor doc made in frames doc.

My new App J on tensor doc connection has material now in the Question 1 docs, so let's try now to assemble App J and then return to the above G proofing. // It is done, but no eq nums yet.

Update 3/14/17

I did due diligence by reviewing my three question 1 docs which relate to App J, and all seems well. Will now add eq nums to App J. // done! Appendix J on *Tensor* connection is now finished!

Now back to **proofing** Appendix G.

G.4 now all OK
G.5 all OK and I like the trace proof.

Next, want to proof the new Appendix H.

H.1 OK, reading all detail on this proofing pass.
H.2 OK and very good.
H.3 OK, I like the exercises, reader sees it happen in detail.
H.4 OK, I get components of **ω** in both Frame S then in Frame S', then verify
H.5 OK, the long way to compute ω in both Frames

Appendix H is very good and works better being smaller and focused on its issues. It really is about the Euler Angles and Computation of ω, these are two very well-defined subjects.

Now onward to Appendix I !

I.1 Appearance of the Inertia Tensor
opening text is good
rest of section is very good, laying out the inertia tensor and related subjects
I.2 OK, EOM before diag.
I.3 A OK
I.4 OK, the equations of motion are ready to roll.
I.5 OK

It took me a LONG time to get through I.5 on the rigid rotor. Added a few things, did a few more checks.

Question: If **L** = I**ω** and of **L** = constant and I = constant, why isn't **ω** constant for the free rotor?

$(L)'_1 = I'_1(\omega)'_1$
$(L)'_2 = I'_1(\omega)'_2$
$(L)'_3 = I'_3(\omega)'_3$

But I don't know that the $(L)'_i$ are constant, so instead do

$L_i = I_{ij}\omega_j$

Can you invert I_{ij} ? If it is diagonalizable, and has 3 non-zero eigenvalues, why not? Use the usual formula for inverting a matrix. Then you get

$\omega_i = (I^{-1})_{ij}L_j$

This proves that **ω** = constant, so how can it precess in Frame S?? Paradox of the Day!

Found confirmation on my $\dot{\varphi} = L/I'_1$ in T&M text

We can rewrite $\dot{\phi}$ by substituting sin α from Equation 11.143 and sin θ from Equation 11.141:

$$\dot{\phi} = \omega \frac{L_2}{I_1\omega}\frac{L}{L_2} = \frac{L}{I_1} \qquad \textbf{(11.148)}$$

That was a missing verification, glad to have it.

Status at 8:30PM. I have inserted eq nums through the top section, Read to start on the final section on oblate Earth torque which I see "needs work".

I am looking for another reference on the torque on an oblate spheroid. Williams gets it from the potential of same, but gives no references. But here is a reference on the potential

http://farside.ph.utexas.edu/teaching/336k/Newtonhtml/node108.html

Thus, the gravitational potential outside a uniform spheroid of total mass M, mean radius a, and ellipticity ϵ, is

$$\Phi(r,\theta) = -\frac{GM}{r} + \frac{2}{5}\frac{GMa^2}{r^3}\epsilon P_2(\cos\theta) + \mathcal{O}(\epsilon^2).$$

Update 3/21/17

Everything is done in terms of content. Need to proof and add overviews for new appendices.

Start with an update of both Visio and Maple indices!

Visio index: major update, went through ALL of everything, this is done!

Maple index: First reviewed everything thru App F, made one small addition.
Finished complete Maple index update through App J! All done!

Appendix I Rigid Body Full review (all 37 pages, soup to nuts).

1.1 The Appearance of the Inertia Tensor
opening text OK
Angular Momentum, the Inertia Tensor, and Kinetic Energy
rest is all just fine, it is in fact very good!

I.2 Rigid Body Equations of Motion : Part 1
found a fuzzy claim and cleared it up at the end of this section, updated eq nums.

I.3 Diagonalization of the Inertia Tensor in Frame S'
OK

I.4 Rigid Body Equations of Motion : Part 2
OK

I.5 Zero-torque motion of an Axisymmetric Rigid Body (Rigid Rotor)
OK

I.6 Rigid Body with External Torque: Spinning Symmetric Top
OK

I.7 Gravitational Torque on an oblate Earth: Precession of the Equinoxes
Resume here Weds.

Update 3/22/17

Spent this entire day on the Poinsot stuff, decided to add it after all. This took LOTS of web study, but I think I have the basics nailed down and written up. I now need to start again on proofing all of appendix I due to shifting of material and changes in eqnum. It is now 43 pages! It was supposed to be just a little thing to "make the connection to" rigid body stuff. BUT I wanted to get this done anyway. Since 8:30PM I will pause for the day.

Update 3/23/17

Did a proofing and edit of the new Section I.5, commenting on not scaling ω into **ρ**. Improved my rolling picture, think it is better now.

So I will now do a full proofing of the 42 page Appendix I.

Did a global eq num check, one cosmetic fix, all is correct.

New opening text added to summarize the section, good.

1.1 The Appearance of the Inertia Tensor
opening text, edited a bit, it is good and OK
rest of section is fine, OK

I.2 Rigid Body Equations of Motion : Part 1
very good, and very careful with notation and meaning of objects.
These details are totally glossed over in most sources.

I.3 Diagonalization of the Inertia Tensor in Frame S'
all is OK here. It is reading along OK, small corrections being made

I.4 Rigid Body Equations of Motion : Part 2
OK, added a new equation for no torque and added G comments on solvability.

I.5 Zero-torque motion of a Rigid Body : Ellipsoids and Poinsot
very good, OK, all done! I think I have said it all here.

I.6 Zero-torque motion of an Axisymmetric Rigid Body (Rigid Rotor)
Finding the Frame S' components of $\boldsymbol{\omega}$ OK
The behavior of the Rigid Rotor in Frame S : solving for the Euler angles OK
The behavior of the Rigid Rotor in Frame S : solving for $\boldsymbol{\omega}$ OK
The Earth as an oblate rigid rotor OK
This is a very detailed section, but I think the eq refs are now all fixed up.
It is a reasonable treatment I think, much more detailed than Goldstein who shows Poinsot but not the cones, and Marion who shows the cones (briefly) but not Poinsot. A long section.

I.7 Rigid Body with External Torque: Spinning Symmetric Top
OK, took me a long time to carefully read all this stuff. Now on page 31 of 44.

I.8 Gravitational Torque on an oblate Earth: Precession of the Equinoxes
OK, reads well, not too long, works in the Pope, now to page 34.

I.9 Derivation of the Oblate Earth torque formula
Sturm Liouville Transforms . OK interesting for reader, not hard, rarely discussed!
Expression for 1/R in Spherical Coordinates Add Ed 3 Jackson refs wherever possible!
Potential of a Source Distribution OK
Calculation of the moment J_2 OK
Calculation of the Torque OK, that was fast. On p 40 of 44.
Description of the perimeter of a slightly oblate or prolate spheroid OK

Calculation of I_3- I_1 OK!

Made it all the way through in one shot. The usual periods added and small cosmetic repairs to reduce the usage of "we".
••• This Appendix I is ready to install!! G and H are also ready because I proofed them just a few days ago and have added nothing to them.

Proof Appendix J. Very good, I like it, of course has meaning only to me.

Loose ends.

1. Add Jackson 3rd ed and add refs to it in App J : DONE
2. Add Levitt DONE.
3. Starting now to update the Overview and Summary sections. Much work needed here. DONE

I am ready to do final assembly! Do spell checks first on the Appendices before adding them.

Installs are done, fixed up the section headers, we are at 402 pages. That is enough! I think previous version was 160 pages.

Next step is to do production work. I was up at 4AM working this AM so will end early at 8 PM today.

Update 3/25/17

Several "issues" have cropped up.

Item 1 is my **unsupported statement** that for the top, one can say $(L)'_3$ = constant. I found a simple and clean way to show this is true, and it is not added into frames doc, done with that one.

Item 2: Where does the **bicycle wheel** fit into things? Seems that it is a top with $\theta = \pi/2$ in the usual experiment. And it does not seem to be precessing. My top equation says

$$\dot{\varphi} = \frac{b - a\cos\theta}{\sin^2\theta}$$

so in this case I would have $\dot{\varphi} = b$, but what is b? Well I know that $p_\varphi = (L)_z = bI'_1$ so the bike wheel can precess around its 90° hooked position at rate

$$\dot{\varphi} = b = (L)_z/I'_1 \qquad (\omega)'_3 = (aI'_1/I'_3)$$

and this is seen in https://www.youtube.com/watch?v=8H98BgRzpOM

I think the catch is that you cannot really achieve 90^0 so the wheel always tips down a bit, so there is always some L_z in the system just from the spinning. OK, I am happy with this.

Item 3: I cannot figure out how to model the mag dipole in a B field.

Update 3/26/17

While putting above stuff on hold, today I want to see how rigid body approach works for the dumbbell satellite. // Wasted a lot of time on this, before I realized that neither Frame of the dumbbell satellite problem is a "body frame" for the dumbbell rotor, so my rigid body equations of motion do not apply to this dumbbell problem. I write four docs in a folder before realizing this now-obvious point.

But in doing so, I may have found out that the rigid body analysis is valid in Special Case #1 and not just the super restricted Special Case #4. This could make me re-edit the Goldstein connection chapter again.

I have made edits to App I to generalize it to Special Case #1 and it looks good up to Section I.7 where I now continue to review things. In Section I.7 on the top I state clearly that I am at that point assuming cosited origins. It they were not cosited, the picture would have to have the Frame S origin translated somewhere out of the picture, say. But then the axes shown would be just unit vectors, but it is confusing because they are labeled with axis names like x and y. Probably the entire analysis would go through OK since it is a "rotational" analysis only.

Conclusion here: leave it exactly as is!

Now take another look at the Goldstein connection chapter. DONE, and I really do think it is Special Case #4 only.

Let's do a global scan to make sure all references to Appendices are to the right Appendix! Done. Only the one in Goldstein section which I already fixed 5 min ago.

I think I am finally ready to add three studies to the rigid body section (somehow)

- electric dipole dumbbell in uniform E field
- electric dipole sphere in uniform E field
- magnetic dipole sphere in uniform B field.

I guess I will just tack these on to the end of appendix I in three new sections.

Update 3/27/17

1. Resolved a paradox with the dipole dumbbell satellite which I thought could only done cone motion, but in fact it is just like the spherical pendulum. I was confusing positional θ,φ with similar angles used do describe the L vector. Took probably 10 hours to figure out that stupidity of notation.

2. A new Paradox has now arisen that I cannot verify $\mathbf{L} = I\boldsymbol{\omega}$ in the simple dumbbell case.

3. This has led to a possible disaster where Goldstein's θ,φ are not the same as my θ,φ. Not sure on this, but somehow our $\hat{\boldsymbol{\theta}}$ unit vectors seem different in how they relate to coordinate φ. Will pursue this next time.

Update 3/28/17

Modified Appendix H to show how Goldstein's Euler angles are related to spherical coordinate angle. This resolves Paradox item 2 noted above. This realization is a major breakthrough for me! I did an edit of the entire Section H in a separate file and will soon install it into frames doc. Using italic φ for Euler angle.

Did a final proofing of new last section of App H with all italic φ, then installed and saved the previous App H.

I now want to review all references to App H and makes sure italic is treated properly! Basically this means a review of App I. // Done, there were two refs.

I will now get back to finishing Section I.10. // Finished it, keep adding new good stuff like a nice cone picture. This is a good new section showing new things. I will proof it tomorrow and then write that final section on the little balls with dipole **p** and mag moment **μ**.

Update 3/29/17

Before moving on with I.10 and beyond, I have more repairs to make regarding φ versus φ. This does not JUST appear when I quote the Frame S ω-equations, it also occurs every time I make use of the Euler R matrix which I see now that I do in several places in Appendix I. Let's review each instance:

Section I.1 no angle φ appears here
Section I.2 no angle φ appears here
Section I.3 no angle φ appears here
Section I.4 no angle φ appears here
Section I.5 no angle φ appears here

Section I.6 yes, angle does appear! In this section first:

<u>The behavior of the Rigid Rotor in Frame S : solving for the Euler angles</u>
So change all φ to φ here as needed.
(I.6.16) has no φ after all, so no change needed.
So done with this subsection.

<u>The behavior of the Rigid Rotor in Frame S : solving for **ω**</u>
Above (I.6.26) big calc involving φ. Result for W involves φ which should be φ.
So in (I.6.26) it is really italic φ, fix just made.
Repair completed, including exercise, nothing significant changes however.
Another repair done just above next section.

The Earth as an oblate rigid rotor
no angle φ appears here

Section I.7 yes, angle does appear! In this section first:
Above (I.7.3) φ appears correctly.
All φ's are OK except maybe at the end of (I.7.20) where last 3 lines need big repairs!
Did these repairs and added eq num refs for each claim, all done.

Section I.8 no angle φ appears here
Section I.9 no angle φ appears here

This concludes my review of Appendix I concerning the use of φ versus φ.

Update 3/30/17

Added section at the end of the symmetric top which comments on the difficulty of the problem, the 4 volume set, and how things don't simplify much for spherical top. Also added two zero-gravity video links, one for the tumbling handle and one for the tumbling book.

I am now ready to get back tot he Section I.10 and whatever follows it. // I have gone through I.10 and am about to add the new stuff.

Update 3/31/17

Reviewed my MRI notes and then added a reasonable ending section which explains how the image is formed, something I know every reader must wonder about. Did many edit passes on this section, think it is now OK.

I will now proof the entire Section I.10 after which I intend to install it!
I.10 all done, did a few edits.
I.11 all done, a few more edits.

I am now ready to install! Done.

Decided to correlate the oblate stuff with GPS page 225 and found a bug! I am missing a mass somewhere in my potential (I.9.31). My result is wrong on dimensional grounds! This will require a back thread to fix all related errors. Resume here tomorrow !!!

Update 4/1/17

Fixed the bug, no problem. Then realized that GPS actually compute the 81,000 year precession rate. I tried to find some fast way to reach their result, but it hinged on very messy trig and an "average" potential. I decided to give it up, but I did add helpful comments to people who might want to follow that GPS pathway. I also greatly improved the drawing that goes with this subject.

So now at 8:30 PM I am once again "free" of any urges to add more stuff, clarifications, extra details, and so on. Want to get this baby launched!!! Did no other proofing today, too bad.

Update 4/2/17

Reviewed docs concerning the GPS work on oblate precession, fixed various errors.
Edited a small part of this in Section I.
Updated both the Visio Index and the Maple Index.

I am now going to proof Appendix I again since so much has been happening there. Appendix I by the way is now 417-355 = 62 pages

Appendix I

opening text OK small edits done

I.1 The Appearance of the Inertia Tensor
opening text OK
Angular Momentum, the Inertia Tensor, and Kinetic Energy OK, tiny edits

I.2 Rigid Body Equations of Motion : Part 1 OK he likes it

I.3 Diagonalization of the Inertia Tensor in Frame S' OK

removed this garbage comment (Remember that the Frame S' inertia tensor (I') is identically zero.)

I.4 Rigid Body Equations of Motion : Part 2 OK

I.5 Zero-torque motion of a Rigid Body : Ellipsoids and Poinsot OK A++

I.6 Zero-torque motion of an Axisymmetric Rigid Body (Rigid Rotor)
Finding the Frame S' components of **ω** **OK**
The behavior of the Rigid Rotor in Frame S : solving for the Euler angles **OK**
The behavior of the Rigid Rotor in Frame S : solving for **ω** **OK**
The Earth as an oblate rigid rotor **OK**

Taking a run break

I.7 Rigid Body with External Torque: Spinning Symmetric Top OK

I.8 Gravitational Torque on an oblate Earth: Precession of the Equinoxes OK

I.9 Derivation of the Oblate Earth torque formula
Sturm Liouville Transforms OK
Expression for 1/R in Spherical Coordinates OK
Potential of a Source Distribution OK

Calculation of the moment J_2 OK
Calculation of the Torque OK
Description of the perimeter of a slightly oblate or prolate spheroid OK
Reader Exercises: OK

I.10 Motion of an electric dipole dumbbell in a uniform E field
opening text OK
Force and Torque OK
Newton's Angular Law and Equations of Motion OK
Conical motion solution OK
Angular Velocity **ω** OK
Inertia tensor I OK
Angular momentum **L** OK

I.11 Rotors involving electric or magnetic dipoles
1. ROTOR WITH FIXED EMBEDDED ELECTRIC DIPOLE IN A UNIFORM E FIELD OK
2. ROTOR WITH FIXED EMBEDDED MAGNETIC DIPOLE IN A UNIFORM B FIELD OK
3. THE CHARGED ROTOR AND LARMOR PRECESSION
opening text OK
Enter Quantum Mechanics OK
MRI Machines (Magnetic Resonance Imaging) OK

And that concludes this full pass (in a single day) through the 62-page Appendix I. Lots of small edits were made.

Since it is small, I will just do Appendix J right here (and since it is relatively new).

Appendix J : Connection with *Tensor Analysis and Curvilinear Coordinates*
opening text OK
Example 1: The Transformation from Cartesian to Polar Coordinates OK
Example 2: Lorentz Transformations OK
Example 3: Rotations OK

References

Read through all of it, fixed some minor details.

Read the entire overview and summary. It seems OK. That is it for today!

Update 4/3/17

First off I will proof App G and H on Euler angles since things were shuffled around in that area.

Appendix G: Rotation Matrices and Related Theorems

G.1 Generators and finite rotation matrices **OK**

G.2 About the general rotation matrix

An explicit expression for the general rotation matrix OK

Finding **n** and θ OK

G.3 The Baker-Campbell-Hausdorff and Sandwich Formulas

Statement and proof of the Baker-Campbell-Hausdorff (BCH) formula OK

Statement and proof of the Sandwich Formula OK

Special cases of the sandwich formula OK

G.4 Two more theorems for the rotation matrix toolbox **OK**

G.5 Generalizations of the Rotation Group

N dimensional representations of the rotation group OK

A Differential operator representation of the rotation group OK

Some Other Lie Groups of Interest OK

Special Groups have Traceless Generators OK

Appendix H: The Euler Angles and Computation of ω

H.1 Euler Angles, Intermediate Rotations, and Unit Vectors **OK**

H.2 Theorem 3: Elimination of the Intermediate Rotations **OK**

H.3 Euler Angles : Triple Concatenation and Transformation of Vectors

triple concat stuff OK

Transformation of Kinematic Vectors

back up! I have added a few things to H.1 so start over with H.1 !

OK, after much fiddling I am now happy up to the start of H.4.

H.4 Euler angles which change in time: computation of ω (Method 1) **OK**

H.5 Computation of ω (Method 2)

This and the following section are going to take a long time to proof and possibly edit.

opening text OK

DETERMINATION OF THE FRAME S' COMPONENTS OF **ω**

First expression for: $[(d\mathbf{e}_n/dt)_{S'}]'_i$ OK

Second expression for: $[(d\mathbf{e}_n/dt)_{S'}]'_i$ OK

Equate the two expressions for: $[(d\mathbf{e}_n/dt)_{S'}]'_i$ OK

Computation of A and Statement of Final Result OK

DETERMINATION OF THE FRAME S COMPONENTS OF **ω**

First expression for: $[(d\mathbf{e}_n/dt)_{S'}]_i$ OK

Second expression for: $[(d\mathbf{e}_n/dt)_{S'}]_i$ OK

Equate the two expressions for: $[(d\mathbf{e}_n/dt)_{s'}]_i$ OK
Computation of B and Statement of Final Result OK
Summary of What Happened OK

This section is a real bear, a monster, and I now "strongly" discourage the reader from trying to read it, and say I just did it just as a stress test. I am not going to proof this section H.5 ever again!! It is just not an important section. Now I suspect I have yet another bear section coming up!

Took a run break, now resume with H.6

H.6 The connection between Euler Angles and Spherical Coordinates OK

This thing is finally OK after I fiddled a while. Had to adjust some external xrefs.

Status at 7PM. Today I proofed and edited Appendices G and H. Yesterday I did Appendix I and J and the Refs. So I can at least cross those four appendices off my proofing list! This covers pages 307 to 429, which is 122 pages of the total of 429, or about 1/3. This is the stuff that has changed in the last month or so.

Appendix D: Center of gravity and torque for a tethered satellite

D.1 Definition of Center of Gravity OK

D.2 Center of Gravity for a 2-mass Dumbbell Satellite OK

D.3 Dumbbell Satellite Center of Gravity with the Far Approximation: Numerical Examples OK

D.4 Dumbbell Satellite Center of Gravity for equal masses and no approximation OK

D.5 Center of Gravity for a single-sphere satellite OK

This appendix needs no further attention. It is certainly an offbeat topic, but nobody ever talks about center of mass, but I do with this tether example.
Overview of D: OK

Appendix E: Spherical Coordinate Unit Vectors and Particle Kinematics
E.1 Angle Conventions OK
E.2 Matrix Approach OK
E.3 The motion of a particle in spherical coordinates OK
E.4 Curvilinear coordinates approach OK
E.5 Polar Coordinates OK
E.6 The Affine Connection OK

OK, enough for App E. I did not check any equations, just the general flow. The last section is irrelevant but I add it to "stretch" the reader a bit. Found one real error and fixed it. Equations in this section have

been tested quite a bit in other sections, so I think things are OK here. I have no external references so reader has to trust me, but has rules for checking anything he or she wants.

My next task will be a pass through Appendix F. This is old stuff, I am just looking for bad connections to the newer stuff that was later added.

Update 4/4/17

F.1 Kinematics of the satellite in rotating Frame S

summary	will do as each section is proofed *****	
opening text		OK
Description of the Figure	pretty good and clear	OK
Naming of coordinates		OK
Some Basic Kinematic Facts	very concise	OK
summary:		OK

F.2 Angular momentum of the satellite and its time derivative in Frame S

very good, short	OK
summary:	OK

F.3 The torque on the satellite in Frame S'

opening section	OK
Far Approximation	OK
summary:	OK

F.4 The fictitious torque on the satellite in Frame S

opening text	result not very simple	OK
How might one interpret this simple result?		OK
summary		OK

F.5 Equations of Motion for the satellite in Frame S (Spherical Coordinates) **OK**

summary:	OK

F.6 Force analysis of the satellite in Frame S (Spherical Coordinates)

opening text	OK
Obtain the three equations of motion	OK
Obtaining the tension in the stick (or tether)	OK
Tidal Force	OK
summary	OK

F.7 Numerical solutions of the equations of motion (Spherical Coordinates)

opening text	OK
Verification of in-plane libration	OK
Verification of out-of-plane libration	OK
A more general solution	OK
summary	

F.8 Force analysis of the satellite in Frame S (Cartesian Coordinates) **OK**
summary: OK

F.9 Verification of the Cartesian equations of motion and stick tension **OK**
summary: OK

F.10 Numerical solutions of the equations of motion (Cartesian Coordinates) **OK**
summary: OK

super overview added OK

summary at start of frames doc OK

This is my last ever (I hope) pass through this 50 page appendix. I spent a lot of time on it and think errors are minimal.

So at this point I have proofed these sections: Overview,Summary, Appendix D,E,F,G,H,I,Refs.

Much remains to be proofed. I guess I will finish all the appendices first, then proof the main body.

Appendix C: The Foucault Pendulum
Nutshell Analysis of the Foucault Pendulum OK
summary: OK

C.1 Drawings, Notation, Coordinates and Basis Vectors **OK**
summary: OK

C.2 Qualitative Solution **OK**
summary: OK

C.3 Equations of Motion for the Foucault Pendulum (Spherical Coordinates) **OK**
summary: OK

C.4 The Simple Pendulum
opening text OK
Exact Solution for the Simple Pendulum OK
summary: OK

C.5 The Spherical and Foucault Pendulums
(a) The equations of motion for the Spherical Pendulum OK
(b) L_z as constant of the motion OK
(c) E as another constant of the motion OK
(d) Exact solution to the Spherical Pendulum (outline) OK
(e) The nature of the general solution for the Spherical Pendulum OK
(f) The Conical Motion solution of the Spherical Pendulum OK

(g) The thin ellipse scenario for the Spherical Pendulum OK
(h) The Intrinsic Airy Precession of the Spherical Pendulum OK
(i) The Foucault Mode of the Spherical Pendulum OK
(j) Interference between the Airy and Foucault precession OK
(k) The Foucault Pendulum at the Pantheon in Paris OK
(ℓ) General Numerical Orbits of the Spherical Pendulum OK
summary: OK

C.6 Equations of Motion for the Foucault Pendulum (Cartesian Coordinates)
opening text OK
Small oscillation limit OK
summary: OK

C.7 Verification of the Cartesian equations of motion and string tension OK
summary: OK

C.8 Numerical solutions of the equations of motion (Cartesian Coordinates) OK

Comments: This is (IMHO) a reasonable monograph appendix on this subject, bringing in many details. It starts now on page 183 and next section starts at 227, so length is 227-183 = 44 pages. It is not very exciting, but I hope the reader likes seeing the equations of motion in θ,φ and in x,y,z and likes seeing the simulations from Maple. The Pantheon case study I like. The links add a lot, if they would only stay working (which they won't).

Appendix B: The G Rule for a Tensor of Rank n
opening text OK
G Rule for a Rank-2 Tensor OK
G Rule for a Rank-n Tensor OK
Summary of G rule for all ranks (tensor form) OK
summary: OK

Appendix A: Derivation of R(ξ) and Properties of Rotation Matrices OK
summary: OK

Status: ALL appendices have been proofed in this latest round!

I will now do a fast read of the main body.

1.1 Basis vectors e_n , e'_n , rotation R, Dirac notation, the Basis Theorem, and Concatenation
opening text OK
The Notation Problem OK
Dirac Notation OK
The Basis Theorem OK
The matrices R and R' OK
Other Dirac Facts OK
the rest OK

1.2 Expansions of a vector and use of primes and parentheses
opening text OK
Matrix Notation to show how components are related. OK

1.3 Active and Passive Views of rotation, and a review of dot products
Basis Vectors, Kinematic Vectors, Active and Passive Views OK
Active View OK
Passive View OK
Rank-2 tensors in Active and Passive View OK
Dot Products and Scalars OK

1.4 When are two vectors equal? OK

1.5 The small rotation of a vector about an axis OK

1.6 The time rate of change of a rotating vector OK

1.7 Rate of change of the basis vectors OK

1.8 Notations for the many time derivatives of vectors r, r', b and L OK

1.9 Angular momentum OK

1.10 No frame label is needed for d/dt of a scalar function OK

1.11 When do operations d/dt and taking a component "commute" ? OK

All done with Section 1. I found no blaring problems, I think I picked them all up on previous passes. It is a motley collection of facts, I must admit. Lots of different ideas thrown together (Dirac, basis vectors, rotations, passive view, $\partial_{\mathbf{S}}$ idea, completeness, tensor and vector transform rules, etc etc etc). This Section used to be short, but it has now grown to 41-10 = 31 pages! I am not sure where you would read this stuff in a text book. It is not calculus, not algebra, it is wherever physics has multiple reference frames.

2. The G Rule for arbitrary vector a and its derivation OK

3. The Apparatus and its Observer at Rest in Frame S' OK

4. The Relationship between the Two Frames S and S'

4.1 Explanation of Fig (4.1.1): Frame S in the plane of paper **OK**

4.2 Explanation of Fig (4.2.1) : Vector ω pointing directly out of paper **OK**

4.3 Comments on $\dot{\mathbf{b}}_S$ and $\dot{\mathbf{b}}_{S'}$ **OK**

4.4 Special Case #1 : ω axis through Frame S origin **OK**

4.5 Special Case #2 : ω axis through Frame S' origin **OK**

4.6 The Turntable **OK**

4.7 The Earth **OK**

4.8 The Flying Camera Platform **OK**

5. The Goal of the next two sections **OK**

6. Determination of velocities

6.1 Velocity $v_{S'}$ **OK**

6.2 Velocity $v \equiv v_S$ **OK**

6.3 Velocity v'_S **OK**

6.4 Velocity Summary **OK**

6.5 Velocities for Special Cases **OK**

6.6 Comments **OK**

7. Determination of accelerations

7.1 Acceleration a'_S **OK**

7.2 Acceleration $a \equiv a_S$ **OK**

7.3 Acceleration $a_{S'}$ **OK**

7.4 Acceleration Summary **OK**

7.5 Relation between $\ddot{\mathbf{b}}_S$ and $\ddot{\mathbf{b}}_{S'}$ **OK** **browsing only!**

8. The Fictitious Forces

I am going to stop here for the day 4.4.17 because I don't want to glaze over. I hope tomorrow to finish my complete proofing of frames doc!

Update 4/5/17

8. The Fictitious Forces

8.1 Development of the Fictitious Forces **very good** **OK**

8.2 Interpretation of the Centrifugal and Euler Fictitious Forces **OK**

Very good, had to repair some crucial primes which dropped down to subscript level,

8.3 Interpretations of the Coriolis Fictitious Force

Qualitative Arm-Waving Interpretation of the Coriolis Force OK (made fixes)

Comments: OK

Superposition Interpretation of the Coriolis Force OK

8.4 Special Case #1 Problems **OK**

8.5 Problems on the surface of the Earth **OK**

8.6 Tethered satellites and Tidal Forces

opening section OK

Comments: OK very good

8.7 Special Case #3 **OK**

8.8 Tides on the Earth

The basic picture OK

Force Equations OK

The relation between r_{12} and Ω OK

Tidal Force at an arbitrary point on the Earth OK

2 versus 3 OK

Tidal force in polar coordinates OK

Equation of the water surface OK

Idealized Tidal patterns for an arbitrary rotation axis of the Earth

This is a tough long section with Maple plots. OK

What is the angle θ_1 for the real Earth?

Very painful with the cones. OK it is the best I can do

Tidal patterns for the rotating Earth (but still a water world in which water flows instantly) OK

Tides on the real Earth OK

Footnote: Equation of the Water Surface by the Potential Method OK

Yikes! This is a VERY detailed treatment of Tides on the Earth, I am amazed at the amount of detail. The section runs 78 to 101 so is 23 pages long. Compare to Section 5 which is 1 paragraph long! One of the figures had been damaged and removed by some kind of PL control key fumble, but I was able to replace it from a backup of frames doc.

9. Notation comparison with Marion (1970) and Thornton & Marion (2003) **OK**

10. Notation comparison with Goldstein (1950) and Goldstein, Poole and Safko (2001) **OK**

11. Angular Momentum and Fictitious Torques; the Reynolds Transport Theorem

11.1 Introduction **OK**

11.2 Expression of $\mathbf{L}^{(c)}$ and $\dot{\mathbf{L}}^{(c)}$ in terms of Frame S' objects **OK**

11.3 Fictitious Torques and Newton's Rotational Law in a non-inertial frame **OK**

11.4 Application: Fictitious Torques in Fluid Dynamics
This is a very tough but short section, strong tie-ins with Lai for verification OK

11.5 Application: Fictitious Forces in Fluid Dynamics **OK**

11.6 Comments on the Reynolds Transport Theorem **OK**

This is very fancy stuff for me to be doing, but I think it might help some person mystified by all that fluid dynamics mumbo jumbo.

12. Summary of the Forward Problem Solution

12.1 Summary of the Forward Problem equations (non-swap notation) **OK**

12.2 Summary of the Forward Problem equations (swap notation) **OK**

13. The Inverse Problem
opening text OK
13.1 Brute Force Method **OK**

13.2 Swap Rules Method **OK**

13.3 Summary of the Inverse Problem Equations (non-swap notation) **OK**

13.4 Summary of the Inverse Problem Equations (swap notation) **OK**

13.5 Why the Swap Rules Work **OK**

14. Rotating Frames in Curvilinear Coordinates **OK**

15. Ant on Turntable Problems
opening text OK
Kinematics common to all Ant Problems **OK**
What do we know about all the basis vectors? OK
Writing the basis vector relations in matrix notation OK, but not great, why the matrix thing?
Relation between Frame S and Frame S' OK
Motion of vector **b** OK

15.1 Problem 1: Ant crawls at constant speed V to the Origin of Frame S' **OK**

15.2 Problem 2: Ant spirals in at constant V and Ω to the Origin of Frame S' **OK**

15.3 Problem 3: Inverse Problem: Ant flies in Frame S at constant velocity V **OK**

15.4 Problem 4: The Projectile Problem of Section 8.3 **OK**

These last sections I just browsed because I did them all in super fine detail not too long ago.

Now Hear This: The concludes my FULL proofing pass of all this 429 page document!

I now move to the production phase perhaps after reading the overview stuff one more time.

Issues:

- Search for any sunken primes like $r_{'}$ (automatically please). I found several in my edit pass. **DONE**
- Decide on where to use bold primes. Maybe view results in old PDF before deciding. **DONE**
- Pagination of course **DONE**
- Picture centering **DONE**
- spell checking chunk at a time **DONE**
- look for thrown equation numbers **DONE** could not find even one on a full pass
- Decide how much space to leave between subsections. Do this on case for case basis.
- Just gaze at newer equations for good spacing of things **DONE**
- Get the section headers updated.
  (1) first, make sure every section has a new section bar at the start **DONE**
  (2) make sure no extra section breaks out in the middle. **DONE**
  (3) update the header texts **DONE**

Update 4/6/17

1. Made a huge semi-automatic pass looking for subscript primes in wrong places. Found many occurrences of **r'** where the prime looked fine but was really subscript style. I fixed and bolded all these primes, perhaps 50 of them, one at a time! Most primes of this type are on like $\partial_{\mathbf{s}'}$.

2. Another manual pass with this rule: whenever **r**,**v** or **a** is bolded and undecorated, I make the prime bold, so you get **r'**, **v'**, **a'** . I probably did not get all of them, but my argument is that the prime is a key notation in my document and I want it to stand out very clearly.

3. Did a pass cosmetically making equations look better, spacing before and after = and + signs eg. Made quite a few edits, always fixing any thrown eq nums that result.

4. Reading of overview and summary: // Done, made a comment that over half content is in the Apps.

5. Spell checking: do this 50 pages at a time in a scratch file. Pre-scroll through to force the spell checker to underline things.

1-53	OK	one write write instance found and fixed
50-100	OK	no errors
100-150	OK	one error fixed
150-200	OK	no errors
200-250	OK	no errors
250-300	OK	no errors
300-350	OK	one error fixed
350-400	OK	three errors fixed
400-429	OK	no errors

The spell check is complete, it turned up only 6 typos, that is since I have done many earlier spellchecks.

Pagination

toc installed some breaks, will have to do this every time the TOC is updated *****

Overview and Summary together done.

Section 1,2,3:	OK thru p 44	
Section 4,5,6,7:	OK thru p 60	
Section 8 :	OK thru p 100	
Section 9,10 :	OK thru p 105	
Section 11,12,13,14:	OK thru 136	
Section 15:	OK to p 172	
Appendix A:	OK	
Appendix B:	OK	
Appendix C:	OK thru p 226	to p 196 very solid

Appendix D:	OK thru p 248
Appendix E:	OK thru p 258
Appendix F:	OK thru p 306
Appendix G:	OK thru p 322
Appendix H:	OK thru p 354
Appendix I:	OK thru p 417
Appendix J:	OK thru p 424
Refs:	OK

I did all the items above and I am ready for a first cut PDF, just to have something to look at for bugs.

PDF First Cycle bookmarks = 135, then pile driving after short delay, then stopped, no go
Try again: this time it is creating pages.

- Fix the blue text in the TOC **DONE**
- test all the links **DONE**
- above I.11.19 maybe extra space after h. **DONE**
- the heading to section 1.8 should not include L, since that is now Section 1.9 **DONE**

All bookmarks are present! Brought it back to Black.

If I don't survive the night, someone please put this PDF up on the xmission site, it is ready to go.

Update 4/7/17

Today I did a critique of sorts in a separate doc, trying to see whether all of that Section 1 stuff was not just hot air, but I think it really is "foundational" in this subject. I am happy with the stuff, and will put it up soon after I fix the above list of items.

Test the links: I will do this directly from the PDF, starting with the Refs section.

I did some reading, fixing cosmetic things as I went. Found a whole chunk of the pendulum solution missing, one of my equation editor accidental launches I am sure. So now I am going to do the actual release version on Alta. It is going OK, I have a release folder ready to roll.

Ouch! Trouble in the PDF with section header text! Just happened to notice perusing the PDF. All headers are OK up to Appendix I first page where it is wrong. Did repairs, Doing another PDF cycle.

All done, it is up on the xmission site, downloads OK. Release folder all set. DONE!!!!
(probably I will redo it in a few days, that always happens).