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10. Notation comparison with Goldstein (1950) and Goldstein, Poole and Safko (2001)

In this we Section compare our notation for rotating-frame kinematics and non-inertial-frame physics to that of Goldstein on (1950) and Goldstein, Poole and Safko (GFS 2001).

These books don't really have much to say about rotating frames, but we think we have been able to "deduce" what it is they do say and they are also close to our "swap" notation.

A careful reading of Goldstein page 135 shows that **r** is in fact the "short vector" whose tail lies at the origin of the rotating Frame S. He refers to this frame, which we think of as attached to the surface of the rotating Earth, as being a "terrestrial" coordinate system "fixed in the earth, which therefore rotates uniformly relative to the inertial system".

He does not discuss the origin location of his inertial frame, but we think we know where it is. In our swap notation we would say that the rotating Frame S has its origin somewhere on the surface of the Earth, and that inertial Frame S' has its origin *at the same point*, so **b** = 0. So the inertial Frame S is a gimbaled frame whose axes always align with the stars while the axes of Frame S rotate. An alternative would be to have both Frame origins be at the center of the Earth. They key fact is that these two frames have the same origin so **b** = 0. In our notation, one implication is that **r** = **r'** since we have **r'** = **r**+**b** in swap notation. When **r** = **r'** , in our velocity table (1.8.4) the left and right sides are exactly the same. Similarly acceleration table (1.8.6) is identical to table (1.8.5). For example, with **b** = 0 we have,

$$\mathbf{v}_{S'} = (\partial \mathbf{r}/\partial t)_{S'} = (\partial \mathbf{r}'/\partial t)_{S'} = v'_{S'} = \mathbf{v}'$$

$$\mathbf{a}_{S'} = (\partial^2 \mathbf{r}/\partial t^2)_{S'} = (\partial^2 \mathbf{r}'/\partial t^2)_{S'} = a'_{S'} = \mathbf{a}' \,. \qquad (10.1)$$

Of course $\ddot{\mathbf{b}}_{S'} = 0$ when **b** = 0. We now gather up some of our equations in swap notation and state them for the case that **b** = 0, making use of (10.1) above:

$\mathbf{r}' = \mathbf{r}$ // $\mathbf{b} = 0$ *(6.1)*$_S$

$\mathbf{v}' = \mathbf{v} + \boldsymbol{\omega} \times \mathbf{r}$. **r** = short vector *(6.5)*$_S$

$\mathbf{a}' = \mathbf{a} + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ S = rotating frame *(7.4)*$_S$

$m\mathbf{a} = \mathbf{F}_{eff} = \mathbf{F}' - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - 2m\,\boldsymbol{\omega} \times \mathbf{v} - m\dot{\boldsymbol{\omega}} \times \mathbf{r}$. *(8.1.5)*$_S$

We start with our equation *(6.5)*$_S$:

$\mathbf{v}' = \mathbf{v} + \boldsymbol{\omega} \times \mathbf{r}$. **r** = short vector *(6.5)*$_S$

$\mathbf{v}_S = \mathbf{v}_r + \boldsymbol{\omega} \times \mathbf{r}$. **r** = short vector // Goldstein p 135 (4-104)

// GPS p 175 (4.88)

The next comparison is (we add $\dot{\boldsymbol{\omega}} \times \mathbf{r}$ to their equations)

$$\mathbf{a}' = \mathbf{a} + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad . \qquad (7.4)_s$$

$$\mathbf{a}_s = \mathbf{a}_r + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad .$$

// Goldstein p 135 (4-105)
// GPS p 175 (4.89)

And finally,

$$m\mathbf{a} = \mathbf{F}_{eff} = \mathbf{F}' - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - 2m\,\boldsymbol{\omega} \times \mathbf{v} - m\dot{\boldsymbol{\omega}} \times \mathbf{r} \qquad (8.1.5)_s$$

$$m\mathbf{a}_r = \mathbf{F}_{eff} = \mathbf{F} - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - 2m\,\boldsymbol{\omega} \times \mathbf{v}_r - m\dot{\boldsymbol{\omega}} \times \mathbf{r} \quad .$$

// Goldstein p 135 (4-106,7)
// GPS p 175 (4.90,1)

Based on these comparisons, we make the following translation table:

our swap notation	Goldstein authors	$\mathbf{b} = 0$
S	r	name of the rotating frame
S'	s	name of the fixed frame (s = space)
∂_S	$(d/dt)_r$	time derivative in the rotating frame
$\partial_{S'}$	$(d/dt)_s$	time derivative in the fixed frame
$\mathbf{r}$	$\mathbf{r}$	position in rotating frame (short vector)
$\mathbf{v}$	$\mathbf{v}_r$	velocity in rotating frame
$\mathbf{a}$	$\mathbf{a}_r$	acceleration in rotating frame
$\mathbf{r}'$	$\mathbf{r}$	position in fixed frame (long vector = short vector)
$\mathbf{v}'$	$\mathbf{v}_s$	velocity in fixed frame
$\mathbf{a}'$	$\mathbf{a}_s$	acceleration in fixed frame
$\mathbf{F}'$	$\mathbf{F}$	force in fixed frame = true force in rotating frame
$\mathbf{F}_{eff}$	$\mathbf{F}_{eff}$	total effective force in the rotating frame
$\mathbf{F}_{fict}$		total fictitious force in the rotating frame
$\mathbf{b}$	0	location of the rotating frame origin (measured in the fixed frame)
$\dot{\mathbf{b}}_{S'}$	0	velocity of the rotating frame origin (measured in the fixed frame)
$\ddot{\mathbf{b}}_{S'}$	0	acceleration of the rotating frame origin (measured in the fixed frame)

Since **b** = 0 is assumed, the Goldstein and GPS texts only treat a special case of "rotating frames of reference".

ATTEMPT USING MY SWAP NOTATION

Recall our two equations in swap notation.

$$\mathbf{a}' = \mathbf{a} + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \ddot{\mathbf{b}}_{S'} \qquad S = \text{rotating frame} \qquad (7.6a)_S$$

S' S Euler Coriolis centripetal frame (**r** = short vector, **r'** = long vector)

$$\ddot{\mathbf{b}}_{S'} = \dot{\boldsymbol{\omega}} \times \mathbf{b} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{b}). \qquad \text{// Special Case \#1} \qquad (7.13)_S$$

Inserting the second into the first and using **r'** = **b**+**r** gives

$$\mathbf{a}' = \mathbf{a} + \dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') \qquad \text{// } \mathbf{r}' = \text{long vector} \qquad (*)$$

We may compare this to the Goldstein equation,

$$\mathbf{a}_s = \mathbf{a}_r + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad . \qquad \text{// Goldstein p 135 (4-105)} \qquad (**)$$
$$\text{// GPS p 175 (4.89)}$$

In my equation (*) I have **r'** = long vector. But in (**) I know that **r** is a short vector because he says this on page 135. This means that (*) and (**) are not compatible.

Meanwhile, let's compare these two items

$$\mathbf{v}_{S'} = \mathbf{v} + \boldsymbol{\omega} \times \mathbf{r} \quad . \qquad \mathbf{r} = \text{short vector} \qquad (6.5)_S$$

$$\mathbf{v}_s = \mathbf{v}_r + \boldsymbol{\omega} \times \mathbf{r} \quad . \qquad \mathbf{r} = \text{short vector} \qquad \text{// Goldstein p 135 (4-104)}$$
$$\text{// GPS p 175 (4.88)}$$

I am OK with $\mathbf{v} = \mathbf{v}_r$ in the rotating frame. And **r** looks right. But in my equation I have

$$\mathbf{v}_{S'} = (d\mathbf{r}/dt)_{S'} = (d\mathbf{r}/dt)_{space}$$

I refer to this as a cross velocity because I am looking at the motion of the short vector r in the rotating frame using a time derivative from the space frame. That is certainly OK to do. With this connection that we have $v_s = \mathbf{v}_{S'}$, we are 100% OK here, which favors this approach.

Maybe he does the same thing with acceleration. He would have

$\mathbf{a}_S = \partial_{S'}\mathbf{v}_{S'} = \partial^2_{S'}\mathbf{r} = (\partial^2\mathbf{r}/dt^2)_{space}$ exactly what G says!

So I want then to connect $\mathbf{a}_S = \mathbf{a}_{S'S'} = \mathbf{a}_{S'}$. What do I know about this accel?

$$\mathbf{a'}_S = \mathbf{a'} + \dot{\boldsymbol{\omega}} \times \mathbf{r'} + 2\,\boldsymbol{\omega} \times \mathbf{v'} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) \quad . \qquad (7.4)$$

$$\mathbf{a}_{S'} = \mathbf{a} + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad . \qquad (7.4)_S$$

Compare this then to

$$\mathbf{a}_S = \mathbf{a}_r + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad .$$

// Goldstein p 135 (4-105) (**)
// GPS p 175 (4.89)

ATTEMPT USING MY NON-SWAP NOTATION

$$\mathbf{a} = \mathbf{a'} + \dot{\boldsymbol{\omega}} \times \mathbf{r'} + 2\,\boldsymbol{\omega} \times \mathbf{v'} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) + \ddot{\mathbf{b}}_S \qquad (7.6a)$$

S S' Euler Coriolis centripetal frame

S' = rotating frame

(**r'** = short vector, **r** = long vector)

$$\ddot{\mathbf{b}}_S = \dot{\boldsymbol{\omega}} \times \mathbf{b} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{b}) \quad . \qquad \text{// Special Case \#1} \qquad (7.13)$$

Inserting the second into the first and using $\mathbf{r} = \mathbf{b} + \mathbf{r'}$ gives

$$\mathbf{a} = \mathbf{a'} + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v'} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad . \qquad (*) \qquad (10.1)$$

This may be compared with the Goldstein equation (to which we have added a $\dot{\boldsymbol{\omega}} \times \mathbf{r}$ term)

$$\mathbf{a}_S = \mathbf{a}_r + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad .$$

// Goldstein p 135 (4-105) (**)
// GPS p 175 (4.89)

In my equation (*) I have **r**= long vector. But in (**) I know that **r** is a short vector because he says this on page 135. This means that (*) and (**) are not compatible.

I am unable to find a way to compare the two notations!!

Meanwhile, let's compare these two items

$$\mathbf{v'}_S = \mathbf{v'} + \boldsymbol{\omega} \times \mathbf{r'} \quad . \qquad (6.5)$$

$$\mathbf{v}_S = \mathbf{v}_r + \boldsymbol{\omega} \times \mathbf{r} \quad .$$

// Goldstein p 135 (4-104)
// GPS p 175 (4.88)

$\mathbf{v}_s = \mathbf{v}_r + \boldsymbol{\omega} \times \mathbf{r}$. // Goldstein p 135 (4-104)
// GPS p 175 (4.88)

All my equations above in swap-notation are these

(**r** = short vector, **r'** = long vector)

$\mathbf{a}' = \mathbf{a} + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \ddot{\mathbf{b}}_{s'}$ S = rotating frame $(7.6a)_s$
S' S Euler Coriolis centripetal frame

$\ddot{\mathbf{b}}_{s'} = \dot{\boldsymbol{\omega}} \times \mathbf{b} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{b})$. // Special Case #1 $(7.13)_s$

Inserting the second into the first and using $\mathbf{r}' = \mathbf{b}+\mathbf{r}$ gives

$\mathbf{a}' = \mathbf{a} + \dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}')$

Now compare this to

$\mathbf{a}_s = \mathbf{a}_r + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$. // Goldstein p 135 (4-105)
// GPS p 175 (4.89)

So this is IT!!

our swap notation	Goldstein authors	
S	r	name of the rotating frame
S'	s	name of the fixed frame (s = space)
$\partial_{s'}$	$(d/dt)_s$	time derivative in the fixed frame
∂_s	$(d/dt)_r$	time derivative in the rotating frame
r		position in rotating frame (short vector)
v	$\mathbf{v}_r$	velocity in rotating frame
a	$\mathbf{a}_r$	acceleration in rotating frame
r'	**r**	position in fixed frame (long vector)
v'	$\mathbf{v}_f$	velocity in fixed frame

$\mathbf{a'}$	$\mathbf{a_s}$	acceleration in fixed frame
$\mathbf{F'}$	$\mathbf{F}$	force in fixed frame = true force in rotating frame
$\mathbf{F_{eff}}$	$\mathbf{F_{eff}}$	total effective force in the rotating frame
$\mathbf{F_{fict}}$		total fictitious force in the rotating frame
$\mathbf{b}$	$\mathbf{R}$	location of the rotating frame origin (measured in the fixed frame)
$\dot{\mathbf{b}}_{\mathbf{S'}}$	$\dot{\mathbf{R}}_{\mathbf{f}}, \mathbf{V}$	velocity of the rotating frame origin (measured in the fixed frame)
$\ddot{\mathbf{b}}_{\mathbf{S'}}$	$\ddot{\mathbf{R}}_{\mathbf{f}}$	acceleration of the rotating frame origin (measured in the fixed frame)