10. Notation comparison with Goldstein (1950) and Goldstein, Poole and Safko (2001)

In this Section compare our notation for rotating-frame kinematics and non-inertial-frame physics to that of Goldstein on (1950) and Goldstein, Poole and Safko (GFS 2001).

Their notation seems to assume the Special Case #1 situation where the rotation axis passes through the inertal frame origin, as for rotating-Earth problems.

ATTEMPT USING MY SWAP NOTATION

Recall our two equations in swap notation.

$$\mathbf{a}' = \mathbf{a} + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \ddot{\mathbf{b}}_{S'} \qquad S = \text{rotating frame} \qquad (7.6a)_s$$

S' S Euler Coriolis centripetal frame (**r** = short vector, **r'** = long vector)

$$\ddot{\mathbf{b}}_{S'} = \dot{\boldsymbol{\omega}} \times \mathbf{b} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{b}). \qquad \text{// Special Case \#1} \qquad (7.13)_s$$

Inserting the second into the first and using **r'** = **b**+**r** gives

$$\mathbf{a}' = \mathbf{a} + \dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') \qquad \text{// } \mathbf{r}' = \text{long vector} \qquad (*)$$

We may compare this to the Goldstein equation,

$$\mathbf{a}_s = \mathbf{a}_r + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad . \qquad \text{// Goldstein p 135 (4-105)} \qquad (**)$$
$$\text{// GPS p 175 (4.89)}$$

In my equation (*) I have **r'** = long vector. But in (**) I know that **r** is a short vector because he says this on page 135. This means that (*) and (**) are not compatible.

Meanwhile, let's compare these two items

$$\mathbf{v}_{S'} = \mathbf{v} + \boldsymbol{\omega} \times \mathbf{r} \quad . \qquad \mathbf{r} = \text{short vector} \qquad (6.5)_s$$

$$\mathbf{v}_s = \mathbf{v}_r + \boldsymbol{\omega} \times \mathbf{r} \quad . \qquad \mathbf{r} = \text{short vector} \qquad \text{// Goldstein p 135 (4-104)}$$
$$\text{// GPS p 175 (4.88)}$$

I am OK with **v** = $\mathbf{v}_r$ in the rotating frame. And **r** looks right. But in my equation I have

$$\mathbf{v}_{S'} = (d\mathbf{r}/dt)_{S'} = (d\mathbf{r}/dt)_{space}$$

I refer to this as a cross velocity because I am looking at the motion of the short vector r in the rotating frame using a time derivative from the space frame. That is certainly OK to do. With this connection that we have $v_s = \mathbf{v}_{S'}$, we are 100% OK here, which favors this approach.

Maybe he does the same thing with acceleration. He would have

$\mathbf{a}_S = \partial_{S'}\mathbf{v}_{S'} = \partial^2_{S'}\mathbf{r} = (\partial^2\mathbf{r}/dt^2)_{space}$ exactly what G says!

So I want then to connect $\mathbf{a}_S = \mathbf{a}_{S'S'} = \mathbf{a}_{S'}$. What do I know about this accel?

$$\mathbf{a'}_S = \mathbf{a'} + \dot{\boldsymbol{\omega}} \times \mathbf{r'} + 2\,\boldsymbol{\omega} \times \mathbf{v'} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) \quad . \qquad (7.4)$$

$$\mathbf{a}_{S'} = \mathbf{a} + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad . \qquad (7.4)_S$$

Compare this then to

$$\mathbf{a}_S = \mathbf{a}_r + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad .$$

// Goldstein p 135 (4-105) (**)
// GPS p 175 (4.89)

ATTEMPT USING MY NON-SWAP NOTATION

$$\mathbf{a} = \mathbf{a'} + \dot{\boldsymbol{\omega}} \times \mathbf{r'} + 2\,\boldsymbol{\omega} \times \mathbf{v'} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'}) + \ddot{\mathbf{b}}_S \qquad (7.6a)$$

S S' Euler Coriolis centripetal frame

S' = rotating frame
(**r'** = short vector, **r** = long vector)

$$\ddot{\mathbf{b}}_S = \dot{\boldsymbol{\omega}} \times \mathbf{b} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{b}) \quad . \qquad \text{// Special Case \#1} \qquad (7.13)$$

Inserting the second into the first and using $\mathbf{r} = \mathbf{b} + \mathbf{r'}$ gives

$$\mathbf{a} = \mathbf{a'} + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v'} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \; . \qquad (*) \qquad (10.1)$$

This may be compared with the Goldstein equation (to which we have added a $\dot{\boldsymbol{\omega}} \times \mathbf{r}$ term)

$$\mathbf{a}_S = \mathbf{a}_r + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad .$$

// Goldstein p 135 (4-105) (**)
// GPS p 175 (4.89)

In my equation (*) I have **r**= long vector. But in (**) I know that **r** is a short vector because he says this on page 135. This means that (*) and (**) are not compatible.

I am unable to find a way to compare the two notations!!

Meanwhile, let's compare these two items

$$\mathbf{v'}_S = \mathbf{v'} + \boldsymbol{\omega} \times \mathbf{r'} \quad . \qquad (6.5)$$

$$\mathbf{v}_S = \mathbf{v}_r + \boldsymbol{\omega} \times \mathbf{r} \quad .$$

// Goldstein p 135 (4-104)
// GPS p 175 (4.88)

$\mathbf{v_s} = \mathbf{v_r} + \boldsymbol{\omega} \times \mathbf{r}$. // Goldstein p 135 (4-104)
// GPS p 175 (4.88)

All my equations above in swap-notation are these

(**r** = short vector, **r'** = long vector)

$\mathbf{a'} = \mathbf{a} + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \ddot{\mathbf{b}}_{S'}$ S = rotating frame *(7.6a)*$_s$

S' S Euler Coriolis centripetal frame

$\ddot{\mathbf{b}}_{S'} = \dot{\boldsymbol{\omega}} \times \mathbf{b} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{b})$. // Special Case #1 *(7.13)*$_s$

Inserting the second into the first and using **r'** = **b**+**r** gives

$\mathbf{a'} = \mathbf{a} + \dot{\boldsymbol{\omega}} \times \mathbf{r'} + 2\,\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r'})$

Now compare this to

$\mathbf{a_s} = \mathbf{a_r} + \dot{\boldsymbol{\omega}} \times \mathbf{r} + 2\,\boldsymbol{\omega} \times \mathbf{v_r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$. // Goldstein p 135 (4-105)
// GPS p 175 (4.89)

So this is IT!!

our swap notation	Goldstein authors	
S	r	name of the rotating frame
S'	s	name of the fixed frame (s = space)
$\partial_{S'}$	$(d/dt)_s$	time derivative in the fixed frame
∂_S	$(d/dt)_r$	time derivative in the rotating frame
r		position in rotating frame (short vector)
v	$\mathbf{v_r}$	velocity in rotating frame
a	$\mathbf{a_r}$	acceleration in rotating frame
r'	**r**	position in fixed frame (long vector)

$\mathbf{v}'$	$\mathbf{v}_f$	velocity in fixed frame
$\mathbf{a}'$	$\mathbf{a}_s$	acceleration in fixed frame
$\mathbf{F}'$	$\mathbf{F}$	force in fixed frame = true force in rotating frame
$\mathbf{F}_{eff}$	$\mathbf{F}_{eff}$	total effective force in the rotating frame
$\mathbf{F}_{fict}$		total fictitious force in the rotating frame
$\mathbf{b}$	$\mathbf{R}$	location of the rotating frame origin (measured in the fixed frame)
$\dot{\mathbf{b}}_{S'}$	$\dot{\mathbf{R}}_f, \mathbf{V}$	velocity of the rotating frame origin (measured in the fixed frame)
$\ddot{\mathbf{b}}_{S'}$	$\ddot{\mathbf{R}}_f$	acceleration of the rotating frame origin (measured in the fixed frame)