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$$= (Gm)[\, h\, \{m_1/r^3_1 + m_2/r^3_2\} + r\cos\theta\, \{m_1/r^3_1 - m_2/r^3_2\}\,]$$

Define

$$A \equiv (Gm)\{m_1/r^3_1 + m_2/r^3_2\}$$
$$B \equiv (Gm)\{m_1/r^3_1 - m_2/r^3_2\}$$

Then

$$F_y = r\sin\theta A$$
$$F_z = hA + r\cos\theta B$$

To find the center of gravity point, we will need the magnitude of F

$$F^2 = F_y^2 + F_z^2 = (r\sin\theta A)^2 + (hA + r\cos\theta B)^2$$
$$= (r^2\sin^2\theta + h^2)\, A^2 + r^2\cos^2\theta\, B^2 + 2hr\cos\theta\, AB$$

$$F = \sqrt{(r^2\sin^2\theta + h^2)\, A^2 + r^2\cos^2\theta\, B^2 + 2hr\cos\theta\, AB}$$

Then from *** we have

$$r_{cog} = \sqrt{\frac{Gm(m_1+m_2)}{F}} = \sqrt{\frac{Gm(m_1+m_2)}{\sqrt{(r^2\sin^2\theta + h^2)\, A^2 + r^2\cos^2\theta\, B^2 + 2hr\cos\theta\, AB}}}$$

We know that

$$\hat{\mathbf{F}} = \frac{\mathbf{F}}{F} = \frac{(Gm)[\, r\sin\theta(m_1/r^3_1 - m_2/r^3_2)]\, \hat{\mathbf{y}} + (Gm)[\, (h+r\cos\theta)m_1/r^3_1 + (h-r\cos\theta)m_2/r^3_2\,]\, \hat{\mathbf{z}}}{\sqrt{(r^2\sin^2\theta + h^2)\, A^2 + r^2\cos^2\theta\, B^2 + 2hr\cos\theta\, AB}}$$

and this vector points generally in the up direction in Fig (D.2.1). We want $\mathbf{r}_{cog}$ to be a vector from mass m to the center of gravity, so then

$$\mathbf{r}_{cog} = r_{cog}\, (-\hat{\mathbf{F}}) = r_{cog}\, (-\frac{\mathbf{F}}{F}) \qquad \text{// vector from mass m to the cog point}$$

and in theory we have all the pieces to compute $\mathbf{r}_{cog}$ as a function of r,θ,h,m_1,m_2. As the reader can see, even for this very simple problem of a system of just two masses, the resulting expression is very complicated.