Other books on Rigid Body Motion PhL 7.20.08

After reading Goldstein rotational stuff, I thought I would look at more elementary books in my possession, just to see what they do on this subject

Resnick and Halliday Physics Part I (1966) Sophomore Harvard 1967

Chapter 11: Rotational Kinematics (18 pages)

Picture page 242 suggests Chasle's theorem. Subject treated here as circular motion of a point in a rigid body, the rotation axis is considered fixed in this chapter. The linear/rotational analogy is established as in the table on page 246. Claim that θ is not a vector, but d$\boldsymbol{\theta}$ is a vector. The reason is that objects like d$\boldsymbol{\theta}$ obey the commutative addition rules, ie, infinitesimal rotations commute $i\theta J_{\mathbf{x}} + i\varphi J_{\mathbf{y}}$. Authors are not clear that $d\boldsymbol{\theta} \equiv d\theta\, \hat{\mathbf{n}}$ by definition. Student might think $d\mathbf{s} = r d\boldsymbol{\theta}$ but really $d\mathbf{s} = \mathbf{r} \times d\boldsymbol{\theta}$. In any event $d\boldsymbol{\theta}/dt = \boldsymbol{\omega}$ is therefore a vector. Nice turntable picture page 250 shows how you would add vector $\boldsymbol{\omega}$'s to get the total $\boldsymbol{\omega}$.

The basic connections to linear motion are derived to be these on page 252

$v = \omega r$ $\quad a_T = \alpha r$ $\quad a_R = v^2/r = \omega^2 r = \omega v$ // scalars

Then in a small-print section, we get these again in vector notation

$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$ $\quad \mathbf{a}_T = \boldsymbol{\alpha} \times \mathbf{r}$ $\quad \mathbf{a}_R = \boldsymbol{\omega} \times \mathbf{v}$ // vectors

Here, the first is just from $d\mathbf{s} = \mathbf{r} \times d\boldsymbol{\theta}$. In Goldstein, it comes from that body/space "ωx rule".

Chapter 12: Rotational Dynamics I

Torque is defined as $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$. Then angular momentum as $\boldsymbol{\ell} = \mathbf{r} \times \mathbf{p}$. It is then shown that $\boldsymbol{\tau} = d\boldsymbol{\ell}/dt$. First we do this with 1 particle, then a system of same.

Inertia $I \equiv mr^2$ for a particle where r is distance to axis, then find that $T = \frac{1}{2} I \omega^2$. Compute I for an annular ring and then for a solid cylinder. Nice table on page 272 shows I for various objects relative to various axes. The parallel axis theorem is stated as $I = I_{cm} + Mh^2$. The term "moment arm" is well shown in the page 274 figure as $r_\perp$. Work is examined and we find that power = $P = \tau\omega$ corresponding to Fv for linear. And we find $\tau = I\alpha$, like F = ma. The full analogy is then shown on page 278 in a table. Finally, on page 281 we see that $L = I\omega$ compare to p = mv.

ON page 282 we have that $T = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} M v_{cm}^2$ as the Chasle partition of energy. An object rolling down a plane is then analyzed first from energy method, then from F = ma rotational method. Then "questions" and 39 problems! Not too much is said about friction, but they do talk about things slipping or not slipping.

Chapter 13: Rotational Dynamics II and Conservation of L

The top is considered in the fast-top limit and the precession frequency found agrees with Goldstein's average on page 171. Nutation is not even mentioned, also not in the index. The rest of chapter concerns conservation of L. He writes on page 303 that $\mathbf{L} = I\boldsymbol{\omega}$ but footnote says this is for ω along a principle axis. Reader is referred for more details to a Mechanics text of Arnold Sommerfeld (1868-1951). The main rotational mechanics ideas as summarized on page 312, and particle spin is mentioned in small print on page 310. There is no mention of an inertia "tensor", nor is the word "matrix" in the appendix.

Berkeley Volume 1 Mechanics Sophomore Harvard 1967

Chapter 8: Elementary Dynamics of Rigid Objects (26 p)

Mentions the Kreisels four volumes in the first paragraph! This chapter was meant to be omitted on first reading of the book due to its complexity. We get L = Iω scalar at first, then various of Goldstein's equations and expressions appear, I as a matrix, and the trace of I is computed, not done in G but trivial. Nice example of a "baton" where **L** and **ω** are not parallel. The "summation convention" is introduced, but uses Greek instead of Latin letters which I don't like much for non-rel stuff! K of rotation, things rolling down planes. Those Euler Equations are derived page 250 bottom. Says that for the precessing earth, the 305 to 420 day problem was solved by Newcomb who showed how elasticity does it, book on this subject referenced. Magnetic moment in B field treatment. The bike and gyroscope are mentioned very briefly. Meant for student with 1D calculus, but no matrix knowledge.

Marion Classical Dynamics 1970 Berkeley Fall 1971

Chapter 12: Dynamics of Rigid Bodies p 359.

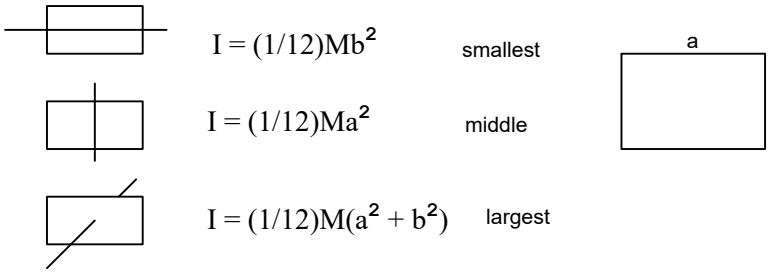
Very Goldstein-like in presentation. The inertia tensor, vector notation looks good. Cube I computed various ways. Shows the secular equation for finding the diagonal moments, but quotes no matrix theorems. Relates the word secular to celestial mechanics history. Shows now obvious fact that a cube has the same three moments for any perp axes through the center, not just for obvious axes sets. Comments on the similarity which diagonalizes I.

Page 384 we have Euler angles and second rotation is about the x axis so same as Goldstein. Computes the famous $\omega_{\mathbf{body}}$ thing on page 386, same as G exactly. Then takes the Lagrange equation and pretty much replicates Goldstein's stuff. Does force-free precession, then does the top with the exact same symbols p_φ and p_ψ. But then he varies a little. On page 396 he thinks of the one dimensional problem for θ and we have an effective potential V(θ) as shown and which you can plot. This makes the notion of

turning points very clear and is like our study of orbits. The same three pictures of G appear on page 399 for the nutation.

Final section talks about stability of rigid body rotations. This is something G did not mention, but S Coleman did mention regarding the tennis racket. And we get the same result quoted that the middle case is unstable. The proof is very simple. Just start with ω along an axis, add perturbations as shown, use Euler to solve, and you get small harmonic motion for the stable cases and expo buildup for the third case. Very nice. Seems odd that G would omit this nice and easily testable result.

Experiment suggested. Rubber band around a thin book. Here is the math from scratch and confirmed web:



So you won't be able to cleanly spin the book with the middle axis. This is a very obvious experiment!! The tennis racket also works well, unstable the long way. Hard to catch the racket though.

Feynman Vol I Chapter 20 also has a small section on rigid body motion, with his usual treatment of details, such as what really happens when you let go of the gyroscope tip.

Some History

Sommerfeld had 6 volumes of lecture notes dated 1943-1952, and they were translated from German to English for Academic Press in 1964. The volume topics were Mechanics, Mechanics of Deformable Bodies, Electrodynamics, Optics, and StatMech/Thermo. I suspect he was the Feynman of his era, or perhaps a Jackson. The bio mentions his being an assistant of Felix Klein, and of their long collaboration on the theory of tops resulting in the 4 volume Theorie de Kreisels.

- Arnold Sommerfeld *Mechanik - Vorlesungen über theoretische Physik Band 1* (Akademische Verlagsgesellschaft Becker & Erler, 1943)
    - Arnold Sommerfeld, translated from the fourth German edition by Martin O. Stern ***Mechanics*** - *Lectures on Theoretical Physics Volume I* (Academic Press, 1964)
- Arnold Sommerfeld *Mechanik der deformierbaren Medien - Vorlesungen über theoretische Physik Band 2* (Akademische Verlagsgesellschaft Becker & Erler, 1945)
    - Arnold Sommerfeld, translated from the second German edition by G. Kuerti ***Mechanics of Deformable Bodies*** - *Lectures on Theoretical Physics Volume II* (Academic Press, 1964)

- Arnold Sommerfeld *Elektrodynamik - Vorlesungen über theoretische Physik Band 3* (Klemm Verlag, Erscheinungsort, 1948)
    - Arnold Sommerfeld, translated from the German by Edward G. Ramberg ***Electrodynamics*** - *Lectures on Theoretical Physics Volume III* (Academic Press, 1964)
- Arnold Sommerfeld *Optik - Vorlesungen über theoretische Physik Band 4* (Dieterich'sche Verlagsbuchhandlung, 1950)
    - Arnold Sommerfeld, translated from the first German edition by Otto Laporte and Peter A. Moldauer ***Optics*** - *Lectures on Theoretical Physics Volume IV* (Academic Press, 1964)
- Arnold Sommerfeld *Thermodynamik und Statistik - Vorlesungen über theoretische Physik Band 5* Herausgegeben von Fritz Bopp und Josef Meixner. (Diederich sche Verlagsbuchhandlung, 1952)
    - Arnold Sommerfeld, edited by F. Bopp and J. Meixner, and translated by J. Kestin ***Thermodynamics and Statistical Mechanics*** - *Lectures on Theoretical Physics Volume V* (Academic Press, 1964)
- Arnold Sommerfeld *Partielle Differentialgleichungen der Physik - Vorlesungen über theoretische Physik Band 6* (Dieterich'sche Verlagsbuchhandlung, 1947)
    - Arnold Sommerfeld, translated by Ernest G. Straus ***Partial Differential Equations in Physics*** - *Lectures on Theoretical Physics Volume VI* (Academic Press, 1964)

The wiki bio is pretty interesting. His students and post docs included Heisenberg, Pauli, Debye, Bethe and Pauling all of whom are Nobels. It says " as of 1928, nearly one-third of the ordinarius professors of theoretical physics in the German-speaking world were students of Sommerfeld.[61] "

Felix Klein, by the way, was a math guy (1849-1925), who did Erlangen (age 23) then ETH then Leipzig then Gottingen where he hired Hilbert! His interests were geometry and group theory and applications of same to physics. Klein Bottle.

Leonhard Euler (1707-1783) -- born in Basel, Suisse, then to St. Petersburg in 1727, lodged with Bernoulli, all funded by Peter the Great to build up Russian science. Became Professor in 1731. Turmoil sent him to Berlin where he worked for 25 years! Fred Great Prussia's niece got Euler tutoring, which results in a historic book of letters (to a German princess, on line and PDF now have, I read a few letters, including about the ether, well written, as you would expect) . As Voltaire appeared, Euler fell out of favor with Fred. Had cataract blindness, but had a true photo memory, continued at 1 paper/week even in 1775. Returned to Russia for good in 1766, Petersburg, then fire killed his wife of 40 years, then he had a brain hem in 1783, RIP.

He invented the idea of f(x). Coined the letter "e" (Euler's number 2.7), Σ for sum, i for imaginary. Worked in early calculus and series expansions. Discovered that $e^{i\theta} = \cos\theta + i\sin\theta$, Euler's formula, and this Euler's identity that $e^{i\pi} = -1$. Invented calculus of variations which led to the Euler-Lagrange equations in the 1750's, ln function, gamma function, etc. Invented hypergeometric series! Hyperbolic trig functions, and some work on primes.

Basically, Euler in 1750 or so invented all the tools that I use every day in the year 2008! Certainly a very Grand Master by any measure. Did both math and physics.

Joseph Louis Lagrange (1736-1813). Succeeded Euler (being 30 years younger) in the top Berlin post. Italian. Then in 1787 he went to France and stayed for good. Lived through the revolution, established self at Ecole Polytechnique.

" Lagrange was one of the creators of the **calculus of variations**, deriving the **Euler–Lagrange equations** for extrema of functionals. He also extended the method to take into account possible constraints, arriving at the method of **Lagrange multipliers**. Lagrange invented the method of solving differential equations known as variation of parameters, applied differential calculus to the theory of probabilities and attained notable work on the **solution of equations**. He proved that every natural number is a sum of four squares. His treatise Theorie des fonctions analytiques laid some of the foundations of group theory, anticipating **Galois**. In calculus, Lagrange developed a novel approach to interpolation and **Taylor series**. He studied the **three-body problem** for the Earth, Sun, and Moon (1764) and the movement of Jupiter's satellites (1766), and in 1772 found the special-case solutions to this problem that are now known as **Lagrangian points**. But above all he impressed on mechanics, having transformed Newtonian mechanics into a branch of analysis, Lagrangian mechanics as it is now called, and exhibited the so-called mechanical "principles" as simple results of the variational calculus."

Lagrange points: this is a concept in 3-body mechanics. If two large masses orbit, there are 5 points of zero force, some more stable than others. L_3 is behind the sun, L_1 between sun and earth, L_2 behind earth so always in dark. The L_4 and L_5 (Trojan) points are symmetrically located as equilateral triangle tips where the two masses form the triangle bases. Goldstein does not mention these either. The 3-body problem is pretty complex you can be sure. Claim is that "halo orbits" around some Lagrange points are stable. We have put lots of stuff already at these points. I found a simple math analysis of these points and their stability and have it as a PDF.