Units for NMR Work PhL 3.28.08

I am "rusty" with SI units, and NMR people tend to use their own conventions, so now is time to get this mess cleared up a bit.

1. Preliminaries. I see I have already written units2.doc which goes into lots of detail with many digressions, but here let's look for something concise. That document eventually arrives at this point:

div**E** = $4\pi k1\ \rho$ curl **E** = - $(1/\alpha)\partial$**B**/∂t (Faraday) k1 = $c^2 \bullet$ k2

div **B** = 0 curl **B** = $4\pi\ \alpha$ k2 **J** + (α k2/k1) ∂_t **E**

where k1 and k2 and α are arbitrary constants except we must have k1/k2 = c^2 , speed of light squared. We usually have $\alpha = 1$, so let's eliminate that one right away, then we have

div**E** = $4\pi k1\ \rho$ curl **E** = - ∂**B**/∂t (Faraday) k1 = $c^2 \bullet$ k2

div **B** = 0 curl **B** = 4πk2 **J** + $c^{-2}\ \partial_t$ **E**

Simpler and more familiar equations are these, showing the same two constants k1 and k2:

F = k1*qQ/r^2 (1) / force between two charges

dF/dx = k2*2iI/D (2) / force between two parallel wires separated by D

Now, in the SI system we use

k2 = $(\mu_0/4\pi) = 10^{-7}$ nt $C^{-2}\ s^2$ // = kg m C^{-2}
k1 = $(1/4\pi\varepsilon_0) = 10^{-7}c^2 \approx 9 \times 10^9$ nt $C^{-2}\ m^2$ // = kg $m^3\ C^{-2}\ s^{-2}$

Both of these constants "carry dimensions" or "have units" in the SI system. From equation (2) above we find that k2 has units nt $C^{-2}\ s^2$ because that makes the units match on both sides of the equation. Then the fact that k1 = k2 c^2 tells us that the units of k1 must be nt $C^{-2}\ m^2$. We can regard this choice of constant k2 as "defining the Coulomb" in terms of other basic units.

The alternate forms on the right go down to basic units, replacing nt = kg m s^{-2} from F = ma.

The **electric field E** in any system of units is defined in terms of F = qE , so units of E are nt/C in SI.
The **potential V** is defined as E = -∇V and this tells us that nt/C = volts/m where we thus define the "volt" as the unit of potential V. Thus, volt = nt m C^{-1}.

The **magnetic field B** from Biot-Savart ($\alpha = 1$) is given by d**B** = k2* (Id**X** x **r**)/r^3 . We conclude that:

dB = (nt $C^{-2}\ s^2$) * C $s^{-1}\ m^{-1}$ = nt C^{-1} s m^{-1} = tesla // = kg $s^{-1}\ C^{-1}$

where here we define the tesla as shown.

Plank's constant $E = \hbar\omega$ so $\hbar$ = joules s = nt m s = kg m^2 s^{-1} = J s . As shown a few lines below, these are also the units of angular momentum.

The **magnetic moment** of a planar current loop of area A is given by **m** = I **A** where A points in the RHR sense based on the current direction. Therefore

magnetic moment = C s^{-1} m^2 /= joule/tesla from a few lines below! = J/T = Amp m^2

Here is another example: the Bohr magneton is $\mu_e = e\hbar/2m_e$ = C * kg m^2 s^{-1} * kg^{-1} = C m^2 s^{-1} which agrees with our magnetic moment dimensions.,

Now consider Levitt page 24 which says

$E = -\boldsymbol{\mu} \bullet \mathbf{B}$ // Levitt page 24

and $\boldsymbol{\mu}$ is a magnetic moment. We *should* then have

joule = C s^{-1} m^2 * tesla

Let's check it out

C s^{-1} m^2 * tesla = C s^{-1} m^2 * nt C^{-1} s m^{-1} = nt-m = joules // checks

Our next item of interest is the **gyromagnetic** ratio:

$\boldsymbol{\mu} = \gamma \mathbf{s}$ s = "the spin"

See also Bleaney page 20 where the spin is called G. In Bleaney, G has units of angular momentum

G = **angular momentum** = r x **p** = **r x mv** = kg m^2 s^{-1} // = nt m s = J s

The second form is more obvious from r x F = dJ/dt = N = torque. So what are the units of **gyro** γ ?

$\gamma = \mu / G = C\ s^{-1}\ m^2 / [kg\ m^2\ s^{-1}] = C / kg$ / = s^{-1} $tesla^{-1}$

Do this another way // above says

$\gamma = \mu / G = J\ T^{-1} / [J\ s] = s^{-1}\ T^{-1}$

where the last form just comes from the tesla units as noted above.

Aside: what are the units of γB?

γB = s^{-1} T^- * T = s^{-1} = ω-units = Larmor frequency.

Now, what is the angular momentum of a proton (ie, what is its spin):

spin $= 1/2\hbar$ = angular momentum units = J s

What is γ for a proton? First, from page 408 of Livesey, we are told that

$\mu_n = e\hbar/2M_p$ = "the nuclear magneton" $= 5.05 \times 10^{-27}$ A m^2

But this is not the proton's actual magnetic moment, it is the scale used for same. From page 410 we find that

$\mu_P = 2.79270\ \mu_N = 2.79270 * 5.05 \times 10^{-27}$ A m^2

We know that the proton's spin angular momentum is $\hbar/2$ which is (1/2) $1.0545 * 10^{-34}$ J s. Therefore we should find that

$\gamma_P = \mu_P / spin_P = 2.79270 * 5.05 \times 10^{-27} * 2 / 1.0545 * 10^{-34}$ A m^2 / [J s]

The units are J T^{-1} / [J s] = T^{-1} s^{-1} . The number is

$2.79270 * 5.05 * 2 / 1.0545 * 10^7 = 26.748477952 \times 10^7 = 267.48 \times 10^6\ T^{-1}s^{-1}$

and this agrees with Levitt page 14, we can just ignore his "rad" thing.

Now let's jump to page 204 of Levitt where we are talking dipole-dipole. We have

$b = - (\mu_0/4\pi)\ \gamma_1\gamma_2\hbar / r^3$

What are the dimensions of this "thing" ?

{ nt C^{-2} s^2 } { s^{-1} T^{-1} }^{2} { nt m s } /m^3

= nt^2 C^{-2} s m^{-2} T^{-2}

Let's replace T = nt C^{-1} s m^{-1} to get

= nt^2 C^{-2} s m^{-2} { nt C^{-1} s m^{-1}}$^{-2}$ = nt^2 C^{-2} s m^{-2} nt^{-2} C^2 s^{-2} m^2 = s^{-1}

So we conclude that b is a frequency of some sort, probably radians/second as he suggests. Basically, in this equation we have $b = \omega_b$ and the I's are dimensionless.

Now on page 149 Levitt tells us **" to get the true Ham from the script Ham, mult by $\hbar$ "**. So the units of a script Ham are always ω units, in agreement with what we just found.

Comments on Levitt page 204 on dipole-dipole:

1) the dimensions of script H are ω
2) the dimensions of b are ω
3) the spin operators are dimensionless.

Now look at Levitt page 175

Equation 7.9 says $\boldsymbol{\mu} = \gamma \mathbf{I}$. What units are implied in this equation ? If I were dimensionless, we woujld want to say

$$\mu_{\mathtt{physics}} = \gamma \hbar \mathbf{I}_{\mathtt{dimless}}$$

But what he has is this:

$$\mu_{\mathtt{NMR}} = \gamma \mathbf{I}_{\mathtt{dimless}}$$

So he is using

$$\mu_{\mathtt{NMR}} = \mu_{\mathtt{physics}} / \hbar$$

This then works right in equation (17.10) where we have

$$H_{\mathtt{NMR}} = - \mu_{\mathtt{NMR}} \bullet \mathbf{B}$$

So he never messes with the definition of γ itself, just μ and H.