Floaters inside the focal point PhL 12.1.02

Page 106 of Goodman, equation 5-22, I think has my answer! In this equation, suppose I put on the "input plane" a small circular aperture that fits within the reduced pupil function. Then we can ignore the pupil function and we have a standard Fraunhofer integral! The "distance" is z = d, as you can see comparing to equation 4-17. Note that d is the distance from the floater to the focal plane. Even though this distance might be very small, we are still getting the Fraunhofer result, due to the spherical wave coming onto the floater from the left. Note also that the field amplitude is boosted everywhere in this input plane by factor f/d compared to what it is at d = f, before we "cone down" the beam. [This is all fine, but 5-22 tells you the diffraction pattern as seen on the focal plane at z=f, not on any other plane!]

Now, for a circular aperture of radius "a", we know that we will get the Airy pattern on the focal plane. That pattern is this:

$$U(r) = \exp(ikd)\exp(ikr^2/2d)\,\pi a^2 * (1/i\lambda d) * 2J_1(kar/d)/(kar/d) \qquad \text{/ since distance is d} \qquad (1)$$

The diameter of the central lobe of the Airy function is $1.22\,\lambda\, d / a$. Suppose we also consider by superposition some normal image a distance d' to the left of the lens. We know from 5-19 that at the focal plane on the right we get the Fourier Transform of this source image scene, multiplied by a radially dependent phase (which only vanishes if d' = f on the left).

Now, suppose we look to the right of the focal plane where the scene image comes into focus. We know what the scene looks like there, the field has a magnification reduction by M, and a luminous gain factor or M as well. Thus, in this short distance, the scene image has completely transformed again from the Fourier transform on the focal plane (plus phase) back to the original image on the image plane.

So what does our Airy pattern look like on this same image plane?

Conjecture: I know that the circular aperture of radius a turns into the above Airy pattern at the focal plane after it propagates a distance d. Thus, if I let it propagate another distance d to the right of the focal plane, the aperture ought to once again appear as a hard circle (albeit upside down perhaps). Reciprocity seems to suggest this. Thus, if you are focused on an image scene such that the image plane happens to lie a distance d behind the focal plane, any floaters which happen to be floating d to the *left* of the focal plane should appear very sharply in focus, providing that d < f. Of course if the floater is an 8 μ blood cell, you probably cannot see anything because the images are too small.

Comment on (1) above. This is supposed to be what you get at the focal plane on the right if you put your circular aperture a distance d to the left of that plane. It is noted that the spherical wavefront effect on the aperture exactly cancels the Fresnel inside phase, so you get the exact Fourier transform. We have to remember, however, that we are then using the Fresnel formula to propagate this distance d, but that result is only accurate for d small compared to the transverse dimensions. Thus, we cannot take the limit as d = 0 and expect to get anything meaningful! [illogical?!] So this approach is not going to answer the question: what happens as a tiny floater approaches the focal plane on the axis!

Idea: Suppose we really did put a small circular aperture at the right side focal plane. We know that the Fourier transform of the original image is sitting there. The effect of the aperture would be to pass only the very low frequency parts of the transform, so on the image plane you would end up only with a crude DC looking image. Conversely, if you put a floater there, you remove some DC component, so the effect

is really to just darken the entire image perhaps a lot. If that image were pure white to start with, the Fourier transform would be a delta function at the floater, and we would then knock out the whole picture.

The Babinet question. We know that a circular aperture in a plane wave gives the Airy, which is bright in the center and then fades off to darkness as radius increases. Babinet says the disk field equals the plane wave minus the aperture field. I think it is safe to evaluate the Airy U(r) at r=0 and we get:

$$U(r=0)_{Airy} = \exp(ikz) * \pi a^2 / (i \lambda z)$$

Compare this with the $U_{plane} = \exp(ikz)$. So Babinet suggests that

$$U_{disk}(r=0) = \exp(ikz) [1 - \pi a^2 / (i \lambda z)] = \text{(plane) - (aperture)}$$

I have never seen this result quoted, at least it is dimensionally correct, and as z gets large, the effect of the disk goes away. I think Babinet does imply coherence. Let x = area/(λz) = A/λz as we have here. Then you can say

$$\text{Intensity}(r=0) = 1 + (A/\lambda z)^2 .$$

If you do this same thing at general r you get this:

$$| \text{field} |^2 = 1 + q^2 + 2 q \sin(kr^2/2 z) \qquad \text{where } q = (A/\lambda z) * 2 J_1(kar/z) / (kar/z)$$

As z gets large, this approaches $1 + (A/\lambda z)^2 + 2 (A/\lambda z) 2\pi r^2 / (2 \lambda z) = 1 + (A/\lambda z)^2 [1 + 2 (r/a)^2]$

How to interpret all these things? The first one alone suggests that at the central line, in the shadow of a disk things are actually brighter than the plane wave! That sure seems odd, but I guess the Poisson spot was also odd in the center of a dark shadow.

Error: now I think the disk should be the negative of the aperture field. I am out of time, heading to NYC!

[Note added: all these topics have been refined since that time, but let's print these notes anyway for future reference.]