Notes on Wyant's PDF on Fresnel Diffraction PhL 12.28.02

Fresnel is pronounced <fre-nel'>, not <fres'-nal>. Fresnel lived 1788-1827 and came up with the lighthouse lens trick, among other things.

Wyant is a Professor of Optical Sciences at Arizona, and is Director of the Optical Sciences Center there, the OSC. This OSC was formed in 1964 from government, industry, and academia. His home page is at www.optics.arizona.edu/jcwyant. He has many interesting documents there, which include Chapters of a "book" for his course Optics 505. My Fresnel notes just Chapter 13 of that book. I have now downloaded all the chapters of this book (some are missing or don't exist yet). I did this because Wyant's notes are excellent in quality!

Fresnel Zones. In Fraunhofer diffraction from a circular aperture, we are in a limit such that the entire hole radiates in phase to an observation point on the axis to the right. That is to say, in the Fresnel integral Goodman 4-17 we ignore the quadratic phase variation term and we get a simple Fourier Transform of the circular hole which is the Airy pattern. In the Fresnel case, we must keep this phase term and deal with it. The source of this factor is of course just the phase path difference that arises when you move away from the central axis of the hole, a fact of simple geometry, although we have kept only the first term in a geometric expansion, which is the "Fresnel" approximation.

So, when the circular aperture is "large" relative to the observation point, we have a complicated phase situation on the hole, and this is where the definition of Fresnel Zones becomes useful. To get the proper motivation, Wyant first does a calculation. Suppose we have a point source on the left (on axis), and we try to compute the diffraction field at an arbitrary point *on-axis* on the right hand side. He uses the point-source version of the Sommerfeld I formula, which appears as 3-43 in Goodman. It has an obliquity factor $\cos\theta$ which is 1 for our on-axis observation point. Using this formula, Wyant directly does the Fresnel integral in closed form, see page 3-4. We end up quickly with an integral of differential rings that make up the hole. If we write the integral in terms of a dimensionless variable q that gives the path excess to a point in the hole (on one of these differential rings) as a multiple of $\lambda/2$, we see an amazing fact. The integrand is nothing but the phase $\exp(i\pi q)$ and this just goes around in a circle as you do the integral. The integral is then simply $1 - \exp(i\pi q(R))$ where R is the hole radius. Now let R vary and think about what happens. When R is very small, we get the Fraunhofer central value [from a point source: note that there is a slight difference in the form of any result with a point source versus a plane wave source. The plane wave is dimensionless and is $\exp(ikx)$, whereas the point source is $\exp(ikr)/r$. This mystery is resolved by multiplying $\exp(ikx)$ times A where A has dimensions of inverse length.] The integral is the sum of the "1" vector and a rotating unit vector that starts off pointed to the left. When R=0, q(R)=0 and the integral is therefore 0. It is again 0 when q = 2,4,6 etc. And it is maximum of 2 (intensity is 4) when q = 1,3,5. We *interpret* this by saying that the hole is divided into the **Fresnel Zones**, and the contribution to the integral of the zones ending at q=1,3,5,7 are all equal and of positive amount, whereas the contributions of the zones ending at q=2,4,6 are all equal and of negative amount. So you interpret the integral by saying that the contributions from the zones *cancel* as you integrate out. It is certainly not "obvious" that the zones exactly cancel in this manner, but you see if must be true in the Fresnel approximation when you just look at this integral. Things are simple here because we are on-axis!

With this as motivation, we can look at how the zones are defined on page 1. The zone n=1 ends at a radius such that the added path length is $\lambda/2$. The zones end at $\rho n = \sqrt{n\lambda L}$ where L is z1 || z2. The areas of the zones are all equal, and are equal to $\pi\lambda L$. See picture.

Wyant next does the circular hole off-axis. He starts with the usual Fresnel formula with that extra phase and assumes a point source on the left (on-axis) as before. We end up with the double-integral shown as (12) on Wyant page 6, where variable nFZones is like the q used earlier, a dimensionless parameter that has integral values at the zone boundaries. Since we have radial symmetry of the hole, he can rewrite the interesting part as shown on that page in Mathematica language. Then assuming various interesting values for nFZones, I guess he lets Mathematica do the integrals numerically and he plots the results. What you see is this: as you get more zones, you move from the simple peaked Fraunhofer result to a more shadow with edge ripples result, visible even at nFZ = 2 on page 7.

The Fresnel Zone Plate.

Suppose we black out all the even zones on a zone plate and stick that in our circular aperture. Well, the zones are only really zones for one particular value of z2 on the right. For that value of z2, if we recompute the on-axis field, our simple integral can be written as shown in (14), where each term gives the same amount [that amount is -2, Wyant has a sign problem here] and the result is that the intensity at our axis position is enhanced by a factor N^2 where N is the number of odd zones in the hole. As we move to a slightly different value of z2 in either direction, the zone plates are no longer "correct" and the we get cancellations and a much smaller field. Since light emitted from the point at z1 on the left seems to be getting focused at a point z2 on the right, the plate acts as a lens that would do this same thing. For such a lens, we would say that focal distance was f = z1 || z2. [By the way, how do you "prove" this fact about a lens? Put a point source z1 to the left, it's wave to the left of the lens is $\exp(i\, k^2\, \rho^2 / 2z1)$. The lens then applies the phase shift $\exp(i\, k^2\, \rho^2 / 2f)$. And that must be $\exp(i\, k^2\, \rho^2 / 2z2)$, QED.] Therefore, the focal length of our Fresnel Zone Plate is $f = L = \rho_1^2 / \lambda$. If you want a focal length of 10 cm, your ρ_1 will be pretty small, so the plate will have lots of zones, and you get a "good" lens since N^2 will be large.

A plate made in this manner is called a binary amplitude zone plate. Instead of blacking out every other zone, you could apply a coating that shifts light an extra π in say the odd zones. In this case, all the zones are additive, and you get an even better lens, but this lens only works for a fixed frequency of light.

Edmunds does not sell zone plates, but they sell something similar which they call a Fresnel Lens. This is a flat lens with thin rings that refract at different angles and thus simulate a large fat lens. The rings are equally spaced, and have nothing to do with Fresnel zones. This Fresnel lens is the same idea as the Fresnel Lens in a lighthouse which is designed to gather up lots of light and waste as little as possible.

Rectangular Aperture. We get here the usual integral form. One usually picks a fixed observation point on axis on the right, and then one "translates" the square aperture to get the result. The result factorizes as usual in the two directions, and some re-scaled integration variables are defined, and the result can be expressed in terms of the Fresnel integral functions. (I fail to follow his last step which seems to be the on axis result). Wyant goes on to define the notion of a Fresnel Zone for this geometry -- you can think of a separate set of zones for the two directions. The zones here of course are just rectangular strip pairs, not rings. He defines two things, one is nFZones, the other w is related to this. All this is just setup for his plots. He does his plots, and you get the idea that you can move smoothly from the Fraunhofer limit which is the sinc function on page 17 to the shadow region on page 20. Of course we are doing a hole here, so the shadow region is bright. If you move one edge of a slot aperture off to infinity, you get the diffraction pattern of an edge which is the famous Cornu spiral thing, and a plot is shown here. He ends up with some computer plots of the field in the square slot and results are similar to the circle.

Summary: This guy has explained the notion of Fresnel Zones and why they are useful concepts for both circular and rectangular geometries, if nothing else to label plots. The Zone Plate idea as a lens was interesting as well. We got to see our Somerfeld I formula for a point source on the left being used many times. He makes the point that you can do FFT to do the integrations and get high speed. He has various Mathematica statements, I could look at what Maple could do as well. I guess my opinion of the author dropped just a little as I wrote these notes, but he has filled in this missing information for me!