Matrix Optics Tutorial Notes PhL 12.29.02

These notes were really quite good, but all in HTML files so could not get a nice print. The basic idea of the matrix is presented clearly right off the bat. In Chapter 3 author shows what the matrix is for simple propagation through a uniform medium, the **translation matrix**. The next matrix case is **refraction** at a curved boundary, such as one surface of a lens. He then shows how we can **concatenate** matrices to describe a complex optical system in Chapter 4.

Equation (22) puts some optical system between two air gaps, just to get us set up for what is yet to come. Then continuing his Chapter 4, author *hypothesizes* that some optical system has "**focusing capability**" as illustrated on the first page of "chapter 4 continued". If it can focus, that tells us certain things about the matrix elements! He assumes that it can focus on both sides, albeit not symmetrically. To make this work in terms of (22), we have to have both diagonal elements be 0, and this relates the distances to the focal planes (known as D and D') to the general optical component matrix elements:

D' = a22/a11 D = a11/a21

We end up then with (22) having two non-zero elements, to which we give names:

upper right element = (a12 - D*a22) = (a12 - a11*a22/a21) = -f ' f ' = **second focal length**
lower left element = a21 = 1/f f = **first focal length**

At this point, a little theorem is proved: if you concatenate any system of translation and refraction matrices, the determinant is always nR/nL Since our hypothetical focusing system is presumably made of these elements, it must have determinant nR/nL as well. But we can see from its form that the determinant can also be written as f ' / f, so we conclude that:

f ' / f = nR/nL

In particular, if you have the same index medium on both ends, then f ' = f and you have only one distinct focal length.

Notice, by the way, that the distance from the "walls" of the optical element to the focal planes is D and D', and these are not the same as f or f '.

In Chapter 5, we draw some focus-capable optical element along with its focal planes, and then we put an object x to the left of the left focal plane, and we try to make an image distance x' to the right of the right focal plane. Notice how these x and x' distances are carefully defined.

The notion of "making an image" means that all rays emanating from a point on the object must meet on a point on the image, so this is going to put a big constraint on the overall matrix for this situation. The upper right element of the overall matrix must be 0, and this tells you that x x' = f f '. You also get an expression for the magnification m = -f/x = -x'/f '. These last are Newton's Equations of image formation.

So I now think that ALL general optical systems described by the four a_{ij} numbers are going to be able to focus things in both directions. After all, if we assume this is true, we know how to calculate the two focal plane distances and the two focal lengths. Of course you might get f = infinity, as would be the case if you considered a system of just "air" (the translation matrix). The single curved surface gives a finite value if nL ≠ nR.

Chapter 5 continues with a discussion of the **cardinal points** of a lens system:

Cardinal Points:
positive and negative **principle points**
positive and negative **nodal points**
focal points

I think by "focal points" he means the location of the focal planes, which we have spotted at D and D' already. I only read about the "positive principle points" in the above set. The analysis shows that these points are both inside the focal planes as shown in the figure on page (39). These are planes where the relative magnification on the two sides is +1. It turns out that H is f to the right of F, and H' is f' to the left of F', as shown in the figure. I have trouble interpreting these points.

The negative principle points are easier to understand. If you have an object 2f to the left of H, the image will be 2f ' to the right of H', and the magnification will be exactly -1. For a thin lens where f = f ' , this occurs when we put the object 2f to the left and get an image (that is, a focussed image) at 2f on the right. The point is that with a general lens, things are not as simple as with a thin lens. If you want to do ray tracing with things like thick lenses, it helps to know where all these cardinal points are. I did not read at all about the nodal points.

In chapter 6 we finally specialize to the **thin lens** and its matrix is shown in (66). Author goes on to consider two thin lenses in series and then to treat the "**telescope**" and the "ocular". So this is where I stopped reading.

Now let's get back to the question of linear systems. We know that all the basic elements we use are linear -- the translation and the low-angle refraction. Thus, we know that any lens system we build out of such elements will be linear. I guess that is the only point. Sure, if you go to large angles, a spherical surface is no longer described in this linear fashion and things fall apart. I suppose you could tune the shape of a spherical surface (for some frequency at least) to compensate for the angle problem, and thereby maintain linearity as best you can.