Better Floater Radiation Models PhL 1.9.03

Up to now I have been using the Babinet idea with a simple hole in a screen and Kirchhoff. In this theory, it appears as if the hole itself "radiates" and creates the downstream Airy pattern, since you integrate over the empty hole. In reality, the Airy pattern is caused by radiation from the screen, and probably mainly from radiation near the edges of the hole. The Kirchhoff integral is a trick using Green's Theorem to tell you the field inside a volume in terms of the field on the surface of the volume. Having a conducting or opaque screen makes this problem doable for a >> λ.

What happens when you try to apply Kirchhoff to a disk directly?

Sommerfeld I applied to infinite-extent sources. Consider the Sommerfeld-I formula with its cosine obliquity factor applied to a plane wave hitting something like an edge or a disk. In this case, you have to take your ξ,η integral high up say in the y direction. The normal internal exp(ikr)/r factor makes a 1/r term, and the cosine = z/r, so you now have $1/r^2$. Let's use ρ as the radial variable in the ξ,η plane, so we now have $1/\rho^2$ inside the integral. If we integrate over that plane, we pick up ρ dρ, and so end up then with dρ/ρ. This integral is logarithmically divergent, so you cannot get an answer! The explanation is of course that in this case the assumption made in ignoring the infinite radial part in developing the Kirchhoff integral was wrong! The fix here is to replace the plane wave with a point source on the left. We then get an extra factor of 1/ρ coming from this source, leaving us with $d\rho/\rho^2$ which integrates to 1/ρ which converges at the large radius end. This I think is how the usual "edge" formula is derived. Physically, the field from the source is now dropping off as you go to larger ρ, allowing convergence. Therefore, I guess you could compute the field of a disk directly in this manner, and the Fraunhofer limit should agree with Babinet! I will do this calculation soon.

Countermand the Above Paragraph! Now that I understand integrals better, I realize that the phasor stops the logarithmic divergence at the high end. Therefore, maybe I should go back and try again with my incident plane wave. At least it does not have the electric field direction ambiguity that you get with a point source which clouds the application of the scalar Kirchhoff theory at large angles.