Some notes on the Fraunhofer diffraction pattern of a circular disk PhL 1.3.03

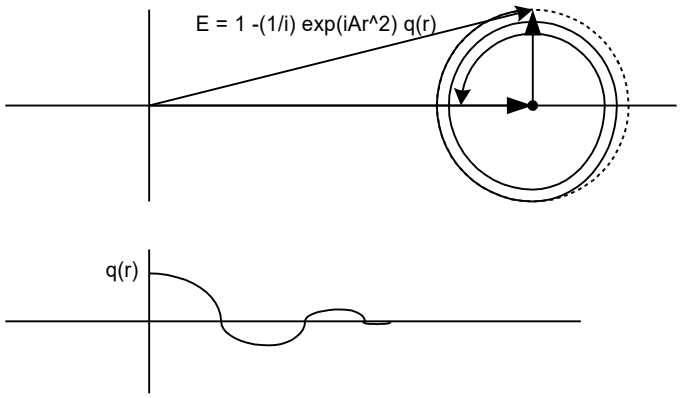
The phase of the incoming unit amplitude plane wave is just exp(ikz), so in the Babinet field difference we can factor this out of both Ein and Airy and we get this result:

$E2 = e^{ikz} * [\ 1\ -\ (1/i)\ e^{iAr\hat{}2}\ q(r)\ ]$ $A = k/2z$

where

$q(r) = B * 2\ J_1(Cr)/(Cr)$ $B = (\pi a^2)/(\lambda z)$ $C = ka/z$

Remember that q(r) is a sinc like function, and B <<1 to be in the Fraunhofer limit. Looking at the form of E2, ignoring the outside overall phase, what you see is a vector to the point "1" to which is added a second vector $i\ e^{iAr\hat{}2}\ q(r)$. As r increases from 0, this second vector which starts at 90 degrees starts to rotate to the left at an accelerating rate (since r^2). The magnitude of the second vector is |q(r)| which starts at value B at r=0 and shrinks and grows as that sinc-like function varies, but the magnitude never reaches this initial value of B again. So here is a Visio graphic to show the construction:



Without doing any more, you can SEE what |E(r)| is going to look like. At r = 0 it has a larger than 1 value, then it drops down to a smaller value, and then oscillates around until eventually it zeros in on value 1 because q(r) approaches 0, being sinc-like. The second smaller vector is going to follow a spiral pattern like that shown in the figure. The starting amplitude is $\sqrt{1+B^2}$ which then starts to decrease during the first turn. Depending on the values of the parameters, the amplitude could then reach an even larger value of 1 + B if it reaches the far side of the circle. So 1+B is an upper bound on the amplitude at all values of r, and 1-B is a lower bound.

Simple algebra gives this result which of course implies the same shape:

$$|E(r)|^2 = 1 - 2\ \sin(Ar^2)\ q(r) + q(r)^2$$

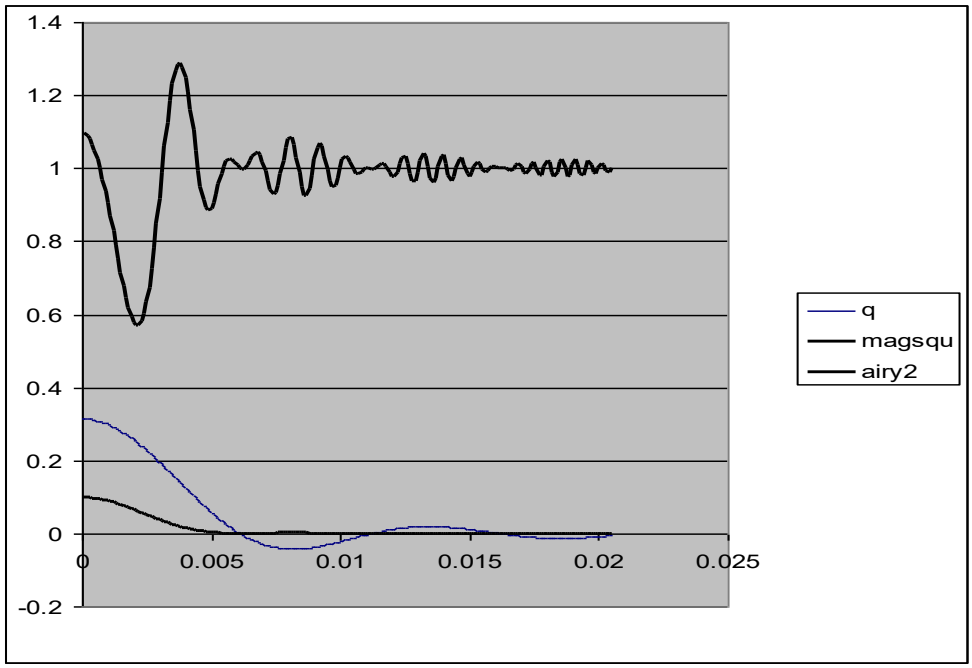
where you can see that $|E(0)|^2 = 1 + q(0)^2 = 1 + B^2$ which is in fact > 1.

So now we get a result which agrees with that PDF file. It says that in the diffraction pattern of a circular disk, the center is slightly brighter than the incident wave, the magnitude then drops off, and then oscillates toward 1, perhaps reaching a value larger than the central value. The central bright area is the

remnant in the Fraunhofer limit of the Poisson bright spot in the geometric shadow in the Fresnel near-field case. Here is a specific Excel plot with the following values:

$\lambda = .5\ \mu$ a = 1 mm z = 20m

A	314159.3 minv2
B	0.314159 dimless
C	628.3185 minv
d	0.0122 lobe width



The thin curve is just the function q(r), sinc-like as noted, and q(0) = B = .3. The lower heavy curve is the Airy intensity pattern you would get from a *hole*, while the upper curve is the intensity pattern resulting from the *disk* of the same diameter according to Babinet, and these two intensities are properly scaled. We can see that the Airy pattern of the hole is fairly subdued, half-width is about 5 mm. In contrast, the pattern of the corresponding disk has a dramatic swing going from about .58 to 1.3 in intensity (this is the square of the field, so $(1-B)^2 = .49$ is the lower bound).