

Survey of Optical Systems

Last time:

Developed tools to analyze optical systems

- ray matrix technique
- thick lens picture

Today, look at several common systems

Won't use matrix methods explicitly

but many lenses thick:

implicitly use thick lens picture

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Outline:

- the eye
- eye glasses
- magnifying glass
- microscope

These and more examples: Hecht 5.7

Next time:

advice for designing systems of your own

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The Eye (Hecht 5.7.1)

Most basic optical system

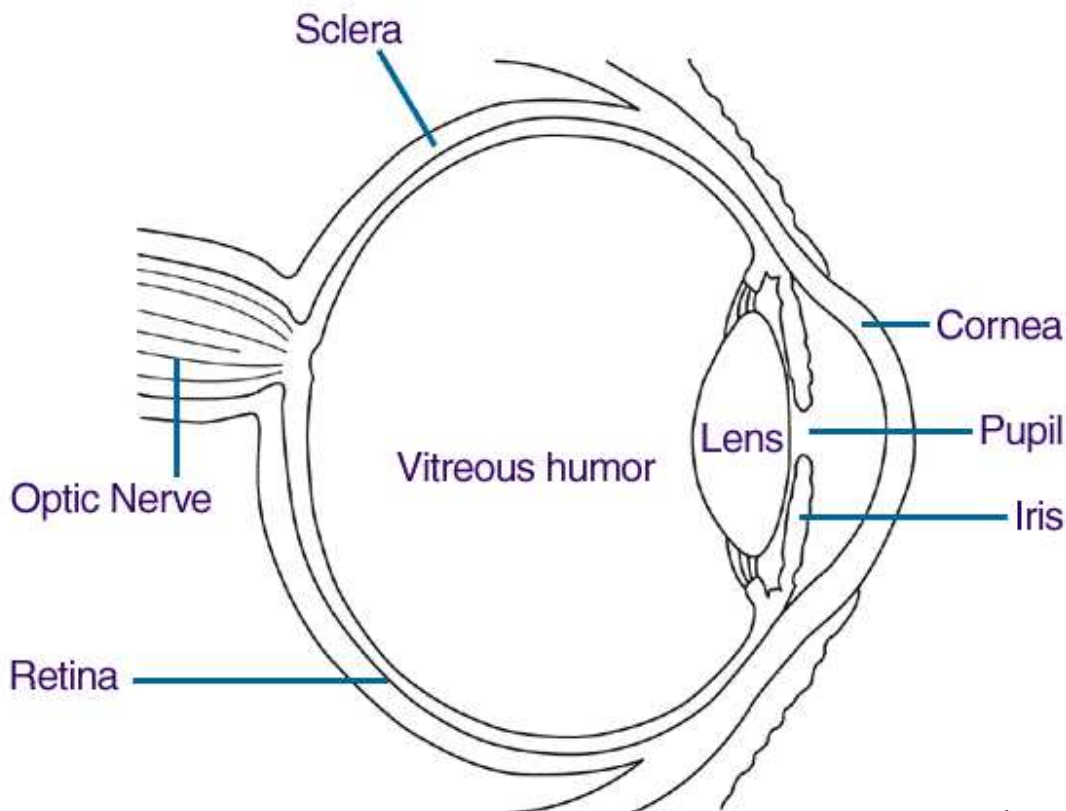
Components:

- cornea: $n = 1.376$
- vitreous humor $\approx$ water: $n = 1.33$
- lens: $n = 1.39 - 1.41$
 - index varies: high in center, low at edges
- iris: variable aperture stop
 - diameter 2-8 mm
- retina: detector

Again, use thick lens picture

lens system has well defined focal length

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Note : detector is in medium $n_i = 1.33$

Lens equation becomes

$$\frac{1}{s_o} + \frac{n_i}{s_i} = \frac{1}{f_o} = \frac{n_i}{f_i}$$

f_o = object focal length

f_i = image focal length

Irrelevant for ray matrix

Modifies thick lens picture:

f_o applies to front focal point

f_i applies to back focal point

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System focal length variable

max object distance = ∞ (“relaxed”)

min object distance ≈ 25 cm (varies)

What focal lengths?

Distance from lens to retina ≈ 24 mm $\approx s_i$

Relaxed eye: $s_o = \infty \Rightarrow f_i = s_i$

For $s_o = 25$ cm: $\frac{n_i}{f_i} = \frac{1}{s_o} + \frac{n_i}{s_i} \Rightarrow f_i = 22$ mm

So $f_i = 22 - 24$ mm

Called “accommodation”

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Most focusing power from cornea-air interface

$$f_i = \frac{n}{n-1}R$$

$$R \approx 9 \text{ mm} \Rightarrow f_i \approx 33 \text{ mm}$$

Remaining surfaces:

$$\frac{1}{f_{\text{total}}} \approx \frac{1}{f_{\text{cornea}}} + \frac{1}{f_{\text{rest}}}$$

So

$$\frac{1}{f_{\text{rest}}} = \frac{1}{24} - \frac{1}{33} = \frac{1}{88 \text{ mm}}$$

for relaxed eye

$f_{\text{rest}} = 66 \text{ mm}$ for accommodated eye

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f_{rest} adjusted by squeezing lens

muscles relaxed: f long

muscles tense: f short

Minimum achievable $s_o = \text{near point}$

depends on flexibility of lens

varies with age

Question: Why can't you see well under water?

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Capabilities of the eye

Resolution:

angular resolution $\Delta\theta \approx 0.017^\circ = 0.3 \text{ mrad}$

Just adequate to resolve crescent of Venus

Corresponds to about $5 \mu\text{m}$ on retina

At $s_o = 25 \text{ cm}$,

spatial resolution $= s_o \Delta\theta = 75 \mu\text{m}$

Also, wide field of view:

corresponds to 100 Mpixels!

Resolution best in center

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Sensitivity:

Fully expanded pupil, can see $I \leq 10^{-10} \text{ W/m}^2$
from point source

Power $= IA$

Area $= \pi(4 \text{ mm})^2 \Rightarrow P \approx 10^{-14} \text{ W}$

Maximum irradiance:

sunlight $I \approx 250 \text{ W/m}^2$

pupil area $\pi(1 \text{ mm})^2$

Max power $= 10^{-3} \text{ W}$

But: sun is not point source

power spread out on retina

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Sun subtends angle $10 \text{ mrad} \approx 30 \times \Delta\theta$

Same intensity from point source:

illuminate area $30^2 \times$ smaller on retina
 $\approx 1000 \times$ higher image irradiance

Max power from point source $\approx 10^{-6} \text{ W}$
($\approx$ damage threshold for laser)

Dynamic range of eye: 10^{-14} to 10^{-6} W
eight orders of magnitude

Instantaneous range lower:
 $\sim$ five orders of magnitude

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Best artificial detectors:

photographic film

high-end CCDs

dynamic range $\approx$ four orders of magnitude

$10 \times$ worse than eye

Upshot:

Can't build a detector nearly as good as the eye

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Eyeglasses (Hecht 5.7.2)

Common problem: focal length of eye isn't right

Too strong = near sighted = myopic:

relaxed eye has $f_i < 24$ mm

$$\frac{1}{s_{o\max}} = \frac{n}{f_i} - \frac{n}{s_i} > 0$$

so can't focus at ∞

Maximum distance of focus = *far point*

Easy to measure

For me, far point ≈ 25 cm

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Also moves near point closer:

for me $s_{o\min} \approx 7$ cm

What is my range of f ? (assuming $s_i = 24$ mm)

$$\frac{1}{f_{\min}} = \frac{1}{1.33 \cdot 70 \text{ mm}} + \frac{1}{24 \text{ mm}}$$

$$\Rightarrow f_{\min} = 19 \text{ mm}$$

and

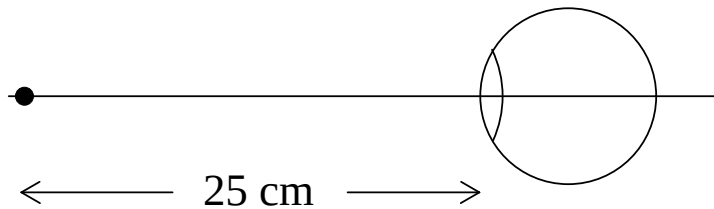
$$\frac{1}{f_{\max}} = \frac{1}{1.33 \cdot 250 \text{ mm}} + \frac{1}{24 \text{ mm}}$$

$$\Rightarrow f_{\max} = 22 \text{ mm}$$

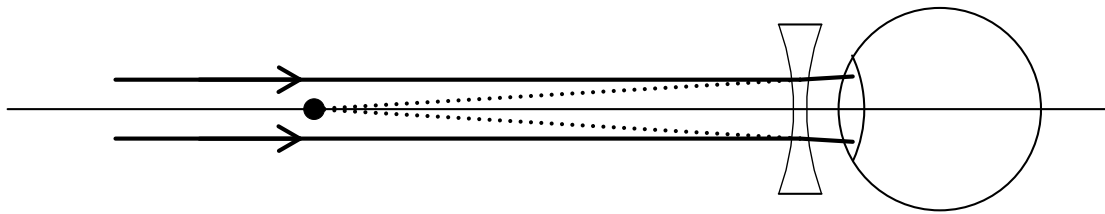
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Fix with eye glasses

Relaxed eye:



Add lens to put image of ∞ at 25 cm:



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What focal length required?

want $s_o = \infty$, $s_i = -25$ cm

So $f = -25$ cm

This is my prescription:

$$\mathcal{D} = \frac{1}{f} = -4 \text{ diopters}$$

How close is near point with glasses on?

s_o such that $s_i = -7$ cm for $f = -25$ cm

$$\frac{1}{s_o} = -\frac{1}{25} + \frac{1}{7} = \frac{1}{10} \text{ cm}$$

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Other vision problems:

Far sighted = hyperopia:
eye's lens too weak

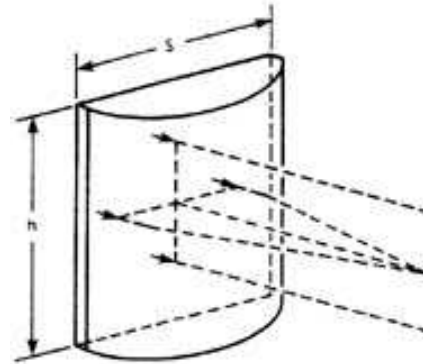
Correct with positive lens

Astigmatism:

asymmetry in lens

f 's different along x, y

Correct with cylindrical lens



Question: If you want to start a fire with your glasses, should you be near-sighted or far-sighted?

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Magnifying glass (Hecht 5.7.3)

At 25 cm, typical eye can resolve $75 \mu\text{m}$

Use a lens to see something smaller...
what kind?

Want erect, magnified image of real object

- Real object: $s_o > 0$
- Erect: $m = -s_i/s_o > 0$ so $s_i < 0$
- Magnified: $m > 1$ so $|s_i| > |s_o|$

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Have

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Want $1/s_o$ positive and large

$1/s_i$ negative and small

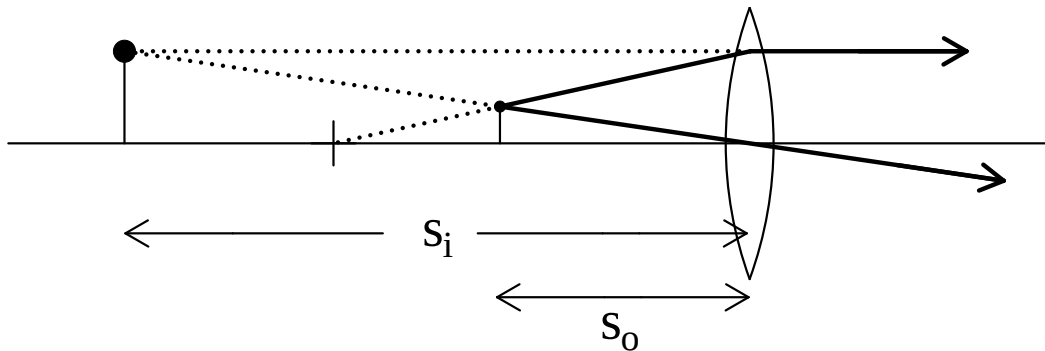
Means f should be positive

Recall: get virtual image with positive lens

when $s_o < f$

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Picture:



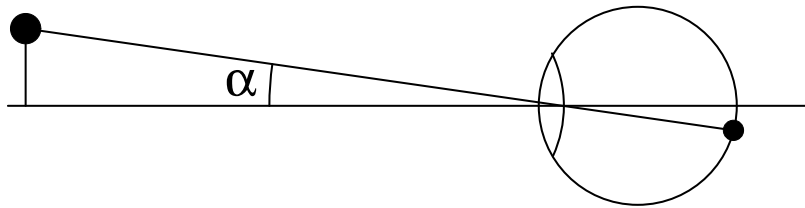
See image is magnified

But also further away...

Resolution improved if image on retina is bigger

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Note size of image on retina proportional to angular size of object



Size on retina $= \alpha f$

Don't really know f , just consider α

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Define *magnifying power* of system (MP)

= angular magnification

$$= \frac{\alpha \text{ with lens}}{\alpha \text{ without lens}}$$

Write as $MP = 5\times$, etc.

Could make α without lens very big:

hold object right up to eye

But can't focus if $s_o < \text{near point}$

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For magnifying glass, microscope, etc
(not telescope)

Define α for standard distance $s_o = 25$ cm

Example:

If I take off my glasses, near point is 7 cm

Object at 7 cm subtends $\alpha = y/7$ cm

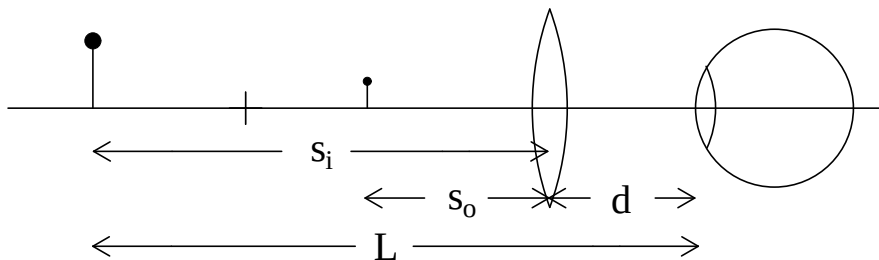
Object at standard 25 cm subtends $\alpha_0 = y/25$ cm

$$\text{Magnifying power} = \frac{\alpha}{\alpha_0} = \frac{25}{7} = 3.6$$

My bare eyes have MP = 3.6×

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What is MP of glass?



Express in terms of practical parameters

d = distance from eye to glass

L = distance from eye to image

f = focal length of glass

Have object size y_o

image size y_i

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Angular size of image $\alpha = \frac{y_i}{L} = \frac{my_o}{L}$

magnification $m = -s_i/s_o$:

$$\alpha = -\frac{s_i y_o}{s_o L}$$

Without glass $\alpha_0 = \frac{y_o}{d_o}$

$$\text{So MP} = \frac{\alpha}{\alpha_0} = -\frac{s_i d_o}{s_o L}$$

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Eliminate s_o, s_i :

Have $s_i = d - L$

and $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$

$$\text{So } \frac{s_i}{s_o} = \frac{s_i}{f} - 1 = \frac{d - L}{f} - 1$$

$$\text{Gives } \boxed{MP = \left(1 + \frac{L - d}{f}\right) \frac{d_0}{L}}$$

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Two reasonable ways to use:

- Make $L \rightarrow \infty$

Achieve by making $s_o \rightarrow f$ (so $s_i \rightarrow \infty$)

View image with relaxed eye, d doesn't matter

$$\text{Get MP} \rightarrow \frac{d_0}{f}$$

Large MP for small f

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Other method:

- Lens close to eye $d \rightarrow 0$

$$\text{MP} \rightarrow d_0 \left(\frac{1}{L} + \frac{1}{f} \right)$$

To get large MP, want L small

minimum $L = \text{near point} = d_0$

$$\text{Then MP} \rightarrow 1 + \frac{d_0}{f}$$

Large if f is small

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Where do we need to put object?

$$\begin{aligned}\frac{1}{s_o} &= \frac{1}{f} - \frac{1}{s_i} \\ &= \frac{1}{f} + \frac{1}{d_0} \\ &= \frac{1}{d_0} \left(1 + \frac{d_0}{f} \right)\end{aligned}$$

So $s_o = \frac{d_0}{\text{MP}}$

Recall eye example: $\text{MP} = d_0/s_o \dots$ same

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Object looks as it would if you could focus at s_o

Lens makes eye stronger
like being near-sighted

Works well if you can hold object up to eye

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Either method works up to about $4\times$
 f down to 6 cm

For higher MP, not paraxial:

- lens aberrations important
- requires more complex lens

Still works:

Method 2: Jeweler's loupe

Get MP up to $30\times$

Impractical if object position fixed
or MP so high you can't hold steady

Already a problem at $10\times$

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Method 1: put lens very close to object
since $s_o \approx f$ and f is small

Problem: exit pupil is small and far away
- can't see very much

Question: Where is the exit pupil in this case?

Solution: compound microscope

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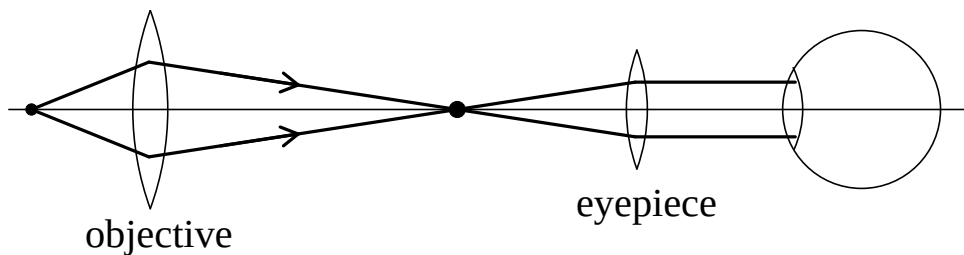
Microscope (Hecht 5.7.5)

Use two lenses:

objective: short f , close to object

eyepiece: collects light, match to eye pupil

Typical system:



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Typically don't care about inversion:

- object creates real inverted image

Objective collects rays at steep angles:

- important to control aberrations

Eyepiece:

- puts final image at ∞
 - view with relaxed eye
- provides additional magnification
- matches exit pupil to eye

Aberrations less important than in objective

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What is magnifying power?

Objective: magnification $= -s_i/s_o$

Angular magnification not appropriate:

intermediate image not viewed by eye

Even so, there is a standard length scale

Want objectives interchangeable:

standard position for object, image

Set by *tube length*

= distance from back focal point to image

= 160 mm

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Then $s_i = 160 \text{ mm} + f$

$$\frac{1}{s_o} = \frac{1}{f} - \frac{1}{s_i}$$

$$\text{MP} = \frac{s_i}{s_o} = \frac{s_i}{f} - 1$$

$$= \frac{160 \text{ mm} + f}{f} - 1 = \frac{160 \text{ mm}}{f}$$

This is magnification written on objective

So 20 $\times$ objective has $f = 8 \text{ mm}$

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$$\text{Total MP} = \text{MP}_{\text{obj}} \cdot \text{MP}_{\text{eyepiece}}$$

$$= \frac{160 \text{ mm}}{f_{\text{obj}}} \cdot \frac{250 \text{ mm}}{f_{\text{eyepiece}}}$$

where MP for eyepiece follows standard convention

Typical objective: 5× to 60×

Typical eyepiece: 5× or 10×

Warning:

Fancy microscopes don't follow these conventions

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Summary:

- Eyes are impressive instruments, both for sensitivity and resolution
- Eyeglasses/contacts work by adjusting location of near and far points
- Effect of magnifying glass is really *angular* magnification
 - Measure with magnifying power
- Microscope uses two lenses to provide more MP than glass

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