Floaters as Dielectric Spheres PhL 1.28.03

Status Report. After doing my back yard experiments with hard black dots, I was puzzled that none of my floaters showed up as such hard black dots. I was able to just detect hard black dots of 1/6" diameter on white paper at a distance of 8 yards in full sunlight in one eye with my best glasses, implying a retina shadow diameter of about 6 μ (I just checked this calculation). So I was left with the obvious question: if floaters are red blood cells of diameter 8 μ which leak from the retinal blood vessels, why don't I see some small hard black dots caused by those that are very close to the retina? I never see such things. My answer to this question was and is: they must be partially transparent. Since my opaque-disk model for the floater did not allow for this, I digressed to find a better model. Now I think the best model is a dielectric sphere which is filled with a cyto-fluid that has an average index that is slightly larger than that of water. For active cells, I have seen numbers that n = 1.37 for the inside fluid of a cell, versus 1.35 for the extracellular fluid. Here are some other numbers I quote from the web

for light (of wavelength 589.3 nm),
n = 1 in a vacuum;
n = 1.0003 in air;
n = 1.333 in water;
n = 1.336 in vitreous humour (inside the eye);
n = 1.413 in the eye's lens;
n = 1.52 in crown glass;
n = 1.61 in flint glass, and
n = 2.42 in diamond.

Since the diffraction pattern of a dielectric object is proportional to Δn, this provides a good mechanism for the transparency I am seeking. So I am now ready to resume my model building.

What is the close-in shadow of a dielectric sphere?

I am thinking of a sphere of radius 4 μ, light with λ = 1/2 μ. Our only real assumption in the final field formulas is that kR >> 2π so we can drop 1/R terms at various stages, such as in B = ∇ x A and in getting E after that. If we stay perhaps 10 μ behind the sphere, we have kR = 2π 10/(1/2) so 5 >> 1 roughly speaking, so our formula should at least be reasonable, and we expect to be in the "geometric shadow" area, which is what we now want to learn about. The field E at the shadow is given by

$$\mathbf{E} = k^2 \chi (4\pi a^3) \frac{e^{ikr}}{r} \left[\frac{\cos(2ka \sin(\theta/2))}{(2ka \sin(\theta/2))^2} - \frac{\sin(2ka \sin(\theta/2))}{(2ka \sin(\theta/2))^3} \right] (\mathbf{E_0} - E_{0r} \hat{\mathbf{r}})$$

We developed the theory from Jackson assuming our radiator was embedded in n=1 vacuum. But now we want it embedded in something that also has a dipole moment like water. Using superposition, we can treat the problem as if the sphere had a χ value that was $\chi_{eff} = \chi 1 - \chi 2$ arising from ε1 - ε2. We know from an earlier document then that $\chi_{eff} = \Delta n/2\pi$ so we rewrite the above formula as:

$$\mathbf{E} = k^2 \Delta n (2a^3) \frac{e^{ikr}}{r} \left[\frac{\cos(2ka \sin(\theta/2))}{(2ka \sin(\theta/2))^2} - \frac{\sin(2ka \sin(\theta/2))}{(2ka \sin(\theta/2))^3} \right] (\mathbf{E_0} - E_{0r} \hat{\mathbf{r}})$$

So now we have some idea of the overall scale of things, thinking that maybe $\Delta n = 1.37\text{-}1.337 = .033$. The bracket factor is on the order of 1 or less. Rewrite again as

$$\mathbf{E} = \mathbf{2}\ (ka)^2 \Delta n \frac{a}{r} e^{ikr} \left[\frac{\cos(2ka \sin(\theta/2))}{(2ka \sin(\theta/2))^2} - \frac{\sin(2ka \sin(\theta/2))}{(2ka \sin(\theta/2))^3} \right] (\mathbf{E_0} - E_{0r} \hat{\mathbf{r}})$$

Let's set $ka = 2\pi a/\lambda = 2\pi * 8 = 16\pi$, and this squared is 2527, so we now have $\mathbf{2}\ (ka)^2 \Delta n = 2*2527*.033 = 167$, so

$$\mathbf{E} = 167 \frac{a}{r} e^{ikr} \left[\frac{\cos(2ka \sin(\theta/2))}{(2ka \sin(\theta/2))^2} - \frac{\sin(2ka \sin(\theta/2))}{(2ka \sin(\theta/2))^3} \right] (\mathbf{E_0} - E_{0r} \hat{\mathbf{r}})$$

Question #1: How can the radiated field be stronger than the incoming field? First of all, as long as $kr >> 1$, you can ignore the dropped gradient term, since the factor there is $(1 - 1/ikr)$. If we agree to stay at least "a" away from the sphere center, this is $2\pi a/\lambda = 6*8 = 48 >> 1$, so that is fine. Second of all, look back at the field from a point dipole:

$$\mathbf{B} = k^2 \frac{e^{ikr}}{r} \hat{\mathbf{r}} \times \mathbf{p}$$

This is max to the side of p where sin=1. There is no limit to how large this B can be as r gets smaller, as long as we keep $kr >> 1$. So I guess large B and E is OK, just seems odd.

Question #2: How can the radiated field be infinite in the forward direction? Answer is it is not, it just looks that way. Fine. In fact, the value in the forward direction is -1/3.

Question #3: What does the form factor look like?

$$f(\theta) = \left[\frac{\cos(2ka \sin(\theta/2))}{(2ka \sin(\theta/2))^2} - \frac{\sin(2ka \sin(\theta/2))}{(2ka \sin(\theta/2))^3} \right.$$

As noted, the forward direction has $f(\theta) = -1/3$, as you learn by putting the first two terms in both sine and cosine expansions. Secondly, the zeros are at $\tan[\ 2ka \sin(\theta/2)] = [\ 2ka \sin(\theta/2)]$. Looking at $\tan x = x$ we find that $x = 4.4935$ for the first solution, so the first zero will be at:

$\sin(\theta/2) = 4.4935/(2ka) = .358\ (\lambda/a) = .0447$ small enough so $\theta/2 = .0447$
and $\theta = .0894 = 5.1$ degrees

and the Excel plot exactly confirms this. So for these values at least, $f(\theta)$ has a negative hump down to -.33, and then hits its zero at 5 degrees, and after that it is very small, goes into decreasing oscillation as you would expect. In general expect the angle of diffraction to be on the order of λ/a radians or 1/8 radian or 7 degrees ball park, so our answer is fine.

Question #4: How wide is the central peak on the retina? Think of this θ as the half angle of the central peak. So diameter on retina of the central peak D is:

$D/z = 2*\tan\theta = 2\theta = 2*.0894 = 0.18$ so $z = 4\mu$ means $D = .7\ \mu$, too small to see.

So we have to increase z from our starting point of $4\mu = a$ and move out to about $z = 6\mu/.7\mu * 4 = 34\ \mu$. At this distance, the central hump will be about 6 μ wide, and maybe then we can see it, based on the back yard experiments. So with $z = 34\mu$, we are saying $r/a = 34/4 = 17/2$. Our formula above becomes

$$\mathbf{E} = 167\,\frac{4}{34}\,e^{ikr}\,(-1/3)\,\left(\mathbf{E_0} - E_{0r}\,\hat{\mathbf{r}}\right) = -6.5\,e^{ikr}\,\left(\mathbf{E_0} - E_{0r}\,\hat{\mathbf{r}}\right)$$

So we are claiming that we should just start seeing the sphere shadow when it is 34 μ away from the retina, at which point the central shadow hump is 6 μ wide. At this point, the radiation field is about 6.5 times stronger than the incident field.

But, hold the phone. In deriving the field formula, we assumed $E_{rad} << E$, and obviously this is not the case, so we have to go fix up our derivation! I will resume here after doing some other things.