

Example

**THE FREE CHARGE DENSITY σ_f ,
THE BOUND CHARGE DENSITY σ_b ,
AND THE ELECTRIC DISPLACEMENT D
AT A DIELECTRIC-CONDUCTOR BOUNDARY**

At the interface between a dielectric and a conductor there is a bound surface charge density on the dielectric and a free surface charge density on the conductor. For example, at the lower plate of the capacitor considered above,

$$\sigma_b = \mathbf{P} \cdot \mathbf{n} = -P, \quad (3-64)$$

since $\mathbf{n}$ is the outward normal to the dielectric surface and points downward, while the electric polarization $\mathbf{P}$ points upward like $\mathbf{E}$. Thus

$$-\sigma_b = P = D - \epsilon_0 E, \quad (3-65)$$

$$= \frac{\epsilon_r - 1}{\epsilon_r} D = \frac{\epsilon_r - 1}{\epsilon_r} \sigma_f, \quad (3-66)$$

and

$$\sigma_f = \frac{\sigma_f}{\epsilon_r}, \quad (3-67)$$

as in Eq. 3-60.

Since $\epsilon_r > 1$, the bound surface charge density σ_b and the free surface charge density σ_f have opposite signs. The bound charge density is also smaller in magnitude than the free charge density by the factor $(\epsilon_r - 1)/\epsilon_r$. This relationship is always true whenever a Class A dielectric is in contact with a conductor, or carries a free surface charge density.

The free charge density σ_f on a conductor in contact with *any* dielectric is always equal to the electric displacement D just inside the dielectric. This follows from Gauss's law, as in the case of the parallel-plate capacitor. Furthermore, if a conducting surface is introduced into a dielectric so as to coincide with an equipotential surface, then the free surface charge density σ_f on the conductor at a particular point is equal in magnitude to the electric displacement D in the dielectric at that point.

Example

DIELECTRIC SPHERE WITH A POINT CHARGE AT ITS CENTER

Let us consider a Class A dielectric sphere of radius R with a point charge Q embedded at the center, as in Figure 3-8. (It is possible to trap negative charges within dielectrics by bombarding them with high-energy electrons. This example is of little practical interest, but it proves to be an excellent illustration of the behavior of dielectrics.)

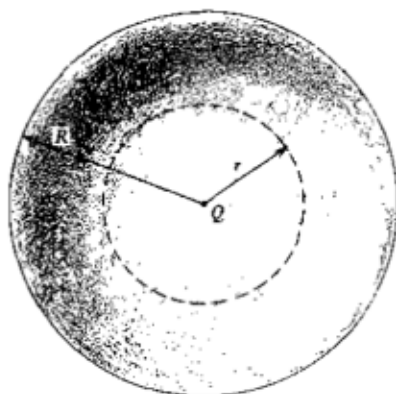


Figure 3-8. Dielectric sphere with a point charge Q at the center.

Again, we can find D from Gauss's law. We draw an imaginary sphere of radius $r < R$ and use it as a Gaussian surface. Then

$$D = \frac{Q}{4\pi r^2} r_1, \quad (3-68)$$

$$E = \frac{Q}{4\pi\epsilon_0\epsilon_r r^2} r_1. \quad (3-69)$$

$$P = \frac{\epsilon_r - 1}{\epsilon_r} D = \frac{\epsilon_r - 1}{\epsilon_r} \frac{Q}{4\pi r^2} r_1. \quad (3-70)$$

At the outer surface of the sphere,

$$\sigma_b = P \cdot n = P = \frac{\epsilon_r - 1}{\epsilon_r} \frac{Q}{4\pi R^2}. \quad (3-71)$$

The total amount of bound charge on the outer surface is thus

$$Q_{b0} = \frac{\epsilon_r - 1}{\epsilon_r} Q. \quad (3-72)$$

Since there is no volume density of free charge, there should be no volume density of bound charge, as we saw in the last section. This is correct:

$$\rho_b = -\nabla \cdot P = -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 P) = 0. \quad (3-73)$$

There is also a bound charge on the surface of the cavity containing the free charge Q at the center. This can be calculated as follows. Let the radius of the cavity be δ . Then, if $-\sigma_{b0}$ is the density of bound charge on the cavity surface, the total bound charge on the cavity is

$$Q_{b0} = -\sigma_{b0} 4\pi\delta^2 = -P_\delta 4\pi\delta^2 = -\left(\frac{\epsilon_r - 1}{\epsilon_r}\right) D_\delta 4\pi\delta^2, \quad (3-74)$$

$$= -\left(\frac{\epsilon_r - 1}{\epsilon_r}\right) Q. \quad (3-75)$$

The dielectric as a whole therefore remains neutral, as must be expected. The net charge at the center is then

$$Q_{\text{net}} = Q - \left(\frac{\epsilon_r - 1}{\epsilon_r} \right) Q = \frac{Q}{\epsilon_r} \quad (3-76)$$

and is smaller than the free charge by the factor ϵ_r . This accounts for the reduction of the electric field intensity within the dielectric by the factor ϵ_r , as in Eq. 3-69.

It is instructive to compare the electric field intensity outside the sphere and that inside. Outside, from Gauss's law,

$$D = \frac{Q}{4\pi r^2} r_1, \quad (3-77)$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} r_1, \quad (3-78)$$

which is to be compared to Eqs. 3-68 and 3-69 for the field inside the sphere. At the surface of the sphere the electric field intensity is discontinuous, the magnitude just outside the surface being ϵ_r times as large as the magnitude just inside the surface. The difference is due to the bound charges, which produce an opposing field within the dielectric.

Example

THE BAR ELECTRET

An electret is the electric equivalent of a bar magnet. In most dielectrics the polarization disappears immediately when the electric field is removed, but some dielectrics retain their polarization for a very long time. Indeed, several polymers have extrapolated lifetimes of several thousand years at room temperature.

One way of charging a dielectric is to place it in a strong electric field at some high temperature. The bound charge density on the surfaces then builds up slowly as the molecules orient themselves. Free charges are also deposited on the surfaces when sparking occurs

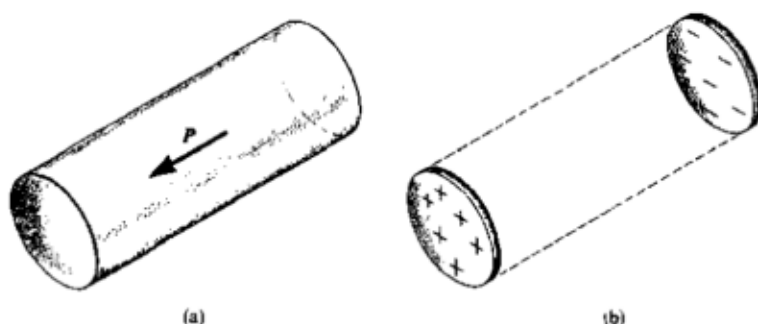


Figure 3-9. (a) Bar electret polarized uniformly parallel to its axis. (b) The E field of the bar electret is the same as that of a pair of circular plates carrying uniform surface charge densities of opposite polarities.