Morse & Feshbach Chapter 1 notes PhL 1.5.12

Purpose here is just to take a very high-altitude survey of this two-volume book set. I want to make sure I could teach a course from it if I had to. I have never just browsed through before. I want to know what is and what is not contained herein.

The set contains 12 **chapters**. Each chapter has **sections** such as 1.1, 1.2 below. Within these sections are some **subsections** with bolded names. The general TOC shows all the sections and subsection titles.

Methods of Theoretical Physics (1953)

Chapter 1: Types of Fields (1-117)

The opening paragraphs discuss PDE's and their solution fields as perhaps **Procrustean** in approach, but I think they mean to say nevertheless there is a good payoff. They will not do "rigor" ahead of "physics" in this book. They note that a PDE model can at the same time be "too smooth" out in the open compared to real physics, and "too sharp" at discontinuities and singularities. Out in the open, things are not as smooth as the model says, and the real world does not have those singularities either. We are warned to think about those "ratios" like mass per volume as $V \to 0$ as being not too large and not too small, the notion of the PDE physics of air being an example. So this seems a good introduction to a 2000 page book largely on PDE's of physics!

1.1 Scalar fields (4). They refer to surfaces of constant ψ value as "**isotimic** surfaces", a word not in the ODE. I would call these contours of the function, and for temperature "isotherms". The "normal lines" are perp to the isotimic surfaces, fine. Then a discussion of the Laplacian and 2D rubber sheet as example. The stretch is minimized, center is average of boundary value, etc. Interesting comments about "bulginess" in 2D and "lumpiness" in 3D, and how $\nabla^2\psi=0$ means cannot have either. Have to think $-\nabla^2\psi = \rho$ to see that large ρ means large ψ at that point, a "lump".

1.2 Vector fields (8). Boldfaced Latin. Dot and Cross product. Axial (pseudo)vectors (presented strangely), **a•bxc** object. Write unit vector as **i** the way I do (but no hat for them). **Flow lines** as lines which follow the field arrows. Their first **stereo pictures**, they do nothing for me, I think this was a bad idea. If course flow lines have perp surfaces which get no name here other than "potential surfaces". $\mathbf{F}\bullet d\mathbf{s} = 0$. Then comes the notion of a flux surface integral of $\mathbf{F}\bullet d\mathbf{A}$. They write this for closed surface using $\oint \mathbf{F}\bullet d\mathbf{A}$ which is not a notation I would have used. A bit on Gauss's integral law for a source point. Then they use the same notation for a line integral $\oint \mathbf{F}\bullet d\mathbf{s}$, a "**circulation** integral". Comments about region of a **vortex**, a place where the field direction lines go around in circles, the opposite of irrotational. Sources and vortex lines are both singularities (point and line, in 3D ie).

1.3 Curvilinear coordinates (21).

Comparison of notations:

M&F PL

$\mathbf{a}_1$	$\hat{\mathbf{e}}_1$	// unit vector along coordinate line, tangent base vector
$\mathbf{a}_i \bullet \mathbf{i} = \alpha_i = \gamma_{i1}$	$\hat{\mathbf{e}}_i \bullet \hat{\mathbf{x}}$	// a particular "direction cosine"
$\mathbf{a}_i \bullet \mathbf{j} = \beta_i = \gamma_{i2}$	$\hat{\mathbf{e}}_i \bullet \hat{\mathbf{y}}$	// a particular "direction cosine"
$\mathbf{a}_i \bullet \mathbf{k} = \gamma_i = \gamma_{i3}$	$\hat{\mathbf{e}}_i \bullet \hat{\mathbf{z}}$	// a particular "direction cosine"
$\hat{\mathbf{a}}_m$ p 30	$\mathbf{e}_m$	// non-unit vectors along coordinate lines
$\mathbf{a}_m$ p 22	$\hat{\mathbf{e}}_m$	// unit vectors along coordinate lines
f^m p 30	F'^m	// contravariant components
f_m p 30	F'_m	// covariant components
F_m p 29	F'^m	// components onto unit vectors
$h_m = \lvert\hat{\mathbf{a}}_m\rvert$	$h'_m = \lvert\mathbf{e}_m\rvert$	// scale factors
$\partial\xi_m/\partial\xi'_n$	$\partial x^m/\partial x'^n = S_{mn} = S^m{}_n$	
$\partial\xi'_m/\partial\xi_n$	$\partial x'^m/\partial x^n = R_{mn} = R^m{}_n$	
$\mathbf{F} = \Sigma_m f^m \hat{\mathbf{a}}_m$	$\mathbf{F} = \Sigma_m F'^m \mathbf{e}_m$	
$\mathbf{F} = \Sigma_m F_m \mathbf{a}_m$	$\mathbf{F} = \Sigma_m F'^m \hat{\mathbf{e}}_m$	
$\mathbf{F} = \Sigma_m f_m \hat{\mathbf{a}}^m$	$\mathbf{F} = \Sigma_m F'_m \mathbf{e}^m$	
$F_m = f^m h_m$	$F'^m = F'^m h'_m$	
$f'^n = \Sigma_m R^n{}_m f^m$ 1.3.12	$F'^n = \Sigma_m R^n{}_m F^m$	// going Cartesian to curvilinear, contravariant
$(F'_n/h_n) = \Sigma_m R^n{}_m (F_m/h_m)$	$(F'_n/h'_n) = \Sigma_m R^n{}_m (F_m/h_m)$	
$f'_n = \Sigma_m R_n{}^m f_m$ 1.3.13	$F'_n = R_n{}^m F_m$	// going Cartesian to curvilinear, covariant
$F'^m = \Sigma_n (h'_m/h_m) R_n{}^m F_m$		// now the unit scaled vector transforms
$F_n' = \Sigma_m \gamma_{nm} F_m$	$F_n' = \Sigma_m \gamma_{nm} F_m$	// transforms using direction cosines

On page 22, M&F say that $\mathbf{a}_i$ are unit vectors, and on page 30 then define their $\hat{\mathbf{a}}_m$ as my $\mathbf{e}_m$ which is very confusing since "today" people think of the hat as indicating a unit vector, whereas they use it for just the opposite sense!

Consider now 1.3.12

$$(f^n)' = \sum_m f^m \frac{\partial \xi'_n}{\partial \xi_m} = \sum_m f^m \left(\frac{h_m}{h_n}\right)^2 \frac{\partial \xi_m}{\partial \xi'_n} \qquad (1.3.12)$$

Translating this to my notation gives

$$F'^n = \Sigma_m R^n{}_m F^m = \Sigma_m F^m (h_m/h'_n)^2 S^m{}_n$$

I feel M&F have an "error" in 1.3.12 because h_n should be h_n' and refers to their ξ' space. I will assume that in the following. Then the equation above seems to require that

$R^n_m = (h_m/h'_n)^2 S^m_n = (h_m/h'_n)^2 R_n^m = (g_{mm} / g'_{nn}) R_n^m$

$\Rightarrow R_n^m = g'_{nn} R^n_m / g_{mm}$

Now in M&F world, coordinate systems are orthogonal, which means $g'_{nn} = 1/g'^{nn}$ since the metric tensor is diagonal. Then the above becomes

$R_n^m = g'_{nn} R^n_m g^{mm}$

For an orthogonal system, this then agrees with my section 7 (n) result

$R_a^b = g'_{aa'} R^{a'}_{b'} g^{b'b}$

So finally I see why they are showing those scale factor ratios as above in 1.3.12.

I think M&F have a general problem in that they try to refer to general x and x' spaces using ξ and ξ', but then should also then have h_n and h_n' corresponding scale factors, but I never see h_n' anywhere.

My main point here is that the M&F unit-scaled vector components F_n are my F^n and transform this way, where I correct for their prime-on-h problem,

$(F'_n/h'_n) = \Sigma_m R^n_m (F_m/h_m)$

$F'_n = \Sigma_m (h'_n/h_m) R^n_m F_m$

and you see this strange extra factor (h'_n/h_m) . Now when they finally on page 54 get around to talking about dyads, the write this as the transformation rule for their matrix

$$
\begin{aligned}
(A_{ij})' &= \sum_{m,n} \frac{h_m h_n}{h'_i h'_j} \frac{\partial x_m}{\partial x'_i} \frac{\partial x_n}{\partial x'_j} A_{mn} \\
&= \sum_{m,n} \frac{h'_i h'_j}{h_m h_n} \frac{\partial x'_i}{\partial x_m} \frac{\partial x'_j}{\partial x_n} A_{mn} \\
&= \sum_{m,n} \frac{h'_i}{h_m} \frac{h_n}{h'_j} \frac{\partial x'_i}{\partial x_m} \frac{\partial x_n}{\partial x'_j} A_{mn} \\
&= \sum_{m,n} \gamma_{im} \gamma_{jn} A_{mn}
\end{aligned}
$$

I can translate the second line this way

$A'_{ij} = \Sigma_{m,n} (h'_i h'_j/h_m h_n) R^i_m R^j_n A_{mn}$

This A_{mn} object has the same scaling idea as their F_m but on both indices. This is a type of object I never would use. It is not really a rank-2 tensor, for example. It is the thing that transforms with the direction cosines directly, just as they show above.

Then on page 55 they do relate this to the corresponding true tensor a_{ij}

$$a^{mn} = A_{mn}/h_m h_n; \quad a_{mn} = h_m h_n A_{mn}; \quad a^m_n = h_n A_{mn}/h_m \qquad (1.6.2)$$

where there should be a full slash before the last h_m on the right.

A **direction cosine** γ_{ij} is like $\hat{\mathbf{x}} \bullet \hat{\varphi}$ in sphericals, n^2 of them. Orthogonal systems. Curvilinear unit vectors called $\mathbf{a}_i$. Notion of **scale factor** h_i and obvious formula for same (orthogonals only). I just added this to my tensor doc in a good place. MF relate the direction cosine to my R and S elements, but then go on to argue that direction cosines never appear anywhere, only the scale factors do (hurray! I never mention direction cosines in my tensor doc). They discuss curvature of coordinate lines and even have a mention of "**torsion**" and "total curvature". Simple derivation of the dV rule and simple J for orthogonals. Example with sphericals as ξ_i. **Euler angles**. Comments on how a vector field transforms between two curvilinear systems. A somewhat strange introduction of contravariant and covariant vectors.

1.4 The Gradient Operator ∇. (31) Expressed in curvilinears. Notion of (**n**•∇) ψ. Unusual placement of discussion of "**infinitesimal rotations**" by angle dω, using cross product notation. If ψ'(x) = $e^{i\boldsymbol{\omega}\bullet\mathbf{L}}$ ψ(x) is a rotated field, then dψ'(x) = i**ω**•L ψ = i**ω**•**r** x **p** ψ = -**ω** • (**r** x ∇)ψ(x), and their point is that ∇ appears. Divergence is next, limit of flux/volume, gets result in curvilinears. Gauss's Theorem is the divergence theorem and is "derived" as superposition acting on the definition of divergence. They use this stuff to **"invert" Poisson's formula** in a simple way for ρ (q) in terms of ψ (ignores BC's!). Page 39 and we are on to the curl, again in curvilinears. Back to those vortex lines. The **A** = curl **B** has lines which are the "vorticity lines" of **B**. Then comes Stokes Theorem. Then vector identities with ∇.

1.5 Vector and Tensor Formalism (44). The use the phrase "**tensor calculus**". They use upper and lower **indices** for the two vector types, orthogonal n=3 world only, the basic R and S transform laws appear now. The ∇ is shown to be covariant. Rank 2 tensors. Contraction. Axial vectors? **Christoffel** symbols introduced, not tensors. **Covariant derivative** written as $f^i{}_{,j}$ and they show this is a mixed rank 2 tensor on page 49. Then $f_{i.j}$. This discussion ends p 52 or so. By their admission, this is very minimalist discussion and I don't think they will ever use it. The unit-vector fields are called "the real fields". Noted that contra and covariant fields' components don't even have same dimensions.

Next topic is vector can be expanded as sum of a gradient and a curl of scalar and vector potentials, **Helmholtz' Theorem** and a proof is given by getting formulas for the potentials φ and **A**. I am not sure I knew this simple fact.

1.6 Dyadics and other vector operators (54).

Notes added starting page 55. What is this dyadic thing? Their presentation is very unclear. I know that a_{ij} is the true rank-2 tensor (see notes above) so they first state that

$$| \mathcal{U} | = a^{n}{}_{n} = \text{trace}(a^{i}{}_{j}) = \text{spur}(a^{i}{}_{j}) \qquad \text{// fine}$$

which I guess they are somehow thinking of the "norm of the dyadic $\mathcal{U}$ ".

Then next write (same a notation used for base vectors and for matrix a, beware!)

$$\langle \mathcal{U} \rangle = \mathbf{a}_i \, \varepsilon_{ijk} \, A_{jk} = \hat{\mathbf{e}}_i \, \varepsilon_{ijk} \, A_{jk} = \hat{\mathbf{e}}_i \, \varepsilon_{ijk} \, h_j h_k a^{jk} \quad = \text{a vector}$$

$$\langle \mathcal{U} \rangle_i = \varepsilon_{ijk} \, A_{jk}$$

If A_{jk} is sym + antisym, this $\varepsilon_{ijk} \, A_{jk}$ picks out only the antisym part. So the vector $\langle \mathcal{U} \rangle$ is proportional to the so-called "dual vector" Lai discusses in association with an antisymmetric matrix.

They next write

$$\mathcal{U} \bullet \mathbf{B} = \Sigma_{mn} \; \hat{\mathbf{e}}_m \, A_{mn} B_n = \Sigma_m \, (A\mathbf{B})_m \hat{\mathbf{e}}_m$$

which is a vector with coefficients $(A\mathbf{B})_m$ where A = matrix, **B** = vector. Therefore

$$(\mathcal{U} \bullet \mathbf{B})_m = (A\mathbf{B})_m = \Sigma_n \, A_{mn} B_n$$

So I can just associate object U with this matrix A really. If you put the vector on the left, you get

$$(\mathbf{B} \bullet \mathcal{U})_n = \Sigma_m \, B_m A_{mn}$$

so we can compare

$$(\mathcal{U} \bullet \mathbf{B})_m = \Sigma_n \, A_{mn} B_n \qquad = [\, A\mathbf{B} \,]_m$$

$$(\mathbf{B} \bullet \mathcal{U})_m = \Sigma_n \, B_n A_{nm} \qquad = [\, \mathbf{B}^{\mathrm{T}} A]_m$$

I am fine with this, the only confusion is that A_{nm} is the strangely scaled tensor which transforms simply with direction cosines for orthogonal systems.

They discuss $\mathcal{U}^*$ in a sense where I would call it $\mathcal{U}^{\mathrm{T}}$ because it is in terms of A^{T} matrix.

Enough for now. I agree that this stuff really is "archaic" but the word dyad keeps appearing even in modern books like Lai.

Continuing a bit, M&F finally get around to juxtaposing two vectors in 1.6.6 where a term is $\mathbf{a}_1 A_{11} \mathbf{a}_1$ which the reader would rearrange as $A_{11} \, \mathbf{a}_1 \mathbf{a}_1$ and then the reader wonders what on earth $\mathbf{a}_1 \mathbf{a}_1$ means. What it really means is $\mathbf{a}_1 \mathbf{a}_1^{\mathrm{T}} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} (1 \; 0 \; 0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ so in fact it is a matrix. In fact

$(\mathbf{a}_1\mathbf{a}_1)_{ij} = (\mathbf{a}_1)_i(\mathbf{a}_1)_j = \delta_{1i}\delta_{1j}$

We see more abutting vectors in 1.6.7 where now a term is $\mathbf{a}_1\mathbf{A}_1$ .This is confusing because notation A_1 is usually used to indicate a component of vector **A**, but here $\mathbf{A}_1$ is a vector with 1 as a label. Later on p 58 they take two vectors like this, called $\mathbf{A}_m$ and $\mathbf{B}_m$ and say you can make a dyadic out of these two vectors and then sum over m to get another more general dyadic

$\mathcal{U} = \Sigma_m \mathbf{A}_m\mathbf{B}_m$

What they never seem to say is this simple fact that Lai says

$\mathcal{U} = \mathbf{ab} \quad \Rightarrow \quad \mathcal{U}_{ij} = a_i b_j$

Now to confuse things even more, on p 58 we see this statement:

The most general symmetric dyadic can be expressed in terms of three orthogonal unit vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$;

$$\mathfrak{A}_s = \mathbf{a}_1 A_1 \mathbf{a}_1 + \mathbf{a}_2 A_2 \mathbf{a}_2 + \mathbf{a}_3 A_3 \mathbf{a}_3 \qquad (1.6.8)$$

Since a symmetric dyadic is specified by only six independent constants (three pairs of components are equal), specifying a definite symmetric dyadic uniquely specifies the three constants A_1, A_2, A_3 and uniquely

specifies the directions in space of the three orthogonal unit vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ [which are given by the Eulerian angles ψ, φ, ϑ, see Eq. (1.3.8)]. Referring to Eq. (1.6.7) we see that for a dyadic to be symmetric it must be possible to find a trio of unit vectors **a** such that the component vector $\mathbf{A}_1$ is parallel to $\mathbf{a}_1$, and so on.

NOW A_1 is a scalar, not a vector, I guess in fairness then have italicized it. Equation 1.6.8 means that if you diagonalize this symmetric matrix, you get $\Lambda = \text{diag}(A_1, A_2, A_3)$. Now suppose we solved this EV equation

$U\,\boldsymbol{\varphi} = \lambda\,\boldsymbol{\varphi}$

and suppose the three eigenvalues are $\lambda = A_1, A_2, A_3$ and suppose the three EV's are $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$. As I show in matrix binder theorem 15D, if R is a matrix whose columns are the $\mathbf{a}_i$, then UR = RΛ which is then the same as $U = R\Lambda R^{-1}$. We can then write this as (R orthogonal)

$$U = (\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3) \begin{pmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{pmatrix} \begin{pmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{pmatrix} = (\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3) \begin{pmatrix} A_1\mathbf{a}_1^T \\ A_2\mathbf{a}_2^T \\ A_3\mathbf{a}_3^T \end{pmatrix}$$

$$= \mathbf{a}_1 A_1 \mathbf{a}_1^T + \mathbf{a}_2 A_2 \mathbf{a}_2^T + \mathbf{a}_3 A_3 \mathbf{a}_3^T$$

and then this is the meaning of M&F's claim above

$$\mathfrak{A}_s = \mathbf{a}_1 A_1 \mathbf{a}_1 + \mathbf{a}_2 A_2 \mathbf{a}_2 + \mathbf{a}_3 A_3 \mathbf{a}_3 \qquad (1.6.8)$$

When we say that the vectors $\mathbf{a}_i$ are the rows of matrix $R^{-1} = R^T$, if notation v means a column vector, as it normally does, we need to add a ^{T} symbol to make a row vector!

Now this is consistent with the basic dyadic idea

$$U = (\mathbf{ab}) \quad \text{means} \quad U = \mathbf{ab}^T = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} (b_1 b_2 b_3) = \text{etc} \qquad (ab)_{ij} = a_i b_j$$

So you need to think of the "right side" vector as a transposed vector. You do NOT want to think of the left vector as transposed because then the result is a scalar!

Now here is how M&F want to write my EV equation

$U\ \mathbf{a}_1 = A_1\ \mathbf{a}_1$	me
$U_s \bullet \mathbf{a}_1 = A_1\ \mathbf{a}_1$	M&F

So everything is just fine. More translation

$U\ \mathbf{v}$	me, matrix U acting on a column vector to its right to create a column vector
$U \bullet \mathbf{v}$	M&F same thing
$\mathbf{v}^T U$	me, matrix acting on a row vector to its left to create a row vector
$\mathbf{v} \bullet U$	M&F same thing
$\mathbf{w}^T U\ \mathbf{v}$	a scalar
$\mathbf{w} \bullet U \bullet \mathbf{v}$	M&F same thing

So when U acts on vectors, you put a ^{T} symbol on the left vector when there is one.
But when U is written as (**ab**), you put a ^{T} on the right vector.

I don't think the fact that their A_{ij} matrix has strange scaling has much to do with dyadics at all! It just happens that A_{ij} in their example is this way. It could be any matrix A_{ij}.

Now look at M&F p 60 :

An *antisymmetric* dyadic has its diagonal components A_{nn} zero, and its nondiagonal components change sign on interchange of subscripts; $A_{mn} = -A_{nn}$. The most general antisymmetric dyadic is specified by only three independent constants. It can always be expressed in the form

$$\mathfrak{A}_a = \mathbf{R} \times \mathfrak{I} = -\mathbf{ij}R_z + \mathbf{ik}R_y + \mathbf{ji}R_z - \mathbf{jk}R_x - \mathbf{ki}R_y + \mathbf{kj}R_x \quad (1.6.10)$$

where $\mathfrak{I} = (\mathbf{ii} + \mathbf{jj} + \mathbf{kk})$ is the idemfactor. Choice of the antisym-

and compare this to Lai who says

$$\mathbf{Ta} = \mathbf{t}^{\mathbf{A}} \times \mathbf{a}. \quad (2.21.1)$$

This vector $\mathbf{t}^{\mathbf{A}}$ is called the *dual vector* of the antisymmetric tensor. It is also known as the *axial vector*.

What happens if we apply the M&F matrix to a vector **a** ?

$$\mathcal{U}_{\mathbf{a}} \bullet \mathbf{a} = U_{\mathbf{a}}\mathbf{a} = T\mathbf{a} = (\mathbf{R} \times \mathcal{I}) \bullet a = \mathbf{R} \times (\mathcal{I} \bullet \mathbf{a}) = R \times (Ia) = \mathbf{R} \times \mathbf{a} = \mathbf{t}^{\mathbf{A}} \times a$$

This sill "idemfactor" is just the identity matrix.

$$I = ii + jj + kk \rightarrow i\,i^{\mathbf{T}} + j\,j^{\mathbf{T}} + k\,k^{\mathbf{T}} = \text{etc} = 3\text{x}3 \text{ identity matrix}$$

Thus, the M&F **R** vector is the "dual vector" of Lai.

It is all archaic malarky! I keep coming back to that conclusion. It is just matrix stuff reformulated in archaic notation. We now from Lai that

$$2\mathbf{t}^{\mathrm{A}} = -\varepsilon_{ijk} T_{jk} \mathbf{e}_i .$$

so $\mathbf{t}^{\mathbf{A}}$ is "half the vector" shown on the RHS. Now M&F say

metric dyadic uniquely specifies the vector R, which is equal to half the rotation vector $\langle \mathfrak{A}_a \rangle$ of the antisymmetric dyadic $\mathfrak{A}_a$. We note that the

so M&F regard the RHS of the above as "the rotation vector of $U_{\mathbf{a}}$". They do not **R** the dual vector.

It seems that a dyadic is a rank 2 tensor scaled by pairs of scale factors and they want to use German cap letters for such a thing. Trace is called spur. As you read this, it seems that a dyadic is just a linear matrix operator. They want to write A:B as matrix product. The rule for A^{-1} is stated**.** I think the real idea is simply that M = $\mathbf{AB^T}$ which let's you make a matrix by combining two vectors in the reverse order of a dot product. Then **1 1** is really the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Seems quite dated and useless to me. They wander off in this dyadic context to talk about **eigenvectors** and the **secular determinant**. They go on and on, doing a lot of usual linear algebra stuff but using their dyadic notation. For example, notion of diagonalizing a matrix. p 64. Then "dyadic fields" and they call this all "dyadic algebra". **Stress and strain** given as example. Shear. Elastic medium. Bulk and Young modulus. So they are doing a quick run-through of **continuum mechanics** in dyad formalism. Then suddenly on p 73 we do "complex numbers" out of the blue and then "**quaternions**". Tensor and versor. And then suddenly we are on to "**abstract vector spaces**" on p 76, coupled harmonic oscillators as an example and I guess the normal mode coordinates as abstract vectors. This section is on "operators" so I guess this fits in. More on the EV problem in terms of E^3. Then on to "**quantum operators**" and how non commuting means you can measure both. We are STILL using the German dyadic notation! A little view of quantum's vector space with complex aspect. The SI appears. Matrices extended to complex elements, and notion of the **Hermitian adjoint** of a dyadic (ie, of a matrix). Hermitian and **unitary** matrices. More on QM operators like momentum, uncertainty principle in terms of Δx and Δp. **Spin** operators. More on quaternions, then on to **rotation operators** p 91. $\mathfrak{D} = e^{i(\mathcal{M}/\hbar)\theta}$ is their notation, but they seem to avoid the word "generator" and no mention of groups.

1.7 Lorentz transformations, 4-vectors, spinors (93). A little review of **special relativity**, p = mdx/dτ with proper time. The ∂_μ gradient is called □ (grad) and then $\partial_\mu\partial^\mu = \Box^2$ **d'Alembertian** (same symbol in BD). The 4D version of the stress tensor. Then on to 2D **spinors** and Dirac 4-spinors (I think). STILL in the horrible dyadic notation. So doing 2D representation rotation group stuff without mentioning same. Horrible! Note: M&F avoid talking about a metric tensor, they just say that invariants are like $\mathbf{p^2}$- E^2 . I think they might use the imaginary time trick in a few places.

Problems (107) 1.1 through 1.33.

Table of vector identities and of horrible dyadic identities (114).

Table of curvilinear coordinate properties (115) Here the general forms are given, then cylindricals and sphericals only.

Bibliography (117). M&M and Goldstein and Dirac get mention.

Comments: Well, based on this chapter alone, I would *never* want to use this 1953 book for a course! The dyadic version of linear algebra is just a disaster IMHO. This chapter is a grab bag trying to do quantum mechanics, relativity, vector algebra and linear algebra. I am reminded of the M&F version of the Legendre functions, a similar disaster. So I guess we relegate these books to "reference" works. I am a

little surprised at this opening chapter, though some of the general ideas are well stated. This chapter is about 1/8th of the first volume! I am glad I surveyed this chapter today, now I have a brief record of what is in there. There is no large chunk of knowledge here that I know nothing about, for example.