Chapter 10: Solution of Laplace and Poisson (1173-1330) 157 pages

10.1 *Solutions in Two Dimensions* 1175

Cartesian Coordinates, Rectangular Prism Heated on One Side. Green's Function. Polar Coordinates, Cylinders Placed in Fields. Flow of Viscous Liquids. Green's Function in Polar Coordinates. Internal Heating of Cylinders. Potential Near a Slotted Cylinder. Elliptic Coordinates. Viscous Flow through a Slit. Elliptic Cylinders in Uniform Fields. Green's Function in Bipolar Coordinates.

So this is the subject of **Potential Theory** without and with sources: electrostatics, static heat flow, special fluid flow, etc. **Chapter 6 of Stakgold**, but in this section all 2D stuff. The idea as usual is to do 3D problems with extrusion symmetry, so "cylinders" really means circles and disks and such things. The slotted cylinder is the 2D wedge problem. After doing problems with circles, M&F ponder how things are different if you have ellipses in 2D (and thus "elliptical cylinders"). Notice that approximation theory of Ch 9 is gone now, we are back to ideal world problems.

Something is wrong with the list shown above! It is not complete, it does not match the actual chapter. Oy. It goes awry after "viscous flow through a slit", so I add here the missing sections

Viscous Flow through a slit (1197)
Green's Function for Elliptic Coordinates (1202)
Potential Inside a Cylinder with a narrow slot (1206)
Parabolic Coordinates (1207)
Bipolar Coordinates (1210)
Two cylinders in a uniform field (1211)
Green's Function for Bipolar Coordinates (1213)

Whereas Stak generally did n dimensions, M&F only do n=2 and n=3. That kept them plenty busy!

10.2 *Complex Variables and the Two-dimensional Laplace Equation* 1215

Fields, Boundary Conditions, and Analytic Functions. Some Elementary Solutions. Transformation of Solutions. Circulation and Lift. Fields Due to Distributions of Line Sources. Grid Potentials and Amplification Factors. Linear Arrays of Source Lines. Two-dimensional Array of Line Sources. Periodic Distribution of Images. Potentials about Prisms. Parallel Plate Condenser. Variable Condenser. Other Rectangular Shapes.

We know that analytic functions have real and imaginary parts that solve Laplace, they are "**harmonic functions".** It is not surprising that potential theory can be formulated and studied in terms of analytic functions. I am most familiar with this subject in terms of "**conformal mapping**" between two spaces, one of which is usually Cartesian 2D. This plays a major role in the M&S book for cylindrical coordinate systems based on 2D conformal systems. But I think this section takes a more general view of the subject formulated in terms of a complex variable. Again, this is all just 2D. Example of a grid in a triode tube!

10.3 ***Solutions for Three Dimensions*** **1252**

Integral Form for the Green's Function. Solutions in Rectangular Coordinates. Solutions in Circular Cylindrical Coordinates. Integral Representation for the Eigenfunction. Green's Function for Interior Field. Solutions for Spherical Coordinates. Fields of Charged Disks and from Currents in Wire Loops. Fields of Charged Spherical Caps. Integral Representation of Solutions. The Green's Function Expansion. Dipoles, Quadrupoles, and Multipoles. Spherical Shell with Hole. Prolate Spheroidal Coordinates. Integral Representation of Spheroidal Solutions. Green's Function for Prolate Spheroids. Oblate Spheroids. Integral Representations and Green's Functions. Parabolic Coordinates. Bispherical Coordinates. Toroidal Coordinates. Ellipsoidal Coordinates.

This is the 3D **meat and potatoes** chapter. The "charged spherical cap" item means a sticky dipole layer on same. And the later spherical hole section means a small hole and approximation, perhaps this was reviewed in MacDonald's writeup. M&F are not doing any kind of complete solution to this problem, no inversion method etc. **Green's function expansion means** my 1/R formulas in various systems. And "integral form for Green's Function" just means a continuous spectrum expansion of 1/R, for example

$$\frac{1}{R} = \frac{1}{2\pi^2}\int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \int_{-\infty}^{\infty} dk_z \left\{ \frac{e^{i[k_x(x-x_0)+k_y(y-y_0)+k_z(z-z_0)]}}{[k_x^2+k_y^2+k_z^2]} \right\} \quad (10.3.6)$$

This is a **famous** section of this book because it deals with Potential Theory *in* a variety of **curvilinear coordinate systems**. **Stak** only did Cartesian, cylindricals and sphericals. For example. **Smythe** (1950) also has some prolate and oblate stuff, but toroidal only in his problems. Smythe's static book design section is similar to M&F in that there is a 2D chapter followed by a 3D chapter. Smythe's TOC detail lists are even more massive that the M&F ones! Good old Smythe!

Problems (1309) 10.1 to 10.38

A. **Trig and hyperbolic** function formulas (1320)
B. **Bessel** Functions (1321) -- I and K are called "hyperbolic Bessel functions" , integrals, etc
C. **Legendre** functions (1325) -- integer order, then imaginary argument as in spheroidals, and special :
Toroidal harmonics (1329) -- what I think I call cylinder functions with order n-1/2

or from the TOC:

Trigonometric and Hyperbolic Functions 1320

Trigonometric Functions. Hyperbolic Functions. Generating Functions Relating Hyperbolic and Trigonometric Functions.

Bessel Functions 1321

General Formulas Relating Bessel Functions. Series Relations. Hyperbolic Bessel Functions. Definite Integrals Involving Bessel Functions.

Legendre Functions 1325

Zonal Harmonics. Legendre Functions of the Second Kind. Functions of Imaginary Argument. Toroidal Harmonics.

Bibliography (1330)

Bateman PDE book, Jeans, Jeffreys Math methods book, Lamb hydrodynamics, MacMillan potential theory, Smythe 1950. It is a fairly short list.