Morse & Feshbach Chapter 11 notes PhL 1.14.12

This is the big WAVES section of M&F, first 2D then 3D. Waves were mentioned in volume 1, and here we have elaboration and applications. I like the constant acoustic emphasis, more down to earth it seems than E&M waves. As usual, lots of obscure curvilinear coordinate systems appear, some approximations appear, lots of weird new atomic functions.

Stakgold's waves Chapter 7 part is more "theoretical", n dimensions at times, propagators. Stak does not use oddball coordinate systems, and does not in general have oddball problems (read "real world" type problems). There is a spectrum from pure math theory with theorems and proofs and toy situations only, and pure handbook application with no theory. Stak is 1/3 from the left end, M&F is 1/3 from the right end. I am sure I could find an acoustic calculations handbook somewhere that might be at the right end.

Nothing on computer work of course in this 1953 book which still has "math tables" in the back. This is an analytic book, not a digital book.

Chapter 11: The Wave Equation (1173-1330) 157 pages

11.1 *Wave Motion on One Space Dimension* **1332**

Fourier Transforms. String with Friction. Laplace Transform. String with Friction. String with Elastic Support. String with Nonrigid Supports. Reflection from a Frictional Support. Sound Waves in a Tube. Tube with a Varying Cross Section. Acoustic Circuit Elements. Free-wave Representations. Movable Supports.

So the two examples here are the string in various forms, and sound compression waves in a tube. Friction means force proportional to velocity. Note various kinds of supports for the string including elastic. As for the tube, ends can be open or closed or something else, same idea of playing with the boundary conditions. The "circuit elements" is a way to model acoustic situations with R,L,C-like circuit elements, I have never played with such a thing, maybe Jeff Davis has.

The next title really should say in Two Space Dimensions to be consistent with the adjacent titles!

11.2 *Waves in Two Dimensions* **1360**

Fourier Transforms and Green's Functions. Rectangular Coordinates. Other Boundary Conditions. Variable Boundary Admittance. Polar Coordinates. Waves inside a Circular Boundary. Radiation from a Circular Boundary. Scattering of Plane Wave from Cylinder. Scattered and Reflected Wave. Short and Long Wavelength Limits. Scattering of

Plane Wave from Knife Edge. Fresnel Diffraction from Knife Edge. Scattering from a Cylinder with Slit. Slotted Cylinder, Neumann Conditions. Waves in Parabolic Coordinates. Eigenfunctions for Interior Problems. Waves outside Parabolic Boundaries. Green's Function and Plane Wave Expansions. Elliptic Coordinates. The Radial Solutions. Approximations for Small Values of h and m. Approximations for h Small and m Large. Expansions for h Large. Waves inside an Elliptic Boundary. Green's Functions and Plane Wave Expansions. Radiation from a Vibrating Strip. Radiation from a Strip Carrying Current. Scattering of Waves from Strips. Diffraction through a Slit, Babinet's Principle.

As expected, they get right into issues of **impedance and admittance** at the boundaries. This just means the wall is not "perfect" in some sense, perhaps not perfectly Dirichlet or Neumann. I see the **drumhead** situation early on in the list. With the **circle** we can do waves inside (EF's), or we can scatter from the outside. The famous "**knife edge**" appears here as in Stak. Wow, a **parabolic** boundary problem. Then we get into "**periodic Mathieu functions**" like $Se_m(h,z)$ which appear in the atoms of **elliptical cylinder** coordinates (p 1407). Notice the **parameters h and m**. M&F seem to do approximations in various ranges for these parameters in the TOC list above. So there is a stress on elliptical stuff here, because circular stuff is the simple case covered in the first part of this section. Babinet gets mention.

11.3 *Waves in Three Space Dimensions* 1432

Green's Function for Free Space. Rectangular Enclosure. Distortion of Standing Wave by Strip. Computing the Eigenvalue. Transmission through Ducts. Acoustic Transients in a Rectangular Duct. Constriction in Rectangular Duct. Wave Transmission around Corner. Membrane in Circular Pipe. Radiation from Tube Termination. Transmission in Elastic Tubes. Spherical Coordinates. Spherical Bessel Functions. Green's Function and Plane Wave Expansion. Waves inside a Sphere. Vibrations of a Hollow, Flexible Sphere. Vibrating String in Sphere. Resonance Frequencies of the System. Radiation from Sphere. Dipole Source. Radiation from Collection of Sources. Radiation from Piston in Sphere. Scattering of Plane Wave from Sphere. Scattering from Sphere with Complex Index of Refraction. Scattering from Helmholtz Resonator. Scattering from Ensemble of Scatterers. Scattering of Sound from Air Bubbles in Water. Spheroidal Coordinates. The Radial Functions. Green's Function and Other Expansions. Oblate Spheroidal Coordinates.

Waveguides, perhaps sound ducts (speaker cabinets!) Corners! Elastic tubes! Scattering from air bubbles in water. Chock full of good 3D wave examples. Flexible sphere radiator. All good stuff.

11.4 *Integral and Variational Techniques* 1513

Iris Diaphragm in Pipe. A Variational Principle. Calculating the Transmission Factor. Hole in Infinite Plane. Reflection in Lined Duct. Fourier Transform of the Integral Equation. Factoring the Transformed Equation. Radiation from the End of a Circular Pipe. Formulas for Power Radiated and Reflected. The Fourier Transform of the Integral Equation. Factoring the Transformed Equation. Radiation from Vibrating Source. Angle Distribution of Radiated Wave. Applying the Boundary Conditions. Scattering of Waves, Variational Principle. Angle-distribution Function and Total Cross Section. Scattering from Spheres. Scattering from a Strip. Scattering of Short Waves.

Scattering from spheres noted (blood cells in eyes). It might be that this entire section is approximation theory or is not, I am not sure.

It would be fun to learn how to do these unusual problems.

Problems (1555) 11.1 to 11.32

Addition theorems are given in each of these sections along with the usual special function data:

A. Cylindrical Bessel functions (meaning J, N, H)

B. Weber functions (as for parabolic coordinates)

C. Mathieu functions (elliptical coordinates) -- various kinds, lots of detail here

D. Spherical Bessel Functions -- lower case j and so on

E. Table of Laplace Transforms

Bibliography

C&H, Morse on acoustics, Rayleigh on sound,