Morse & Feshbach Chapter 2 notes PhL 1.5.12

Chapter 2: Equations Governing Fields (120-274)

2.1 The Flexible String (120). The ODE is derived for the static string, the δ function is derived, the Green's Function is stated. Then the 1D **wave equation** appears with velocity c = T/ρ. Sets ξ = x-ct and η = x+ct. On goes $e^{-i\omega t}$, out comes the **Helmholtz equation**. The wave has a PE and KE. Power flows along a wave. Impedance. The driven string, driven by a support. **Fourier** Transform. They fiddle with a shift matrix (dyadic) like raising operator in HO discussion. Here the string is modeled as discrete segments y_n and a digital wave solution is found p 134. Then the limit to the continuous string reproduces earlier results. Friction modeled as a **viscous** velocity term with k = coefficient. But then if this term dominates, you have the **diffusion equation** and we are off on that one p 137. Comparison of nature of wave and **heat equation** solutions, as per Stak. Another type of friction is **medium elasticity** with force $-\mu^2\psi$ (not a velocity force) and this gives the Klein-Gordon $(\nabla^2 - \mu^2)\psi = (1/c^2)\partial_t^2\psi$ where we somehow associate the mass with this elastic force! Solutions are local, force does not go far, **Yukawa** potential type thing. P 140 compares graphs of solutions on string with these two kinds of friction. Recap at the end. So MF have hit on the wave and heat equations right away, and damped wave of two kinds.

2.2 Waves in an Elastic Medium (142). We are back now in dyadic land, sad to say. But it is written in standard stuff and we have $\rho\,\partial_t^2\mathbf{s} = (\lambda+2\mu)\mathbf{grad}\,(\mathrm{div}\,\mathbf{s}) - \mu\,\mathbf{curl\ curl}\,\mathbf{s}$ and this is a vector wave equation. Not clear what **s** is, I suppose the local 2D displacement. In one limit $\mathbf{s} = \nabla\psi$ and we have longitudinal waves, in another limit $\mathbf{s} = \nabla \times \mathbf{A}$ and we have transverse waves. Next is waves in 3D, and only plane waves keep shape and size. Now back to **stress and strain** and some strange looking integral theorems on page 150. So yes, this section is bringing out my complete ignorance of **continuum mechanics**. I don't even have a folder on this subject, but I do have some downloaded books. The Lai Rubin one looks good, and they verify my dyad conjectures of Ch 1.

So the first PDE example is wave motion in continuum mechanics, oy but OK.

2.3 The motion of Fluids (151). The next foray is **fluid mechanics**, another huge Phil hole. They describe the "**two approaches**" and I think select the one based on small volumes with velocity v (and not the one with individual particles). Then we simplify to **incompressible** fluids and some pretty nasty PDE's start appearing as on p 160. Then **irrotational** flow and eventually the **velocity potential**. Subsonic, supersonic, **Mach** numbers. I think this does illustrate the notion of PDE's with singularities. M&F are just giving a little tour, but the scenery is cloudy to me. (a fast-moving train)

2.4 Diffusion and other "percolative fluid motion" (171). Liquid through a porous solid, solid has a flow resistance. But then they are back to the regular diffusion equation which they note is "parabolic". They throw in the equation of state of a gas and distribution stuff and adiabatic compression, mean free path, we are doing **gas dynamics** now. Example of **Milne**: diffusion of light, finally some related PDEs. A long time is spent on this problem. On and on and on with some "**diffusion theory**", energy loss, etc. In the recap, they note solutions are not time-reversible as with wave equation stuff.

2.5 The EM field (200). So now we are going to get their recap of all of E&M theory. Maxwell's equations, electrostatics, retarded potentials, Lorentz transformation of the fields, gauge transformations, force and energy, what happens at conductor surfaces p 217, waves, transmission lines. The **Proca** equation is the EL equation when you write the EM Lagrangian in the usual way (Proca Romanian) .

2.6 Quantum Mechanics (222). Here we are off again this time on QM. We even get the Poisson Brackets after a while. Operators and matrix elements in dyadic f* • U •f. . Page 238 gives us $\mathcal{B}\bullet\mathbf{b}_m = b_m\mathbf{b}_m$ for eigenfunctions of an operator, notice the dot. Position and momentum space and going back and forth. Hamiltonian and **Hamilton's equations**. They like **e**(E) as eigenfunction of H. Naturally the **harmonic oscillator** makes a showing and I see raising and lowering operators $\mathcal{G}_\pm$ it is painful to watch. Next example is particle in EM field. Eventually **Dirac equation** and finally some 4x4 matrices!

The **recap** on p 265 is **for this entire chapter**. They explain why they spent more time on QM since it was relatively new in 1953.

Problems (267) 2.1 through 2.10

Summary of standard PDE forms (271). Again, the continuum and fluid equations make me uncomfortable, but all the rest are familiar enough.

Bibliography (273). L&L on fields, M&M, **Slater & Frank**, **Sommerfeld** on PDEs. **Morse** did a book on sound it turns out. **Love** is the elasticity guy. **Lamb** did some fluid mechanics. **Holf** mentioned on radiative EQ!! (of the WH method). **Chandrasekhar** also on radiative transfer (ie, star theory). **Stratton**. **Van Vleck**. All those good old books!