**Morse & Feshbach Chapter 4 notes** PhL 1.11.12

**Chapter 4: Functions of a Complex Variable (348-492)** ..... **1**
4.1 Complex Numbers and Variables (349)/ ..... 1
4.2 Analytic Functions (356) ..... 1
4.3 Derivatives of Analytic Functions, Taylor and Laurent Series (374) ..... 1
4.4 Multivalued Functions (398) ..... 2
4.5 Calculus of Residues: Gamma and Elliptic Functions (408) ..... 2
4.6 Asymptotic Series; The Method of Steepest Descent (434) ..... 2
4.7 Conformal Mapping (443) ..... 2
4.8 Fourier Transforms (453) ..... 2
Problems for Chapter 4 ( 50 problems!) ..... 2
Tabulation of Properties of Functions of a Complex Variable (480) ..... 2
Table of Special Function Data (486) ..... 3
Bibliography ..... 3

## Chapter 4: Functions of a Complex Variable (348-492)

Intro: M&F ask the question **why complex** is needed since physics is real, and their answers are good. This section is about scalar functions of a single complex variable.

### 4.1 Complex Numbers and Variables (349)/

Discussion of **i** as 90 degree rotation operator in the z plane. Rotation operator as first example and the de Moivre relation. Basics of complex variable math, including gradient. 2D electrostatics. **Cauchy-Riemann** equations for u and v as definition of analytic function. Contour integration. All just intro.

### 4.2 Analytic Functions (356)

This section follows Ahlfors I would say. **Derivative's** role in definition of analytic function, then proof of the C-R equations. **Conformal mapping** idea, **angles** preserved, mapping in general 2D→2D. More detail on **complex integration**. **Cauchy theorem** says closed line integral gives 0 for analytic function. Hence allowed to deform a contour. Integration around a simple pole. Integration formulas for real and imaginary parts p 371, examples are **Hilbert Transform** p 372. Principle part integral above 4.2.17 and their notation is to put $\mathcal{P}$ in front of the integral. Then on to their favorite subject: impedances. The Poisson Formula tells you f(z) inside a circle on which you know f. More Hilbert Transform similar stuff.

### 4.3 Derivatives of Analytic Functions, Taylor and Laurent Series (374)

f analytic => f' analytic. **Taylor** is the analytic thing, **Laurent** if you have some poles to worry about. Notion of essential singularity on page 380. **Liouville's theorem** (as used in WH technique!) p 381. **Meromorphic** function is analytic except for a finite number of poles (or multiple poles I assume). Power series and circle of convergence. Example is F(a,b,c,z). **Analytic continuation** on p 389, a subject I like. Uniqueness of the continuation. **Branch points** merely mentioned. Methods of doing analytic continuation. Power series stepping is pretty painful, not a preferred method. Some long examples here, not sure what they are.

**4.4 Multivalued Functions (398)**

Here we have f(z) = $z^{1/2}$ and we have the usual branch point and **cut** (they call it a branch line, not a cut at this point). Branches of a function. **Riemann surfaces** p 401 with attempt to draw 2-sheeted surface in the stereo pictures. Now they allow the word "cut" in the context of the sheets. They pick z(f) = th(f)/f and study it in detail, showing contours and so on. [ they do not use the word "sheet" ]

**4.5 Calculus of Residues: Gamma and Elliptic Functions (408)**

Evaluation of integrals. Phrase "**calculus of residues**" just means deforming around the poles etc. Examples given where you evaluate integral by deforming around a cut. Can you **invert a power series** function (p 411)? A method is given using a complex integral. Notion of converting a **series to an integral** which I am quite familiar with. **Integral representations**. **Gamma function** defined by an integral representation. Product representation for gamma ( we have a little "special functions" section here fore the Gamma function). The dlnΓ/dz is written as $\psi_1(z)$ = Γ'/Γ. Duplication formula. Beta function p 425. Comment on series and product expansions for periodic functions p 426 top. What about functions with periods in both directions? (**doubly periodic** functions). We are then off on a discussion of the **Jacobi elliptic** functions which have this property, and the corresponding **elliptic integrals** which are the inverses of these functions. **Theta functions** appear here as well.

**4.6 Asymptotic Series; The Method of Steepest Descent (434)**

The Method gives **asymptotic series**. So here is the M&F section on this interesting subject.

**4.7 Conformal Mapping (443)**

**Schwarz-Christoffel** theorem as in Jackson lecture handout of Days of Olde. Use in electrostatics to study fringing fields. The **Method of Inversion** !!!! (short section)

**4.8 Fourier Transforms (453)**

Their **convention** is that F(k) = $1/\sqrt{2\pi}$ $\int$ dx $e^{+ikx}$ f(x), the expo's sign agrees with Stak but not the constant in the projection. Validity of FT? Some discussion p 456 of **Lebesgue** integration ideas. The Fourier Integral Theorem. **Parseval**. Where is the FT analytic? Lots of **strip** stuff. Then the usual convolution integral which they call "the **faltung**" of two functions (folding). Diagonalization. Then on to the Poisson Sum Formula which I learned in Stak. Then the **Laplace** Transform. The **Mellin** Transform.

**Problems for Chapter 4 ( 50 problems!)** 4.1 through 4.51 !! 50 problems!

**Tabulation of Properties of Functions of a Complex Variable (480)**

A little summary which includes some FT lookup tables on p 484, Schaum-like. A little table of Mellin transforms.

**Table of Special Function Data (486)**

Gamma related functions, the Jacobi elliptic functions, and Theta functions as well.

**Bibliography**

W&W.  Ahlfors was 1953, same year as M&F and did not make the cut. Copson 1935 book. Watson on Bessel. Sneddon on FT's. Most of the references I have never heard of!