Morse & Feshbach Chapter 9 notes PhL 1.14.12

This is the first chapter of volume II. I will review the detailed M&F TOC contents only and will not skim all 1000 pages as I did for the first volume, mainly because I know this stuff pretty well.

Before delving into a chapter like this, I would have to have some particular problem in mind, some motivating goal. Otherwise yes, it is just a cookbook of approximation techniques, pretty dry reading.

Chapter 9: Approximate Methods (999-1172) 173 pages

In the first volume we treated **toy** idealized problems. In the real world, serious problems can never be solved exactly in this manner, so approximation methods are always needed. One class of methods is to do perturbation around a problem which is solvable (separable, say), and of course the perturbation has to be relatively small in some sense. You might perturb the boundary conditions ("**surface perturbation**") or the ODE itself ("**volume perturbation**"). Example of two electron SE for atom is given with the e^2/r_{12} term as the perturbation. Acoustic examples (Morse area) might have columns in a room to worry about.

This chapter shall treat three approximation methods, perturbation being just the first:

In this chapter, we shall consider three essentially different methods for obtaining approximate solutions: (1) the *perturbation* method, (2) the *variational* method, and (3) the V-I method, where V-I is short for *variation-iterational*. In the perturbation method, the volume or surface perturbations are assumed to be small and expansions in powers of a parameter measuring the size of the perturbation may be made, the leading term being the solution in the absence of any perturbation.

9.1 *Perturbation Methods* 1001

The Usual Perturbation Formula. Convergence of Series. Multidimensional Problems. An Example. Feenberg Perturbation Formula. Secular Determinant. An Example. Fredholm Perturbation Formula. An Example. Variation—Iteration Method. Convergence of the Method. An Example. Improved Perturbation Formulas. Nonorthogonal Functions.

The **Usual Formula** is what we always do in QM to get a perturbation series with first order, second order, and so on. M&F treat this in a very general manner which looks good and you see the usual "energy difference denominators". M&F write lots of the lower levels of this series. **Feenberg** came up with some improvement in the series regarding denominators and this involves use of a certain secular determinant. M&F then get into the "**Fredholm**" formulas which I think are those involving the trace approximation and which I have written up somewhere. As usual, M&F provide real world solid examples. So yes. this would be a good place to study "perturbation theory" in a systematic way, not just for quantum mechanics. You might have trouble finding a big *general* section on perturbation theory in other books. You want the general method to apply to any PDE ODE or IE not just say the SE.

9.2 *Boundary Perturbations* 1038

Perturbation of Boundary Conditions, f Small. Perturbation of Boundary Conditions, f Large. An Example. Formulas for Small ω. Long, Narrow Rectangle. Perturbation of Boundary Shape. Evaluation of Integrals. Convergence. Improving the Convergence. Perturbation of Boundaries for Dirichlet Conditions. A Special Class of Boundary Perturbation.

Again Morse did acoustics research where all this stuff comes in. You might do volume and surface perturbation theory at the same time! Here is what "f" means:

$$(\partial\psi/\partial n) + f(S)\psi = 0 \qquad (9.2.1)$$

So perhaps you have a Neumann exact solution and you need to add in a little Dirichlet on your boundary with small coefficient f. Large f of course means you are perturbing around the Dirichlet solution. This general form is like Stak's "radiative" BC. Note that f is not a constant, it is a function of location on the surface S. As usual, examples are given.

9.3 *Perturbation Methods for Scattering and Diffraction* 1064

Boundary Conditions for Scattering. Scattering Cross Section. Scattering from Spherically Symmetric Region—Phase Shifts. Integral Equation for Scattering. Integral Equation for One-dimensional Problem. Integral Equation for Three Dimensions. Born Approximation. Higher Born Approximations. Fredholm Series. An Example. A Three-dimensional Example. Long-wavelength Approximation. Long-wavelength Approximation for the Schroedinger Equation. Convergence. Short-wavelength Approximation; WKBJ Method. Relation to the Integral Equation. Case of Well-separated Classical Turning Points. WKBJ Method for Bound Systems. Penetration through a Barrier. WKBJ Method for Radial Equations. WKBJ Phase Shifts. Case of Closely Spaced Classical Turning Points. Short-wavelength Approximation in Three Dimensions.

This is a very famous subject and relevant for the particle folks. It is one thing to do approximations to find EF's for the "bound states" of a problem. But here we have continuous spectrum "scattering" type approximation. The **phase shifts** of the **partial wave** expansion are here. Higher **Born terms**. Fredholm scattering series. The famous **WKB method** to which they add the letter J for some reason. This subject appears in great detail in Schiff for example where you see **Schiff** Chapter 9 "Approximation methods in collision theory", and of course the whole **Goldberger Watson Collision Theory** book I have. This is a monstrous subject, but one can at least hope to get down the basic ideas.

9.4 *Variational Methods* 1106

Variational Principle for Eigenvalue Problems. Variational Principles for Resonant Frequencies and Energy Levels. Vibration of a Circular Membrane. Nonlinear Variational Parameters. Rayleigh-Ritz Method. Application to Perturbation Theory. Integral Equation and Corresponding Variational Principle. An Example. Variational Principle for Phase Shifts. Variational Principle for the Phase Shift Based on an Integral Equation. Variational Principle for the Transmission Amplitude. Variational Principle for Three-dimensional Scattering Problems. Variational Principles for Surface Perturbations. Variational Principle Based on the Integral Equation for Boundary Perturbations. A Variational Principle for Radiation Problems. Variation-iteration Method. An Extrapolation Method. Lower Bounds for λ_0. Comparison Method for Lower Bounds. An Example. $\mathfrak{L}$ Not Positive-definite. Variational Principles for the Higher Eigenvalues. Method of Minimized Iterations.

Stak did some of this for finding eigenvalues, iterative methods, fit things with parameters, find bounds. I am pretty sure I could read and understand this section if I wanted to. Notice that the "variation iteration" method appears in the TOC block above.

Problems (1158) 9.1 to 9.11

A. Appendix Reviews ALL the methods in a lot of detail.

Tabulation of Approximate Methods 1162

Bound States; Volume Perturbation. Iterative-perturbation Series. Feenberg Series. Fredholm Formula. Variational Principles for Bound States. Variation-iteration Method. Perturbation of Boundary Conditions. Perturbation of Boundary Shape. Perturbation Formulas for Scattering. Variational Principles for Scattering. Scattering from Spherically Symmetric Object.

Bibliography (1170)

Condon and Morse's QM book. Feenberg's note in 1948. Schiff is here. Pauling and Wilson QM.