Thomson Scattering PhL 10.20.08

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Summary (all in Jackson units)

• The general sinusoidal-time electric-dipole power radiation formula is this:

$$dP/d\Omega = (c/8\pi)\, k^4\, p^2 \sin^2\psi \qquad \text{where} \qquad \sin\psi \equiv \hat{\mathbf{r}} \times \hat{\mathbf{p}} \qquad \cos\psi \equiv \hat{\mathbf{r}} \bullet \hat{\mathbf{p}}$$

where radiating dipole is **p**, and observer is a distant location **r**. If the observer is located at spherical coordinates r,θ,φ (large r), and if the incoming radiation has $\hat{\mathbf{k}} = \hat{\mathbf{z}}$, the above can be written as

$$dP/d\Omega = (c/8\pi)\, k^4\, p^2\, [\, 1 - \sin^2\theta \cos^2(\varphi-\varphi_{\mathbf{p}})\,] \qquad \text{where } [\, 1 - \sin^2\theta \cos^2(\varphi-\varphi_{\mathbf{p}})\,] = \sin^2\psi$$

where $\varphi_{\mathbf{p}}$ indicates the direction of incoming polarization $\hat{\mathbf{p}}$ in the x-y plane.

• If a free electric charge is irradiated with an E&M plane wave, the charge acquires an electric dipole moment of this magnitude

$$p = E_o\, e^2/(m\omega^2) = E_o\, e^2/(mk^2c^2)$$

and when this is inserted into the formula above, we get

$$dP/d\Omega = (c/8\pi)\, E_o^2\, [\, e^2/mc^2]^2 \sin^2\psi$$

and we see no frequency dependence in the scattering.

• If we define a cross section according to $dP = S_o d\sigma$ where $\mathbf{S_o} = (c/8\pi)\, |E_o|^2\, \hat{\mathbf{k}}$, we get the following differential cross section for Thomson scattering

$$d\sigma/d\Omega = [\, e^2/mc^2]^2 \sin^2\psi = r_o^2 \sin^2\psi \qquad e^2/mc^2 = r_o = \text{"classical electron radius"}$$

• If the initial polarization is $\boldsymbol{\varepsilon} = \hat{\mathbf{p}}$, the final polarization is $\boldsymbol{\varepsilon}' = \{\, \hat{\mathbf{p}} - \cos\psi\, \hat{\mathbf{r}}\, \}/\sin\psi$ (both unit vectors) and $\boldsymbol{\varepsilon} \bullet \boldsymbol{\varepsilon}' = \sin\psi$.

• If we average over the continuum of initial polarization directions $\hat{\mathbf{p}}$, the cross section becomes.

$$d\bar{\sigma}/d\Omega = r_o^2/2\,(1 + \cos^2\theta) = r_o^2/2\,(2 - \sin^2\theta)$$

• If we then integrate over $d\Omega$ to get the total cross section, the result is

$$\bar{\sigma} = (8\pi/3)r_o^2 = (8/3)\,(\pi\, r_o^2)$$

which is very small because $r_\circ$ is very small.

History:

"J.J." Thomson 1856-1940 came of with the theory of Thomson scattering in 1906, and this theory applies any place you have free charges -- plasmas. From Google books,

> 3 The argument was first presented by Thomson in his book *Electricity and Matter* (London: Archibald Constable and Company, 1906), the published version of his Silliman Memorial Lectures delivered at Harvard in 1903. This same formula was subsequently used by Thomson in his book *Conduction of Electricity through Gases* (Cambridge: Cambridge University Press, 1907) to derive the expression for the cross-section for the scattering of X-rays by free electrons, the *Thomson cross-section*.

There are many avenues leading to these results, so today let's pick a simple one.

1. Dipole Radiation Formulas

On page 178 of Chapter 6 Jackson shows Maxwell's equations. On page 179 he shows that you can in effect replace the fields with the potential A^μ and you end up with some ugly coupled equations for the potential. He notes that the fields are unchanged by a gauge transformation $A^\mu \rightarrow A^\mu + \partial^\mu\Lambda$ and this allows us to select a Lorentz Gauge in which $\partial_\mu A^\mu = 0$ as in 6.36. In this gauge the messy equations decouple and become (using H-L units just for the moment)

$$\Box\, A^\mu = +\, J^\mu \qquad \text{using the BD metric } g^{\mu\nu} \qquad (\partial_a\, \partial^a) = \Box$$

This allows the Green's function method to be used to find A^μ as an integral of J_μ. Starting on page 183, Jackson computes this Green's function for the equation $\Box\psi = f$ just to be general. The Green's is the usual $\delta(t - r/c)/r$ as shown in 6.64 page 185 and this leads to the solution ψ as a d^4x integral over f times the Green's function as shown in 6.65. If you do the time integral, you get 6.66 which is then a d^3x integral of 1/r times the "retarded" f.

We then jump ahead to Jackson Chapter 9 page 269 where we see our solution of $\Box\, A^\mu = +\, J^\mu$ written in this manner 9.3 (now in Jackson units)

$$\mathbf{A}(\mathbf{x}) = 1/c \int d^3x' \ \mathbf{J_{ret}}(\mathbf{x}') / |\ \mathbf{x} - \mathbf{x}'|$$

This is a very general result which we shall apply to our very simple Thomson scattering problem. It is best at this point to express **A** as a d^4x' integral as shown in 9.2. Then when we install a sinusoidal time function for the current **J**(t') as in 9.1, we get an integral of $\delta(t' + r/c - t)dt' \exp(-i\omega t') = \exp(-i\omega t) \exp(+ikr)$ since $k = \omega/c$. We then define $\mathbf{A}(x,t) = \mathbf{A}(x)e^{-i\omega t}$ and then we end up with 9.3. It says this:

$$\mathbf{A}(\mathbf{x}) = 1/c \int d^3x' \ \mathbf{J}(\mathbf{x}') \ e^{ikr}/r$$

and you see *where* that famous e^{ikr}/r factor comes from: (1) the 1/r is present in the Green's function, (2) the e^{ikr} comes from the retardation effect so we wind up phase as we move away. Another thing we notice is that **A** points in the same direction as **J** ! Now if we go far away from a localized current distribution, we can make the Born approximation and pull the e^{ikr}/r out of the integral to get

$$\mathbf{A}(\mathbf{x}) = e^{ikr}/rc \int d^3x' \ \mathbf{J}(\mathbf{x}')$$

as shown page 271 9.13. Jackson does a parts integration on the above integral, uses the continuity equation (which picks us up a factor of $i\omega = ikc$ from the time derivative of ρ, and parts another minus) , and we end up with what we have been looking for here, our dipole radiation formula. The Born approximation just made above is thus equivalent to keeping only the dipole radiation part of the multipole radiation series. Our final dipole result then is this

$$\mathbf{A}(\mathbf{x}) = -ik\mathbf{p}\ e^{ikr}/r \qquad \mathbf{p} = \int d^3x' \ \mathbf{x}' \ \rho(\mathbf{x}') \ = \text{electric dipole moment of charge distributions}$$

which is 9.16. Again, **A** points in the same direction as **p**.

Using this for **A**, we can then compute the fields in the radiation zone to get

$$\mathbf{B} = k^2 \ e^{ikr}/r \ \hat{\mathbf{r}} \times \mathbf{p} \ = p\, k^2 \ e^{ikr}/r \ \hat{\mathbf{r}} \times \hat{\mathbf{p}} \ = \ (p\, k^2 \ e^{ikr}/r) \sin\psi \ \hat{\mathbf{t}}$$

$$\mathbf{E} = \mathbf{B} \times \hat{\mathbf{r}} \ = (p\, k^2 \ e^{ikr}/r\ ) \ \{\ \hat{\mathbf{p}} - \cos\psi \ \hat{\mathbf{r}}\ \} \ // \text{ see below} \qquad \sin\psi \equiv \hat{\mathbf{r}} \times \hat{\mathbf{p}} \qquad \cos\psi \equiv \hat{\mathbf{r}} \bullet \hat{\mathbf{p}}$$

as shown 9.19. The Poynting vector on page 205 is $\mathbf{S} = (c/8\pi)\mathbf{E} \times \mathbf{B}^*$ (with energy density $U = (1/8\pi)E^2$). We have an outgoing plane wave so $\mathbf{S} = (c/8\pi)\ |B|^2 \ \hat{\mathbf{r}}$ and $\mathbf{S} \bullet d\mathbf{A} = (c/8\pi)\ |B|^2 d\Omega$ since $d\mathbf{A} = r^2 d\Omega \ \hat{\mathbf{r}}$. The power flow into solid angle $d\Omega$ is then

$$dP/d\Omega \ = r^2 \ (c/8\pi) \ |B|^2 = r^2 \ (c/8\pi)^{\,2} \ k^4 \ p^2 \ /r^2 * \sin^2\psi = (c/8\pi) \ k^4 \ p^2 \ \sin^2\psi \qquad // \text{ Jackson 9.23 p272}$$

and this is the usual dipole antenna radiation result which is a donut pattern which peaks to the sides of the dipole antenna, but we express things in terms of **p** instead of current **J**.

2. Application to Thomson scattering: radiation hits a free charge

Suppose we zap a free charged particle (m, e in Jackson units) with an E field plane wave. We have $\mathbf{F} = e\mathbf{E}_o = m\mathbf{a}$ so $\mathbf{a} = (e/m)\mathbf{E}_o$. Assuming $e^{-i\omega t}$ time dependence for E we get $\mathbf{v} = \mathbf{a}/(-i\omega)$ and then $\mathbf{r} = \mathbf{a}/(-i\omega)^2 = -a/\omega^2 = -(e/m\omega^2)\,\mathbf{E}_o$. The dipole moment is $\mathbf{p} = e\mathbf{r}$ (in general it is $\Sigma q_i \mathbf{r}_i$) so we get $\mathbf{p} = -(e^2/m\omega^2)\,\mathbf{E}_o$.

So now we have a dipole moment to throw into our formulas above, with $\omega = kc$. We have

$p = E_o\, e^2/(m\omega^2) = E_o\, e^2/(mk^2c^2)$ = electric dipole magnitude

Then the above radiation formula is

$dP/d\Omega = (c/8\pi)\, k^4(E_o^2\, e^4/(m^2k^4c^4))\sin^2\psi = (c/8\pi)\, E_o^2\, [\, e^2/mc^2]^2\, \sin^2\psi$

where again $\cos\psi = \hat{\mathbf{r}} \bullet \hat{\mathbf{p}}$ and we can think of $\hat{\mathbf{p}} = \hat{\boldsymbol{\varepsilon}}$ = the polarization of the incoming plane wave scattering beam. The incoming beam had $\mathbf{S}_o = (c/8\pi)\, |E_o|^2\, \hat{\mathbf{k}}$ so set $dP = S_o d\sigma$ to define a cross section, then we get

$d\sigma/d\Omega = [\, e^2/mc^2]^2\, \sin^2\psi$ $\quad \sin\psi \equiv \hat{\mathbf{r}} \times \hat{\mathbf{p}}$ $\quad \cos\psi \equiv \hat{\mathbf{r}} \bullet \hat{\mathbf{p}}$

$= r_o^2\, \sin^2\psi$ $\quad e^2/mc^2 = r_o$ = "classical electron radius" $= 3 \times 10^{-13}$ cm

You could hardly imagine a simpler formula. The distance dimension for the cross section has to come from the three constant shown, and the resulting r_o gets the name shown. The number r_o gives the general distance scale of E&M scattering from an electron. Notice that the scattering is independent of the frequency of the incoming light! Our general dipole radiation formula has $k^4\, p^2$, but then our specific $p \sim 1/k^2$ and the k's cancel.

3. Concerning the $\boldsymbol{\varepsilon} \bullet \boldsymbol{\varepsilon}'$ factor.

We know what our radiation zone fields look like for dipole radiation

$\mathbf{B} = k^2\, e^{ikr}/r\ \hat{\mathbf{r}} \times \hat{\mathbf{p}} = (p\, k^2\, e^{ikr}/r)\ \hat{\mathbf{r}} \times \hat{\mathbf{p}}$

$\mathbf{E} = \mathbf{B} \times \hat{\mathbf{r}} = (p\, k^2\, e^{ikr}/r)\ (\hat{\mathbf{r}} \times \hat{\mathbf{p}}) \times \hat{\mathbf{r}} = (p\, k^2\, e^{ikr}/r)\ \{\, \hat{\mathbf{p}} - (\hat{\mathbf{r}} \bullet \hat{\mathbf{p}})\, \hat{\mathbf{r}}\, \}$

$= (p\, k^2\, e^{ikr}/r)\ \{\, \hat{\mathbf{p}} - \cos\psi\, \hat{\mathbf{r}}\, \}$

Thus, we know the direction of the final polarization unit vector $\boldsymbol{\varepsilon}'$

$\boldsymbol{\varepsilon}' = \{\, \hat{\mathbf{p}} - \cos\psi\, \hat{\mathbf{r}}\, \}/\mathrm{sqrt}(1+\cos^2\psi - 2\cos^2\psi) = \{\, \hat{\mathbf{p}} - \cos\psi\, \hat{\mathbf{r}}\, \}/\mathrm{sqrt}(1-\cos^2\psi) = \{\, \hat{\mathbf{p}} - \cos\psi\, \hat{\mathbf{r}}\, \}/\sin\psi$

and therefore

$\boldsymbol{\varepsilon} \bullet \boldsymbol{\varepsilon}' = \hat{\mathbf{p}} \bullet \{ \hat{\mathbf{p}} - \cos\psi\, \hat{\mathbf{r}} \} / \sin\psi = (1 - \cos^2\psi)/\sin\psi = \sin\psi$

which is the result I was looking for,

$\boldsymbol{\varepsilon} \bullet \boldsymbol{\varepsilon}' = \sin\psi$

so we can then say

$d\sigma/d\Omega = = r_o^2 \sin^2\psi = r_o^2 (\boldsymbol{\varepsilon} \bullet \boldsymbol{\varepsilon}')^2$ // Thomson scattering

where $\boldsymbol{\varepsilon} = \hat{\mathbf{p}}$ and $\boldsymbol{\varepsilon}'$ = the polarization of the scattered beam going out in direction $\hat{\mathbf{r}}$. Notice that electron spin does not enter our all-classical calculation at all. The above result agrees with the non-rel limit given in **BD1** page 131 equations D and E.

There is a distinction to notice however between the above formula and the BD one. In our formula here, $\boldsymbol{\varepsilon} = \hat{\mathbf{p}}$ is the assumed initial polarization, and given that fact, we get exactly $\boldsymbol{\varepsilon}'$ as the polarization of the photon after scattering. We don't sum over final polarizations because our $\boldsymbol{\varepsilon}'$ is preselected to be in the exact direction of the final polarization. We could sum in the direction perp to this and that would add 0. Below we will average over the continuum of all possible positions of $\boldsymbol{\varepsilon} = \hat{\mathbf{p}}$, whereas in BD we will average by taking ½ the sum of initial polarizations. We expect results to be the same.

4. The Thomson Cross section averaged over incoming polarization.

Suppose you have two unit vectors $\hat{\mathbf{r}}$ and $\hat{\mathbf{p}}$ in spherical coordinates. Then you can express each in Cartesian coordinates as

$\hat{\mathbf{r}} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$
$\hat{\mathbf{p}} = (\sin\theta_{\mathbf{p}}\cos\varphi_{\mathbf{p}}, \sin\theta_{\mathbf{p}}\sin\varphi_{\mathbf{p}}, \cos\theta_{\mathbf{p}})$

then

$\hat{\mathbf{r}} \bullet \hat{\mathbf{p}} = \sin\theta\cos\varphi\, \sin\theta_{\mathbf{p}}\cos\varphi_{\mathbf{p}} + \sin\theta\sin\varphi\sin\theta_{\mathbf{p}}\sin\varphi_{\mathbf{p}} + \cos\theta\cos\theta_{\mathbf{p}}$

Now suppose $\hat{\mathbf{p}}$ is tilted down to $\theta_{\mathbf{p}} = 90$ degrees and has no z component. Then we have

$\hat{\mathbf{r}} \bullet \hat{\mathbf{p}} = \sin\theta\cos\varphi\, \cos\varphi_{\mathbf{p}} + \sin\theta\sin\varphi\sin\varphi_{\mathbf{p}} = \sin\theta\, (\cos\varphi\, \cos\varphi_{\mathbf{p}} + \sin\varphi\sin\varphi_{\mathbf{p}}) = \sin\theta\, \cos(\varphi-\varphi_{\mathbf{p}}) = \cos\psi$

We have in mind a plane wave in the $\hat{\mathbf{z}}$ direction, where we know that polarization $\hat{\mathbf{p}}$ would then have to be in the x-y plane so the above would apply. Then we have

$\cos\psi = \hat{\mathbf{r}} \bullet \hat{\mathbf{p}} = \sin\theta\, \cos(\varphi-\varphi_{\mathbf{p}})$

$\sin^2\psi = 1 - \sin^2\theta\, \cos^2(\varphi-\varphi_{\mathbf{p}})$

Now, we know our $d\sigma/d\Omega \sim \sin^2\psi$ from above, and we know that $\varphi_{\mathbf{p}}$ gives the direction of the incident polarization in the x-y plane. If we want a cross section which is averaged over incoming polarization, we do this:

$$< (\boldsymbol{\varepsilon} \bullet \boldsymbol{\varepsilon}')^2> \;=\; <\sin^2\psi> \;=\; 1 - \sin^2\theta <\cos^2(\varphi-\varphi_{\mathbf{p}})> \;=\; 1 - \tfrac{1}{2}\sin^2\theta \;=\; \tfrac{1}{2}(1+\cos^2\theta)$$

since [and see Portis page 539 for verification]

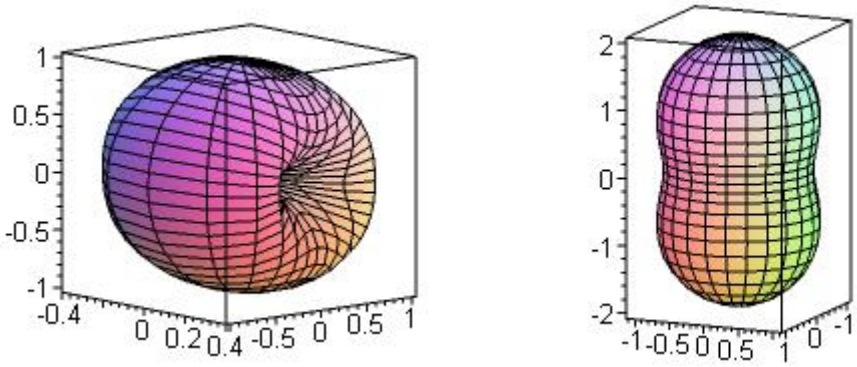
$$<\cos^2(\varphi-\varphi_{\mathbf{p}})> = (1/2\pi)\int_0^{2\pi} d\varphi_{\mathbf{p}}\cos^2(\varphi-\varphi_{\mathbf{p}}) \;=\; (1/2\pi)\int_0^{2\pi} d\varphi_{\mathbf{p}}\,(1/2)\,[\,1+\cos(2[\varphi-\varphi_{\mathbf{p}}])\,] = \tfrac{1}{2}$$

so we then have

$$d\bar{\sigma}/d\Omega = r_o^2\,(\boldsymbol{\varepsilon} \bullet \boldsymbol{\varepsilon}')^2 \;=\; r_o^2 <\sin^2\psi> = r_o^2/2\,(1+\cos^2\theta) = r_o^2/2\,(2-\sin^2\theta)$$

If we take the non-rel limit of **BD1** page 131 F, we set k=k' and the above formula is replicated! Recall that θ is the spherical angle coordinate of observation point **r** where incoming photons have $\hat{\mathbf{k}} = \hat{\mathbf{z}}$.

In the above computation, for each initial polarization $\hat{\mathbf{p}}$ we get a "donut" radiation pattern with axis along $\hat{\mathbf{p}}$ (left plot showing $\sin^2\psi = 1 - \sin^2\theta\cos^2(\varphi-\varphi_{\mathbf{p}})$). That is, no radiation in the $\pm\,\hat{\mathbf{p}}$direction. When we average these donuts around azimuth, everything is additive on the top and bottom so we get peaks in the directions θ=0 and θ = π, namely $\pm\,\hat{\mathbf{z}}$ where sinθ = 0. But things are reduced on the sides due to the effect of the donut hole nulls, so on the right is a plot of $r = (2 - \sin^2\theta)$ with 2 on the top and bottom, but only 1 on the sides. The photon is coming up from the bottom, so scattering is most in the forward and backward directions.



5. The total cross section (averaged over polarization)

Now we can integrate the result of the previous section over all solid angle to get

$$\bar{\sigma} = (r_o^2/2)\int d\Omega\,(1+\cos^2\theta) = \pi r_o^2 \int_{-1}^{1} dx\,(1+x^2) = 2\pi r_o^2 \int_0^1 dx\,(1+x^2) = 2\pi r_o^2\,(1+1/3)$$

$= (8\pi/3)r_o^2 = (8/3)\,\pi\, r_o^2$

in agreement with Portis page 540 (20) and BD1 page 132 A.

6. Doing the polarization sums to get BD page 131 equation F.

Assuming incoming along $\hat{\mathbf{z}}$ so we have these incoming polarizations

$\hat{\boldsymbol{\varepsilon}}^{(1)} = \hat{\mathbf{x}}$
$\hat{\boldsymbol{\varepsilon}}^{(2)} = \hat{\mathbf{y}}$

The outgoing photon has these possible states:

$\hat{\boldsymbol{\varepsilon}}'^{(1)} = \hat{\boldsymbol{\varphi}}$
$\hat{\boldsymbol{\varepsilon}}'^{(2)} = \hat{\boldsymbol{\theta}}$

Then I can put ½ *in front* of KN, then just manually sum on these polarizations:

$$\Sigma_{pol}\,(\boldsymbol{\varepsilon}\bullet\boldsymbol{\varepsilon}')^2 = (\hat{\boldsymbol{\varepsilon}}^{(1)}\bullet\hat{\boldsymbol{\varepsilon}}'^{(1)})^2 + (\hat{\boldsymbol{\varepsilon}}^{(1)}\bullet\hat{\boldsymbol{\varepsilon}}'^{(2)})^2 + (\hat{\boldsymbol{\varepsilon}}^{(2)}\bullet\hat{\boldsymbol{\varepsilon}}'^{(1)})^2 + (\hat{\boldsymbol{\varepsilon}}^{(2)}\bullet\hat{\boldsymbol{\varepsilon}}'^{(2)})^2$$

$$= (\hat{\mathbf{x}}\bullet\hat{\boldsymbol{\varphi}})^2 + (\hat{\mathbf{x}}\bullet\hat{\boldsymbol{\theta}})^2 + (\hat{\mathbf{y}}\bullet\hat{\boldsymbol{\varphi}})^2 + (\hat{\mathbf{y}}\bullet\hat{\boldsymbol{\theta}})^2$$

$$= \sin^2\varphi + \cos^2\theta\cos^2\varphi + \cos^2\varphi + \cos^2\theta\,\sin^2\varphi =$$

$$= 1 + \cos^2\theta$$

Of course constant terms in the NK formula will do this under the same summation

$\Sigma_{pol}\, a = 4a$

So, if the K-N formula were this:

$KN = K * (a + 4(\boldsymbol{\varepsilon}\bullet\boldsymbol{\varepsilon}')^2 - 2)$

Then our result would be

$KN = K/2 * (4a + 4 + 4\cos^2\theta - 8) = 2K * (a + 1 + \cos^2\theta - 2)$

$= 2K * (a + \cos^2\theta - 1) = 2K\,(a - \sin^2\theta).$

This then explains how we get from BD 7.74 to BD F.

7. Range of applicability

We used **F** = m**a** to derive the electrons motion and dipole moment. This means the electron velocity is v << c . Looking at item 2 above, we have **a** = (e/m)$\mathbf{E_o}$ and **v** = **a**/(-iω) so this suggests that we need

$(e/m)E_o/\omega \ll c$ or $E_o \ll \omega\,(mc/e) = \hbar\omega\,(mc/\hbar e)$

Another overlooked factor is quantum mechanics and the recoil of the electron, so we have been assuming $\hbar\omega \ll mc^2$. Combining these we get

$E_o\,(\hbar e/mc) \ll \hbar\omega \ll mc^2$

We noted that the electron is supposed to be free, as in a plasma, but atomic electrons can be considered "free" and to do Thomson scattering as long as the incoming photon energy is not near any bound state transition energies. In general, x-ray photons do Thomson scattering from the electrons in atoms, but you have to modify the formulas somewhat introducing a "form factor".

8. Comparison with Rayleigh Scattering.

In the Thomson case, we have p = er = -e/mω^2 E_o . This $1/\omega^2$ dependence cancels the k^4 factor present in the general dipole radiation formula, so Thomson scattering is independent of wavelength. It is also pretty weak because the classical electron radius is very small.

In the Rayleigh case, we have P = χ_eE or some such formula indicating we have an induced dipole moment in an atom or molecule or larger object. In this case, the susceptibility χ_e is often only weakly dependent on ω, so when we apply the general dipole radiation formula, we get $d\sigma/d\Omega \sim k^4 \sim \omega^4$ so Rayleigh scattering has a very strong 4th power frequency dependence

$$I = I_0 \frac{8\pi^4 \alpha^2}{\lambda^4 R^2}(1 + \cos^2\theta).$$

and this is why the sky is blue and all that good stuff. Here we see the same $(1 + \cos^2\theta)$ angular factor we had above in the Thomson case, $r_o^2/2\ (1 + \cos^2\theta)$, which arises from doing the polarization average. In Rayleigh, when the scattering object gets larger than the photon wavelength, the problem gets harder and it is called Mie scattering.