Cross Products under Rotations PhL 10.18.11

(a) Set up the Problem.

Consider

$\mathbf{Q} = \mathbf{B} \times \mathbf{C}$ $\qquad Q_a = \varepsilon_{abc} B_b C_c$

Presumably if **B** and **C** transform as vectors, so does **Q**. How do we show this fact.

$$Q'_a = \varepsilon_{abc} B'_b C'_c = \varepsilon_{abc} (R_{bb'} B_{b'}) (R_{cc'} B_{c'}) = (\varepsilon_{abc} R_{bb'} R_{cc'}) B_{b'} B_{c'}$$

On the other hand, we expect to have

$$Q'_a = R_{aa'} Q_{a'} = R_{aa'} \varepsilon_{a'bc} B_b C_c$$

Thus, we need to show that

$$(\varepsilon_{abc} R_{bb'} R_{cc'}) B_{b'} B_{c'} = R_{aa'} \varepsilon_{a'bc} B_b C_c = R_{aa'} \varepsilon_{a'b'c'} B_{b'} C_{c'}$$

Thus we need to show that

$$(\varepsilon_{abc} R_{bb'} R_{cc'}) = \varepsilon_{a'b'c'} R_{aa'}$$

This last equation is true as long as R is a rotation. I have solved this problem in N dimensions in Section 5 of document Matrix Theorems Addendum.doc in the matrix folder! It took a long time.

Update 2.1.17.

I just installed a simpler proof into Appendix A of Frames doc.