Comments on some courses I taught in 1978-1979. PhL 1.10.08

First, a quote from my "my physics teaching career" review: (this was about 30 years ago!!!)

> In Fall 1978 the courses were more serious. This was a 3-quarter Quantum Mechanics class with numbers Physics 545, 546 and 547. I think I was just a TA during the Fall 1978 quarter of **Physics 545.** The teacher for Fall was Henk Monkhorst whom I don't remember (but Jim Ball does; he failed to get on as assistant prof and had to leave). The class had 9 kids including Paul Arner, Tom Case, Deb Sawyer, Chris Stone. For Physics 545 I did maybe 5 lectures from Saxon's QM book, Bill Lee was the reader. I think I made up the problem sets.
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> Winter quarter was **Physics 546**, continuing Saxon's QM book. Only 7 students including those just mentioned. I was now the full-bore teacher and organizer of this and all remaining courses.
>
> Spring quarter 1979 was **Physics 547**, now only 6 students left (Clayton Williams quit). This is where we did full-bore group theory, angular momentum, and all that fancy stuff, and I have lots of good notes left over. This I did enjoy.

So I just today and yesterday read through my notes for Physics 547 and I find much material that is relevant to my current study of NMR and Levitt. I wanted to review this stuff now because I am trying to consolidate all my math angular momentum matrix related stuff in one place, and I felt there might be some "gems" hiding in these notes that I wanted locate. At the time, I was quite an expert in this subject, having read about group theory in a more general sense. In retrospect, it was good to do this now, and there were many useful results which I have tried to review below.

Interesting that this subject has not changed for 30 years and I guess will never change. This was a major discovery of humankind -- the notion of a rotation group and other groups and what they imply for the physical world. I don't think many physicists really know this stuff.

Physics 545 Notes

Nothing in these notes on angular momentum, I did not even teach this quarter's course, assisted.

Physics 546 Notes

1. The usual commutator notation is this:

$$[L_i, L_j] = i\hbar\varepsilon_{ijk}L_k$$

where only k is a summation index, and i and j are any fixed indices. Another way to write this same thing is

$$i\hbar L_k = \varepsilon_{kij}L_iL_j \qquad \text{which is the same as} \qquad i\hbar\mathbf{L} = \mathbf{L} \times \mathbf{L}$$

where now k is an arbitrary fixed index, and i and j are summation indices. These are forms I don't use very often, but they are surely valid. Notice that $\varepsilon_{kij}L_iL_j$ is not 0 by the usual rule of antisymmetric tensor contracted on symmetric, because L_iL_j is not symmetric since the operators don't commute! Also, a normal vector crossed with itself is normally zero in geometry, same idea.

2. I think I derived these facts:

$$L_+ = L_x + iL_y$$
$$L_- = L_x - iL_y$$

$$L_+L_- = L^2 - L_z^2 + \hbar L_z$$
$$L_-L_+ = L^2 - L_z^2 - \hbar L_z$$

The second show the notion that you don't raise or lower with these combinations.

3. In my lecture notes I show a few "**cone pictures**".

Physics 547 Notes

Lecture 2. An interesting group is **SO(2)**. A rotation appears this way:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \qquad \text{or } \mathbf{r'} = R_z(\theta)\,\mathbf{r}$$

The matrix R has det(R)=1 which is why this is a "**special**" group, the S of SO(2). Here is how you can find the generator of the rotations:

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \approx \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - i\delta\theta \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = 1 - i\,\delta\theta\ \sigma_2$$

So in SO(2), it is σ_2 which generators what we usually call $R_z(\theta) = e^{-i\theta\sigma_2}$. Also we can say

$$\sigma_2 = i \frac{dR_z(\theta)}{d\theta}\Big|^{\theta=0}$$

The **generator** tells you about group elements near the identity, and this local behavior determines large group elements as well, this is the essence if a **Lie Group**. At first I thought it was odd that σ_2 was the generator of this R_z, but now it seems fine. We are doing SO(2), not SO(3).

Lecture 3.

In SO(3), the general rotation is written $R_z(\phi)R_y(\theta)R_z(\phi')$ which I call the **Euler Angles**. Again, detR = 1 so this is "special" S as in SO(3). If you just add the third dimension to the SO(2) rotation, you get this

$$R_z(\phi) = \begin{pmatrix} C & -S & \\ S & C & \\ & & 1 \end{pmatrix} \qquad \text{so generator becomes} \qquad J_z = \begin{pmatrix} & -i & \\ i & & \\ & & \end{pmatrix}$$

$$R_x(\phi) = \begin{pmatrix} 1 & & \\ & C & -S \\ & S & C \end{pmatrix} \qquad \text{so generator becomes} \qquad J_x = \begin{pmatrix} & & \\ & & -i \\ & i & \end{pmatrix}$$

$$R_y(\phi) = \begin{pmatrix} C\theta & & S\theta \\ & 1 & \\ -S\theta & & C\theta \end{pmatrix} \qquad \text{so generator becomes} \qquad J_y = \begin{pmatrix} & & i \\ & & \\ -i & & \end{pmatrix}$$

where you find the generators just by looking at an infinitesimal rotation. Notice that y is different from the other two. These are my "active" rotations. The last line is a newer matrix form in Courier I just made today. I also talk about this rotation $R_{\hat{n}}(\theta) = e^{-i\theta(\hat{n} \bullet J)}$ but I don't given an expanded form for it.

Lecture 4.

The generators of the 2 dimensional representation of SO(3) are $J_i = (1/2)\sigma_i$. For the finite rotations, I get here the results shown page 577 of Levitt.

Reminder that the **Lie Group** is really completely determined by the **Lie Algebra**.

Lecture 6.

When you write U = exp(-iθG), a **Hermitian** generator gives a **unitary** transformation. All our angular momentum generators are Hermitian (transpose then asterisk).

You can take a Lie Algebra and sandwich some U around it and get a new set of generators that are ***equivalent***. Similarly, you can sandwich U around the R's of the Lie Group to get an equivalent representation of the Lie Group elements. This explains a **mystery** that we might not have noticed.

$$J_3 = \begin{pmatrix} & -i & \\ i & & \\ & & \end{pmatrix} \qquad \text{when we do rotations on 3-vectors}$$

$$J_3' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad \text{when we talk about } |j,m\rangle \text{ states for } j = 1.$$

Lecture 7.

When we talk about j=1/2, since $tr(\sigma_i) = 0$, we know that detU = 1 for $U(\boldsymbol{\theta}) = e^{-i(\boldsymbol{\theta} \bullet J)}$) where we are using $\mathbf{J} = \boldsymbol{\sigma}/2$. So matrices U are special and are unitary, so this is **SU(2)** where now our three parameters are the three real components of vector $\boldsymbol{\theta}$. I make this claim:

$$R_{ij}(\theta) = (1/2)\ tr\ [\ \sigma_i U(\boldsymbol{\theta}) \sigma_j U^{+}(\boldsymbol{\theta})]$$

which relates the 3 dim representation of SU(2) to that of SO(3). Also U and -U give the same R, so we have a 2:1 **homomorphism** between SU(2) and SO(3). They have the same abstract Lie Algebra.

Lecture 8.

The matrix group **O(n)** = O(n,R) of real orthogonal matrices has n(n-1)/2 parameters, so the O(3) group would have 3 parameters. Group elements can have det = +1 or -1 trivial to show. If you restrict to +1, you then get SO(3).

The matrix group **U(n)** of complex unitary matrices has n^2 real parameters and det = $e^{i\phi}$ in general. Again, if you restrict to det = +1, you get **SU(n)**, and then you have n^2 - 1 parameters. So SU(2) has 3 parameters and SU(3) has 8 parameters (the eightfold way).

You can have other matrix groups like **GL(n,C)** which means "general linear" with complex numbers, and this group has $2n^2$ real parameters.

Lecture 9

Earlier we talked about rotating 3-vectors and the generators used J_i and 3x3 matrices $R_{ij}(\theta)$. When you rotate the angular momentum kets, we do it this way (an equivalent representation)

$R_y(\theta) \mid j,m\rangle = d^j_{m'm}(\theta) \mid j,m'\rangle$ m' = summation index as usual

where the prime is on the *first* index, which is not what you might expect. But then we get

$\langle j,m''| R_y(\theta) \mid j,m\rangle = d^j_{m''m}(\theta) = \langle j,m''| e^{-i\theta J_y} \mid j,m\rangle$

and then the convention seems more reasonable. If you insert the full Euler Angles, you get this:

$\langle j,m''| R_z(\phi)R_y(\theta)R_z(\phi') \mid j,m\rangle = e^{-im''\phi} e^{-im\phi'} d^j_{m'm}(\theta) = D^j_{m'm}(\phi, \theta, \phi') = D^j_{m'm}(g)$

So in general we can write

$R(g) \mid j,m\rangle = D^j_{m'm}(g) \mid j,m'\rangle$ m' = summation index as usual

Lecture 10

This entire lecture is about Hilbert spaces which are the **direct sums** of other Hilbert spaces. My notation is this: [I also show quick hand drawings which help a lot, hard to make in Word. see below Visio]

$\mathbf{a_1} \oplus \mathbf{b_1}$ = a vector in the space $\mathcal{H}^{a\oplus b}$
$\mathbf{a}_1 + \mathbf{a}_2$ = a vector in the space $\mathcal{H}^{a}$

$\mathbf{A_1} \oplus \mathbf{B_1}$ = an operator in the space $\mathcal{H}^{a\oplus b}$

Rule for addition of vectors in $\mathcal{H}^{a\oplus b}$: $\mathbf{a_1} \oplus \mathbf{b_1} + \mathbf{a_2} \oplus \mathbf{b_2} = (\mathbf{a_1} + \mathbf{a_2}) \oplus (\mathbf{b_1} + \mathbf{b_2})$

And similarly for adding operators: $\mathbf{A_1} \oplus \mathbf{B_1} + \mathbf{A_2} \oplus \mathbf{B_2} = (\mathbf{A_1} + \mathbf{A_2}) \oplus (\mathbf{B_1} + \mathbf{B_2})$

You don't "multiply" vectors in $\mathcal{H}^{a\oplus b}$, you take their scalar product, so write that as

$$< \mathbf{a_1} \oplus \mathbf{b_1} , \mathbf{a_2} \oplus \mathbf{b_2}>_{a\oplus b} = < \mathbf{a}_1 , \mathbf{a}_2>_a + <\mathbf{b}_1 , \mathbf{b}_2>_b$$

If you define the scalar product in this way, you can show that this really is a scalar product.

You do multiply operators with no symbol used to show it:

$$\mathbf{A_1} \oplus \mathbf{B_1} \quad \mathbf{A_2} \oplus \mathbf{B_2} = \mathbf{A_1A_2} \oplus \mathbf{B_1B_2}$$

And you can act on a vector with an operator:

$$\mathbf{A} \oplus \mathbf{B} \ \ \mathbf{a_1} \oplus \mathbf{b_1} = \mathbf{Aa_1} \oplus \mathbf{B_1b_1}$$

If you indicate a vector or operator just in space $\mathcal{H}^a$, then you are assuming it has a 0 matching element in space $\mathcal{H}^b$ and vice versa. So consider this shorthand notation:

$$< \mathbf{a}_1 , \mathbf{b}_1>_{a\oplus b} = < \mathbf{a_1} \oplus \mathbf{0} , \mathbf{0} \oplus \mathbf{b_1}>_{a\oplus b} = < \mathbf{a}_1 , \mathbf{0}>_a + <\mathbf{0} , \mathbf{b}_1>_b = 0 + 0 = 0$$

example: $<j_1 m_1 , j_2 m_2> = 0 \quad \text{if } j_1 \neq j_2$

Here are some drawings I tried to make in Visio:

Here is a column vector representation of a direct sum vector:

$$a \oplus b = \left\langle \begin{pmatrix} a \\ b \\ c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \right\rangle \qquad a = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad b = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$A \oplus B = \left(\begin{array}{c|c} A & 0 \\ \hline 0 & B \end{array} \right)$$

And on the right above is a matrix representation of a direct sum of two operators. It is a 5x5 matrix in block diagonal form where A is a 3x3 matrix and B is a 2x2 matrix, just an example.

It is possible but not very useful to try to write out specific matrix elements in this picture. For example:

$[\mathbf{A} \oplus \mathbf{B}]_{mn} = A_{mn}$ if m,n both ≤ 3, else $= B_{mn}$ if m,n both in range $4 \leq 5$, else $= 0$.

$= A_{mn}\, \theta(3-m)\theta(3-n) + B_{mn}\, \theta(m-4)\theta(n-4)$ // imprecise but just an idea

Really, this clumsy thing is the definition of a direct sum of two operators, and a similar thing would be the definition of the direct sum of two vectors.

What about the action of a constant multiplier?

$\alpha[\mathbf{a_1} \oplus \mathbf{b_1}] = \alpha\mathbf{a_1} \oplus \alpha\mathbf{b_1}$ and same for operators $\alpha[\mathbf{A_1} \oplus \mathbf{B_1}] = \alpha\mathbf{A_1} \oplus \alpha\mathbf{B_1}$

This follows from our clumsy evaluation of the matrix elements just shown, or from the pictures where you see that everything has to scale together.

This lecture then goes on to generalize to a direct sum of 3 or more Hilbert Spaces.

I then quote a few obvious theorems, one of which is this regarding commutators:

$[\ \mathbf{A_1} \oplus \mathbf{B_1}\ ,\ \mathbf{A_2} \oplus \mathbf{B_2}\ ]\ =\ [\ \mathbf{A_1}, \mathbf{A_2}\ ] \oplus [\ \mathbf{B_1}, \mathbf{B_2}\ ]$

Lecture 11.

Using the above theorem, I consider a direct sum of generators

$$J_i^{j1 \oplus j2} \ = \ J_i^{j1} \oplus J_i^{j2}$$

The result is a "representation" of the Lie Algebra of SO(3) or SU(2) but since it can be put into block diagonal form by a unitary transformation (in this case unity), it is a "**reducible representation**" . All the basic reps of SO(3) that we talk about are **irreducible**. Same talk applies to the Lie Group elements, and we thus have the "irreducible unitary representations of SO(3)".

Lecture 12

Here we talk about **direct product** (cross product, Kronecker product) representations. Here are some of the rules:

$\mathbf{a_1} \otimes \mathbf{b_1}$ = a vector in the space $\mathcal{H}^{a \otimes b}$
$\mathbf{a}_1 + \mathbf{a}_2$ = a vector in the space $\mathcal{H}^{a}$

$\mathbf{A_1} \otimes \mathbf{B_1}$ = an operator in the space $\mathcal{H}^{a \otimes b}$

Rule for addition of vectors in $\mathcal{H}^{a \otimes b}$: $\mathbf{a_1} \otimes \mathbf{b_1} + \mathbf{a_2} \otimes \mathbf{b_2} \ \neq\ (\mathbf{a_1} + \mathbf{a_2}) \otimes (\mathbf{b_1} + \mathbf{b_2})$

but $\mathbf{a_1} \otimes \mathbf{b_1} + \mathbf{a_1} \otimes \mathbf{b_2} = \mathbf{a}_1 \otimes (\mathbf{b_1} + \mathbf{b_2})$

And similarly for adding operators: $\mathbf{A_1} \otimes \mathbf{B_1} + \mathbf{A_2} \otimes \mathbf{B_2} \neq (\mathbf{A_1} + \mathbf{A_2}) \otimes (\mathbf{B_1} + \mathbf{B_2})$

but $\mathbf{A_1} \otimes \mathbf{B_1} + \mathbf{A_1} \otimes \mathbf{B_2} = \mathbf{A_1} \otimes (\mathbf{B_1} + \mathbf{B_2})$

So right away, things are much different from the direct sum situation.

You don't "multiply" vectors in $\mathcal{H}^{a\otimes b}$, you take their scalar product, so write that as

$$< \mathbf{a}_1 \otimes \mathbf{b}_1 , \mathbf{a}_2 \otimes \mathbf{b}_2>_{a\otimes b} = < \mathbf{a}_1 , \mathbf{a}_2>_a \ <\mathbf{b}_1 , \mathbf{b}_2>_b \qquad \text{// normal multiplication}$$

If you define the scalar product in this way, you can show that this really is a scalar product.

You do multiply operators with no symbol used to show it:

$$\mathbf{A}_1 \otimes \mathbf{B}_1 \quad \mathbf{A}_2 \otimes \mathbf{B}_2 \quad = \mathbf{A}_1\mathbf{A}_2 \otimes \mathbf{B}_1\mathbf{B}_2$$

And you can act on a vector with an operator:

$$\mathbf{A} \otimes \mathbf{B} \ \ \mathbf{a}_1 \otimes \mathbf{b}_1 \quad = \mathbf{A}\mathbf{a}_1 \otimes \mathbf{B}_1\mathbf{b}_1$$

These last two results are similar to those of the direct sum world.

It is possible and in fact useful to write out specific matrix elements in this picture. For example, we can represent each index as a pair of indices.

$$[\mathbf{A} \otimes \mathbf{B}]_{ij,\ i'j'} = A_{ii'} B_{jj'}$$

We can take the above as the definition of the direct product of two operators (aka cross product or Kronecker product) and then use it to prove various things claimed above. We also have

$$[\mathbf{a} \otimes \mathbf{b}]_{ij} = a_i b_j \qquad \text{// for vectors}$$

What about the action of a constant multiplier?

$\alpha[\mathbf{a}_1 \otimes \mathbf{b}_1] = \alpha\mathbf{a}_1 \otimes \mathbf{b}_1 = \mathbf{a}_1 \otimes \alpha\mathbf{b}_1$ and same for operators $\alpha[\mathbf{A}_1 \otimes \mathbf{B}_1] = \alpha\mathbf{A}_1 \otimes \mathbf{B}_1 = \mathbf{A}_1 \otimes \alpha\mathbf{B}_1$

This follows easily from our definition, and you see that it differs a lot from the direct sum rule.

Example $\mathcal{H}^a \otimes \mathcal{H}^b = \mathcal{H}^{a\otimes b}$ where $a = j_1$ and $b = j_2$, two angular momentum representations.

Usual vector notation is this: $|j_1 m_1> \otimes |j_2 m_2>$ usually written as $| j_1 m_1; j_2 m_2>$

I then quote some theorems:

$$e^{A\otimes 1} = e^A \otimes 1 \qquad \text{// based on } [A \otimes 1]^n = A^n \otimes 1$$

$$[\ \mathbf{A}_1 \otimes \mathbf{B}_1 \ , \ \mathbf{A}_2 \otimes \mathbf{B}_2 \ ] \ = \ \mathbf{A}_1\mathbf{A}_2 \otimes [\ \mathbf{B}_1, \mathbf{B}_2 \ ] \ + \ [\ \mathbf{A}_1, \mathbf{A}_2 \ ] \ \mathbf{B}_2\mathbf{B}_1$$

$e^{[A\otimes 1 + 1\otimes B]} = e^{A} \otimes e^{B} = e^{A*B}$ // note my definition of the * operator

The first two are trivial, the last one I prove in my "* operators" notes.

Lecture 13

Due to the last theorem above, doing an **A*B operation** on generators results in a A⊗B result for finite group elements. Example:

define $\mathbf{J}^{a\otimes b} \equiv \mathbf{J}^{a} * \mathbf{J}^{b}$ // $= \mathbf{J}^{a} \otimes 1 + 1 \otimes \mathbf{J}^{b}$

Then you find that: $R^{a\otimes b} \equiv R^{a} \otimes R^{b}$

I show that the newly defined generators $\mathbf{J}^{a\otimes b}$ satisfy the usual Lie Algebra, so they are defining some kind of representation of the Lie Algebra, so $R^{a\otimes b}$ is some kind of representation of the Lie Group.

I then quote the basic **theorem of reducibility of direct product angular momentum representations**. There is a shuffle (unitary transformation) required to get the result into block form, however.

Lecture 14

Here we talk about that "shuffle" and the **CG coefficients**. I think this is my grand finale. You just have to read this lecture to see it. I show that the CG coefficients are a matrix which is the unitary transformation which brings the direct product representation to block diagonal form:

$$C^{j_1 j_2}_{m_1 m_2,\, jm} = \langle j_1 m_1, j_2 m_2 \mid jm \rangle$$

where the superscript $j_1 j_2$ tells us we are talking about THIS direct product representation, and the lower indices have one foot in the direct product world, and one foot in the block diagonal world. I then must have handed out the particle data group summary page of these coefficients which has everything on a single page for the lower combinations.

Lecture 15

Misc items.

One has to do with det(1+λK) but I don't know what this relates to! I think if λ is very small, we can say that:

$$\det(1+\lambda K) \approx 1 + \lambda\, \mathrm{tr}(K) \qquad \text{where K is any square matrix}$$

This fact is then used to show that

$$\det(e^{A}) = e^{\mathrm{tr}(A)}$$

in a very compact proof. This is a general matrix theorem and I have here a proof of the thing.

Second item is about **Stern-Gerlach** experiment. What does a beam to if polarized at some angle θ. This is two pages.

Lecture 16

Discussion of "**what it means for a particle to have angular momentum j**". If it is described by a set of states that transform in the right way, then it has angular momentum j. Comparison is made to why **p** is a "vector" -- because its components transform as a vector.

Next is a very fast run-through of **LS versus j-j coupling in atomic physics**. If two electrons don't interact, then each has some j from is ℓ and its s, and then you combine these.

Lecture 17

Claim is that if **A** is a **vector operator**, then this must be true: [that is, the vector A belongs to or transforms according to the j=1 representation of SO(3)]

$$[J_i, A_j] = i\varepsilon_{ijk}A_k$$

This is true for operators **L, R, P** for example. ($\hbar$ = 1 here). [Recall that there is a similar but fancier formula defining the **irreducible tensor operators** and one of these has ℓ = 1] .

Lecture 18

A rotational **scalar** does this:

$$[J_i, H] = 0 \qquad \text{// H = Hamiltonian is an example}$$

This means that a state will have these "**quantum numbers**" : E, j, m . For example, the single-electron states of **hydrogen** have all three of these quantum numbers.

Lecture 19

Discuss the **translation group** in one dimension, generator turns out to be $G = -i\,\partial/\partial x = \hat{\mathbf{p}}_x$. So momentum is the generator for translations. Full group is T_3, **abelian** since the three generators commute.

Lecture 20

If Hamiltonian has no LS interaction terms, you can diagonalize J, L, S, H and J_3 say. Then we add an interaction term of a certain form L•S which is really $L_i \otimes S_i$ summed on i. It is easy then to compute shift in energy levels from perturbation theory. I then go on to do **spectroscopic notation** where these things are all good quantum numbers, and I show that two levels are **split** by the interaction.

At this point, we come into a **second series of lectures**, the binder tab says "Lec 2". I don't know what the dividing line was. I guess I was done with the big angular momentum push.

1. I open with discussion of **scattering** theory from Saxon I suppose, including the famous spinless phase shifts δ_ℓ, getting to a discussion of **cross section** $d\sigma/d\Omega$ and the **optical theorem**. This seems to be a long 15 page set of notes.

2. Next section shows how **symmetry** can simplify a **diagonalization** of the Ham problem.

3. Next I am off with **Green's functions** in scattering theory, the propagator, and finally we get to the **Born Approximation** idea. This is all in Saxon I think.

4. Another section showing that you look for **conserved quantities** and label your state accordingly. Mention of benzene and the D6 group (this would be a Tinkham topic, it looks as if I assigned this job to Debbie Sawyer.

5. Suddenly we are doing **4-vectors** and the $R^\mu{}_\upsilon$ rotation matrix for p^μ . And then the action of **boosts** as well. We then come up with three rotation generators J, and three boost generators K. We write the Lie Algebra of these 6 things, and this group is **SO(3,1)** or the **Lorentz Group**. Equivalent to SL(2,C) I claim, there are 6 parameters. There are two Casimirs $J \bullet K$ and $J^2 - K^2$. Claim that SO(3,1) equiv SU(2) $\oplus$ SU(2) where we know there would be two Casimirs. The representations are labeled (j_1, j_2) therefore. I then claim that the (0,1/2) representations are called "**spinor**" and that (0,1/2) $\oplus$ (1/2,0) is "**Dirac spin**". This is pretty fancy stuff for this class I think.

Then we add the four T translation generators. This makes the **Poincare Group** with 10 generators. And this group also has 2 Casimirs. One is $p^2 = p_\mu p^\mu$. The other is some combination mess of many generators and is called $s^2 = s_\mu s^\mu$ and is related to spin. I claim that particles must have quantum numbers for these **two Casimirs** in eigenstates. In fact,. the irreducible representations of the Poincare Group are called "particles". **A particle must have momentum and spin.**

At this point we started reading some **Feynman lectures**.

Then start the **graphical method** of doing group representations as dot diagrams. We did some SU(3) at this point. My last item is a chart showing matter and radiation and quantum versus classical.