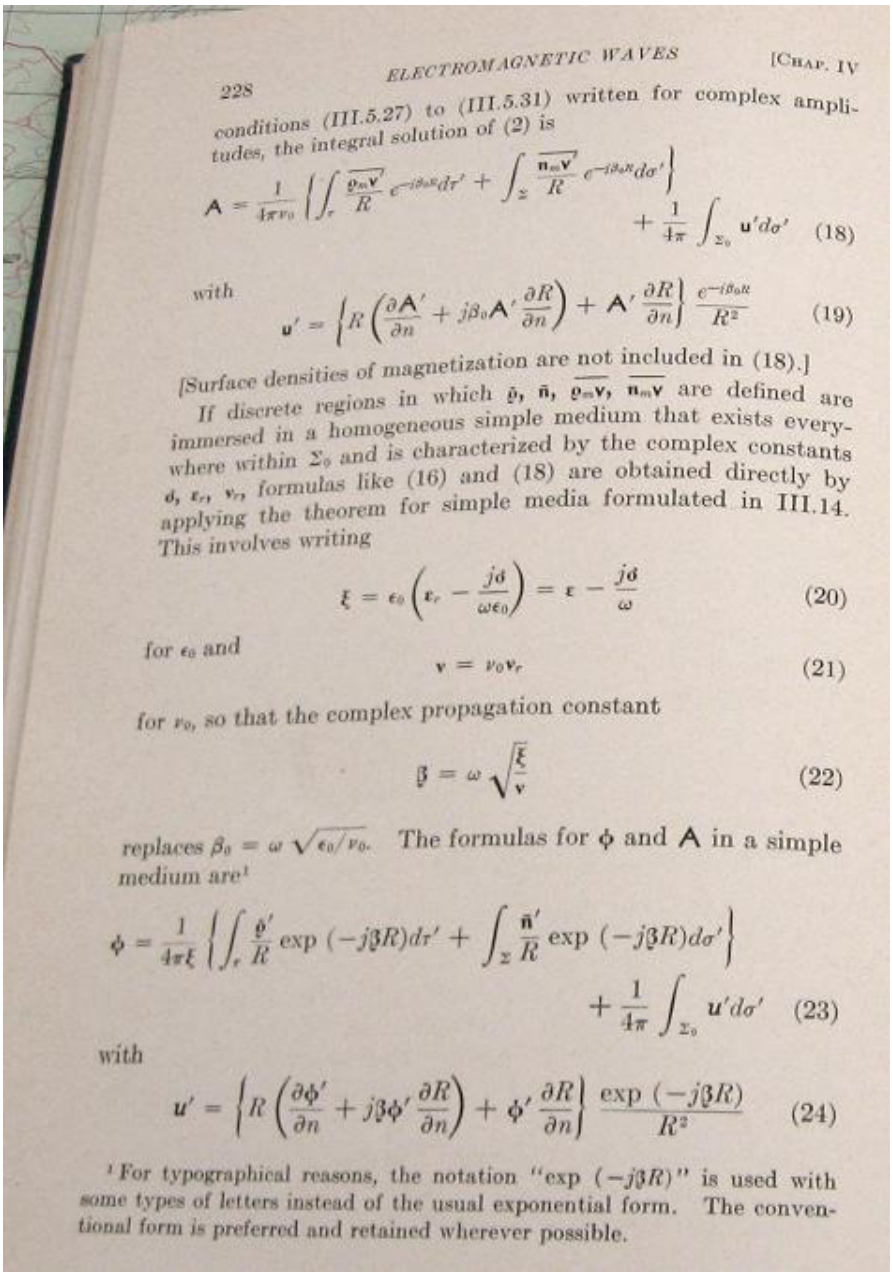
King Notes from Electromagnetic Engineering PhL 10.19.13

Here are the things King has to say about ξ in his 1945 book Electromagnetic Engineering which I got from the ARC. Will archive this as "reviewed", 11.15.13.

The page below shows the usual ν_0 version of A where he breaks current integral into a volume integral term and then a surface integral term dσ' . He also shows the boundary condition terms which arise from the Green's Theorem thing. It has the normal gradient of various things as you expect. The boundary is called Σ_0. He then arm-waves in the middle paragraph, defines ξ as usual as ε + σ/jω and then suddenly you see ξ appearing in the final integral! This is his unexplained magic trick. He references III.14 as if that section were going to explain things in more detail. We shall see.



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conditions (III.5.27) to (III.5.31) written for complex amplitudes, the integral solution of (2) is

$$\mathsf{A} = \frac{1}{4\pi\nu_0}\left\{\int_\tau \frac{\overline{\varrho_m \mathbf{v}}}{R} e^{-j\beta_0 R} d\tau' + \int_\Sigma \frac{\overline{\mathbf{n}_m \mathbf{v}}}{R} e^{-j\beta_0 R} d\sigma'\right\} + \frac{1}{4\pi}\int_{\Sigma_0} \mathbf{u}' d\sigma' \quad (18)$$

with

$$\mathbf{u}' = \left\{R\left(\frac{\partial \mathsf{A}'}{\partial n} + j\beta_0 \mathsf{A}' \frac{\partial R}{\partial n}\right) + \mathsf{A}' \frac{\partial R}{\partial n}\right\} \frac{e^{-j\beta_0 R}}{R^2} \quad (19)$$

[Surface densities of magnetization are not included in (18).]

If discrete regions in which $\bar{\varrho}$, $\bar{\mathbf{n}}$, $\overline{\varrho_m \mathbf{v}}$, $\overline{\mathbf{n}_m \mathbf{v}}$ are defined are immersed in a homogeneous simple medium that exists everywhere within Σ_0 and is characterized by the complex constants δ, ϵ_r, ν_r, formulas like (16) and (18) are obtained directly by applying the theorem for simple media formulated in III.14. This involves writing

$$\xi = \epsilon_0\left(\epsilon_r - \frac{j\delta}{\omega\epsilon_0}\right) = \epsilon - \frac{j\delta}{\omega} \quad (20)$$

for ϵ_0 and

$$\nu = \nu_0 \nu_r \quad (21)$$

for ν_0, so that the complex propagation constant

$$\beta = \omega\sqrt{\frac{\xi}{\nu}} \quad (22)$$

replaces $\beta_0 = \omega\sqrt{\epsilon_0/\nu_0}$. The formulas for ϕ and A in a simple medium are[1]

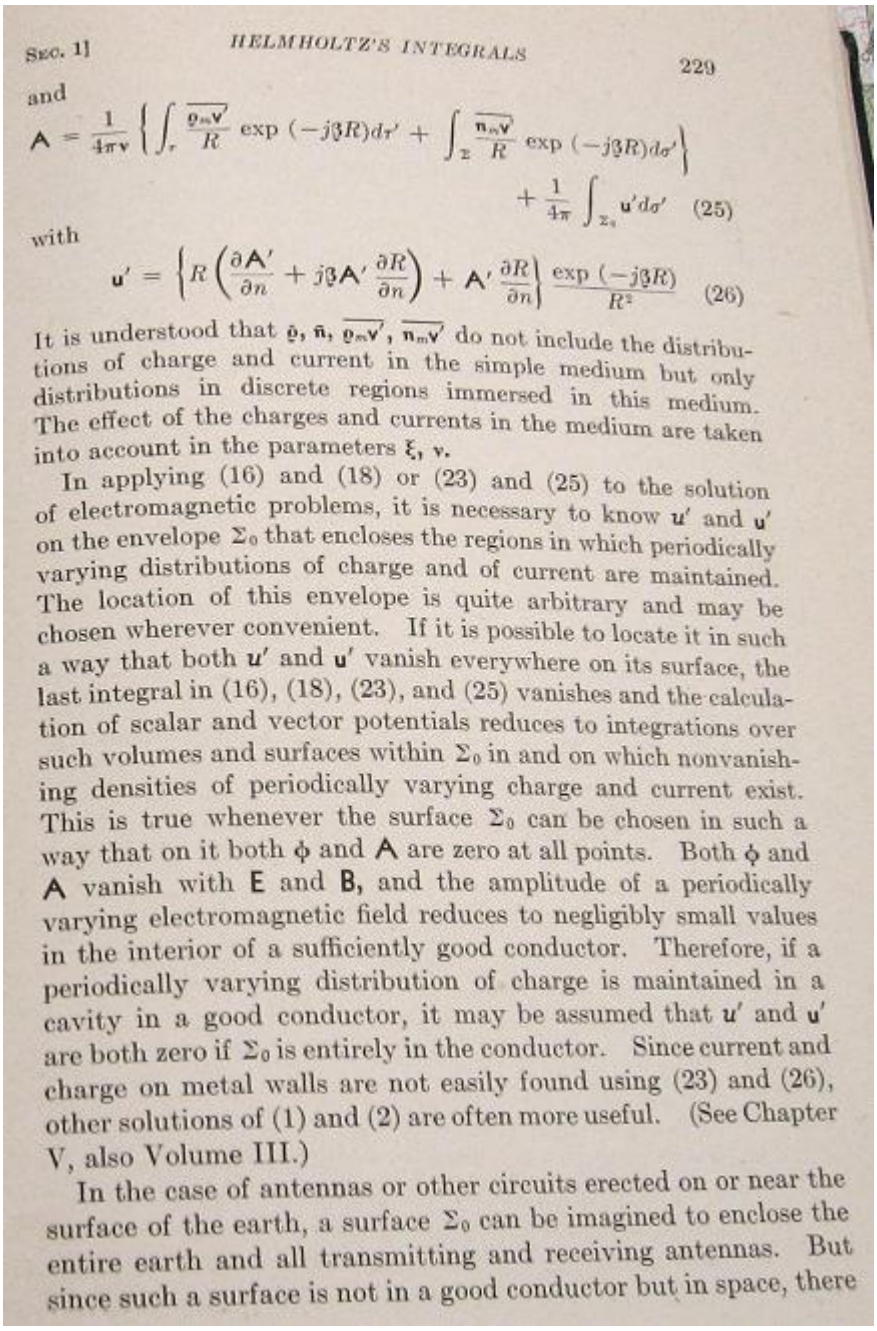
$$\phi = \frac{1}{4\pi\xi}\left\{\int_\tau \frac{\bar{\varrho}'}{R} \exp(-j\beta R) d\tau' + \int_\Sigma \frac{\bar{\mathbf{n}}'}{R} \exp(-j\beta R) d\sigma'\right\} + \frac{1}{4\pi}\int_{\Sigma_0} \mathbf{u}' d\sigma' \quad (23)$$

with

$$\mathbf{u}' = \left\{R\left(\frac{\partial \phi'}{\partial n} + j\beta\phi' \frac{\partial R}{\partial n}\right) + \phi' \frac{\partial R}{\partial n}\right\} \frac{\exp(-j\beta R)}{R^2} \quad (24)$$

[1] For typographical reasons, the notation "exp $(-j\beta R)$" is used with some types of letters instead of the usual exponential form. The conventional form is preferred and retained wherever possible.

Here is the corresponding A integral on the next facing page. More fiddly words that don't mean much to me. He then discusses those boundary terms and how we like them to go away.



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and

$$\mathsf{A} = \frac{1}{4\pi\nu}\left\{\int_\tau \frac{\overline{\varrho_m \mathsf{v}'}}{R}\exp(-j\beta R)d\tau' + \int_\Sigma \frac{\overline{\mathfrak{n}_m \mathsf{v}'}}{R}\exp(-j\beta R)d\sigma'\right\} + \frac{1}{4\pi}\int_{\Sigma_0} \mathfrak{u}' d\sigma' \quad (25)$$

with

$$\mathfrak{u}' = \left\{R\left(\frac{\partial \mathsf{A}'}{\partial n} + j\beta \mathsf{A}'\frac{\partial R}{\partial n}\right) + \mathsf{A}'\frac{\partial R}{\partial n}\right\}\frac{\exp(-j\beta R)}{R^2} \quad (26)$$

It is understood that ϱ, $\mathfrak{n}$, $\overline{\varrho_m \mathsf{v}'}$, $\overline{\mathfrak{n}_m \mathsf{v}'}$ do not include the distributions of charge and current in the simple medium but only distributions in discrete regions immersed in this medium. The effect of the charges and currents in the medium are taken into account in the parameters ξ, ν.

In applying (16) and (18) or (23) and (25) to the solution of electromagnetic problems, it is necessary to know $\mathfrak{u}'$ and $\mathfrak{u}'$ on the envelope Σ_0 that encloses the regions in which periodically varying distributions of charge and of current are maintained. The location of this envelope is quite arbitrary and may be chosen wherever convenient. If it is possible to locate it in such a way that both $\mathfrak{u}'$ and $\mathfrak{u}'$ vanish everywhere on its surface, the last integral in (16), (18), (23), and (25) vanishes and the calculation of scalar and vector potentials reduces to integrations over such volumes and surfaces within Σ_0 in and on which nonvanishing densities of periodically varying charge and current exist. This is true whenever the surface Σ_0 can be chosen in such a way that on it both ϕ and A are zero at all points. Both ϕ and A vanish with E and B, and the amplitude of a periodically varying electromagnetic field reduces to negligibly small values in the interior of a sufficiently good conductor. Therefore, if a periodically varying distribution of charge is maintained in a cavity in a good conductor, it may be assumed that $\mathfrak{u}'$ and $\mathfrak{u}'$ are both zero if Σ_0 is entirely in the conductor. Since current and charge on metal walls are not easily found using (23) and (26), other solutions of (1) and (2) are often more useful. (See Chapter V, also Volume III.)

In the case of antennas or other circuits erected on or near the surface of the earth, a surface Σ_0 can be imagined to enclose the entire earth and all transmitting and receiving antennas. But since such a surface is not in a good conductor but in space, there

Here is a later page in the book where I think he is doing transmission lines. Again we have the fancy version of the A_z integral as you see with β_c in the exponent and the fancy ν presumably. This page is not too important for this discussion since we have already seen the fancy integrals.

scalar and vector potential differences as well as the total current and not in terms of scalar potential difference and total current alone as is customary. It is possible to eliminate the vector potential only if the condition (19.2) $\beta_c b << 1$ is a good approximation. The vector potential due to a total axial current I_z in a straight conductor extending from $z = 0$ to $z = s$ is (IV.6.4)

$$A_z = \frac{1}{4\pi\nu}\int_0^s I'_z \frac{\exp(-j\beta R)}{R}\,dz' \tag{1}$$

where

$$R = \sqrt{(z'-z)^2 + r^2} \tag{2}$$

is the distance from the element dz' on the axis of the conductor to the point where A_z is calculated. This expression is a good approximation on the surface of the conductor.

On the equipotential surface $r = a$ of conductor 1, the vector potential due to the total axial currents I_1 and I_2 in both conductors is

$$A_{1z} = \frac{1}{4\pi\nu}\int_0^s \left\{ I'_1 \frac{\exp(-j\beta R_a)}{R_a} + I'_2 \frac{\exp(-j\beta R_b)}{R_b} \right\} dz' \tag{3}$$

with

$$R_a = \sqrt{(z'-z)^2 + a^2} \tag{4}$$

$$R_b = \sqrt{(z'-z)^2 + b^2} \tag{5}$$

The primed coordinates locate the element of integration; the unprimed the point where A_{1z} is calculated. The primed currents are at z'. On conductor 2,

$$A_{2z} = \frac{1}{4\pi\nu}\int_0^s \left\{ I'_2 \frac{\exp(-j\beta R_a)}{R_a} + I'_1 \frac{\exp(-j\beta R_b)}{R_b} \right\} dz' \tag{6}$$

The vector potential difference is

$$W_z = A_{1z} - A_{2z} = \frac{1}{4\pi\nu}\int_0^s (I'_1 - I'_2) \left\{ \frac{\exp(-j\beta R_a)}{R_a} - \frac{\exp(-j\beta R_b)}{R_b} \right\} dz' \tag{7}$$

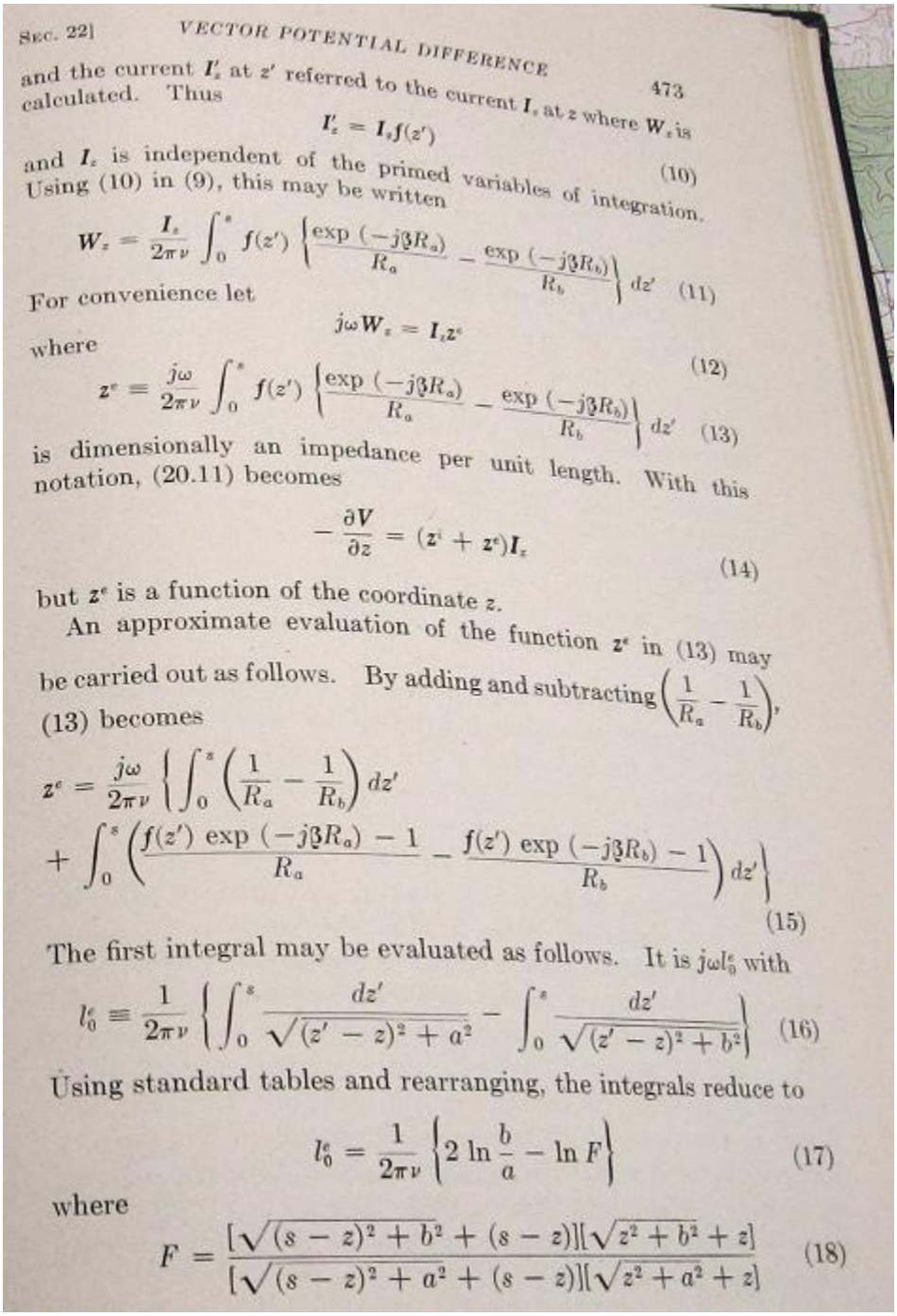
Since it is assumed that

$$I'_2 = -I'_1 = -I'_z \tag{8}$$

$$W_z = \frac{1}{2\pi\nu}\int_0^s I'_z \left\{ \frac{\exp(-j\beta R_a)}{R_a} - \frac{\exp(-j\beta R_b)}{R_b} \right\} dz' \tag{9}$$

Now let a distribution function $f(z')$ for current be introduced

This is just the page facing the above, nothing of direct interest.



and the current I_z' at z' referred to the current I_z at z where W_z is calculated. Thus

$$I_z' = I_z f(z') \tag{10}$$

and I_z is independent of the primed variables of integration. Using (10) in (9), this may be written

$$W_z = \frac{I_z}{2\pi\nu} \int_0^s f(z') \left\{ \frac{\exp(-j\beta R_a)}{R_a} - \frac{\exp(-j\beta R_b)}{R_b} \right\} dz' \tag{11}$$

For convenience let

$$j\omega W_z = I_z z^e \tag{12}$$

where

$$z^e \equiv \frac{j\omega}{2\pi\nu} \int_0^s f(z') \left\{ \frac{\exp(-j\beta R_a)}{R_a} - \frac{\exp(-j\beta R_b)}{R_b} \right\} dz' \tag{13}$$

is dimensionally an impedance per unit length. With this notation, (20.11) becomes

$$-\frac{\partial V}{\partial z} = (z^i + z^e) I_z \tag{14}$$

but z^e is a function of the coordinate z.

An approximate evaluation of the function z^e in (13) may be carried out as follows. By adding and subtracting $\left(\frac{1}{R_a} - \frac{1}{R_b}\right)$, (13) becomes

$$z^e = \frac{j\omega}{2\pi\nu} \left\{ \int_0^s \left(\frac{1}{R_a} - \frac{1}{R_b} \right) dz' + \int_0^s \left(\frac{f(z')\exp(-j\beta R_a) - 1}{R_a} - \frac{f(z')\exp(-j\beta R_b) - 1}{R_b} \right) dz' \right\} \tag{15}$$

The first integral may be evaluated as follows. It is $j\omega l_0^e$ with

$$l_0^e \equiv \frac{1}{2\pi\nu} \left\{ \int_0^s \frac{dz'}{\sqrt{(z'-z)^2 + a^2}} - \int_0^s \frac{dz'}{\sqrt{(z'-z)^2 + b^2}} \right\} \tag{16}$$

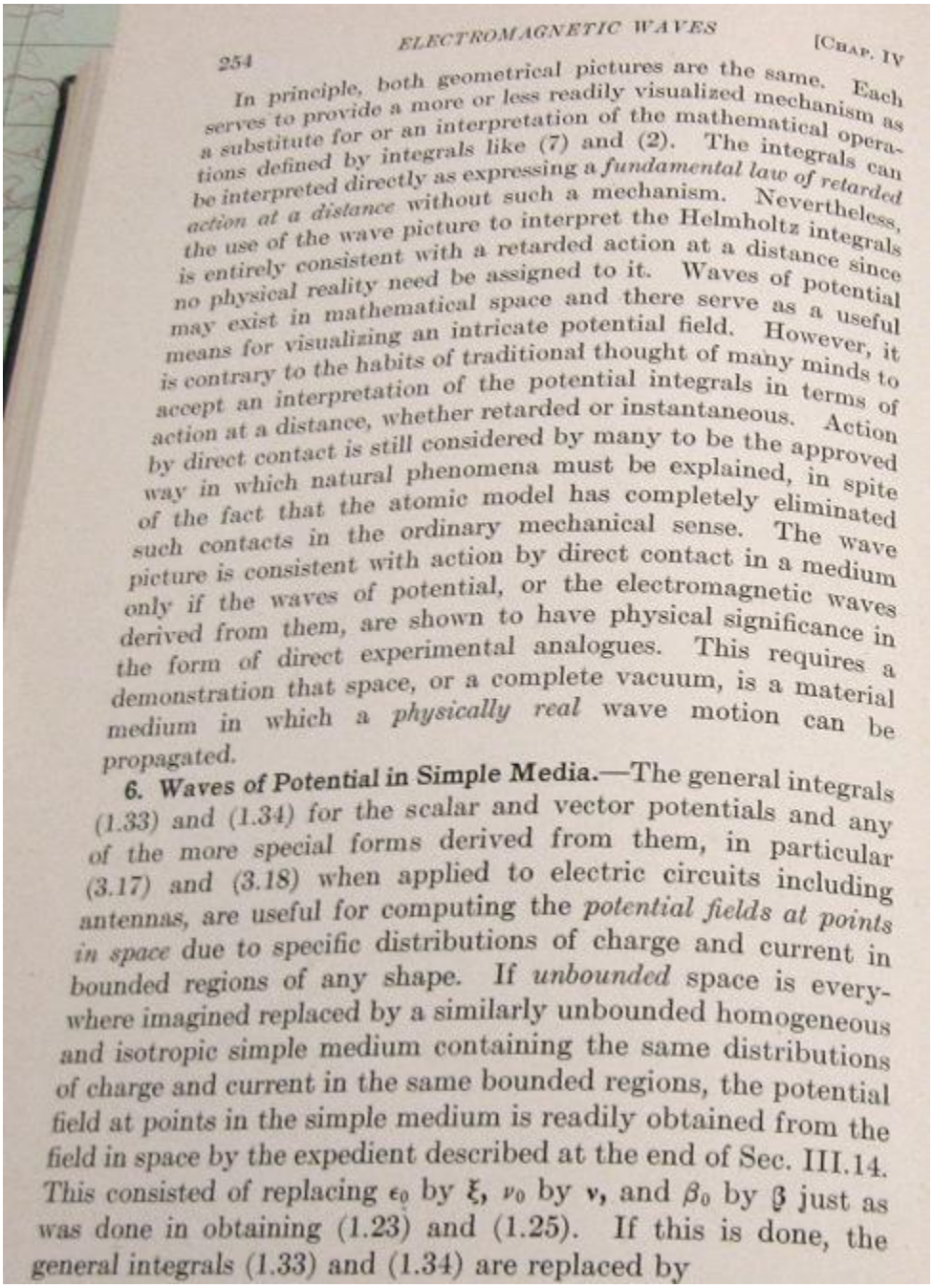
Using standard tables and rearranging, the integrals reduce to

$$l_0^e = \frac{1}{2\pi\nu} \left\{ 2 \ln \frac{b}{a} - \ln F \right\} \tag{17}$$

where

$$F = \frac{[\sqrt{(s-z)^2 + b^2} + (s-z)][\sqrt{z^2 + b^2} + z]}{[\sqrt{(s-z)^2 + a^2} + (s-z)][\sqrt{z^2 + a^2} + z]} \tag{18}$$

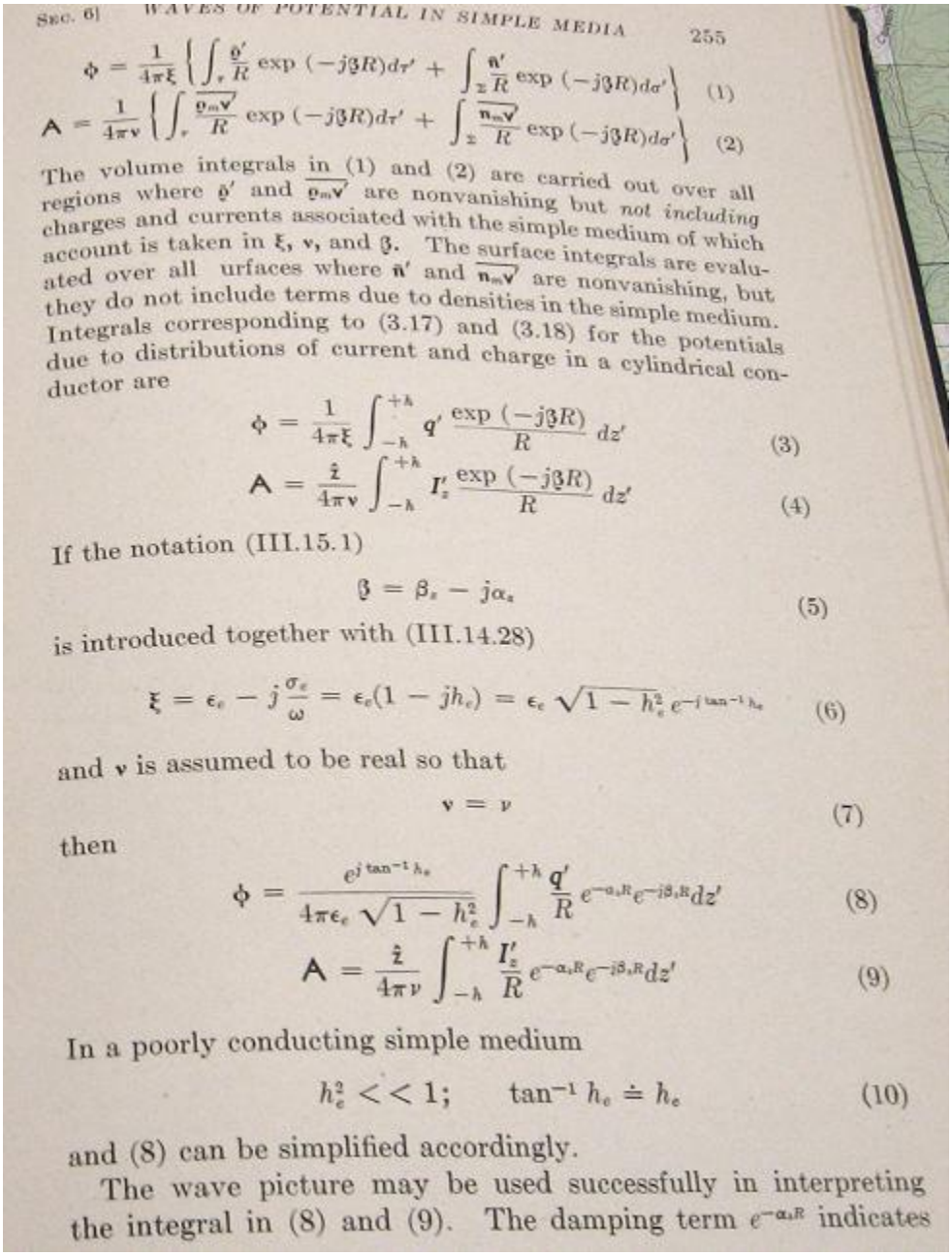
Here is where he claims that you can just magically replace $\varepsilon_0 \rightarrow \xi$ and so on, bottom of page.



In principle, both geometrical pictures are the same. Each serves to provide a more or less readily visualized mechanism as a substitute for or an interpretation of the mathematical operations defined by integrals like (7) and (2). The integrals can be interpreted directly as expressing a *fundamental law of retarded action at a distance* without such a mechanism. Nevertheless, the use of the wave picture to interpret the Helmholtz integrals is entirely consistent with a retarded action at a distance since no physical reality need be assigned to it. Waves of potential may exist in mathematical space and there serve as a useful means for visualizing an intricate potential field. However, it is contrary to the habits of traditional thought of many minds to accept an interpretation of the potential integrals in terms of action at a distance, whether retarded or instantaneous. Action by direct contact is still considered by many to be the approved way in which natural phenomena must be explained, in spite of the fact that the atomic model has completely eliminated such contacts in the ordinary mechanical sense. The wave picture is consistent with action by direct contact in a medium only if the waves of potential, or the electromagnetic waves derived from them, are shown to have physical significance in the form of direct experimental analogues. This requires a demonstration that space, or a complete vacuum, is a material medium in which a *physically real* wave motion can be propagated.

6. Waves of Potential in Simple Media.—The general integrals (1.33) and (1.34) for the scalar and vector potentials and any of the more special forms derived from them, in particular (3.17) and (3.18) when applied to electric circuits including antennas, are useful for computing the *potential fields at points in space* due to specific distributions of charge and current in bounded regions of any shape. If *unbounded* space is everywhere imagined replaced by a similarly unbounded homogeneous and isotropic simple medium containing the same distributions of charge and current in the same bounded regions, the potential field at points in the simple medium is readily obtained from the field in space by the expedient described at the end of Sec. III.14. This consisted of replacing ϵ_0 by ξ, ν_0 by ν, and β_0 by β just as was done in obtaining (1.23) and (1.25). If this is done, the general integrals (1.33) and (1.34) are replaced by

And here you see the fancy integrals. He says the integral is only over boundary ρ and J and *does not* include things like J in the medium. The medium takes care of itself. That is an interesting point and I will ponder it more later. I don't think I got that idea right before.



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$$\phi = \frac{1}{4\pi\xi}\left\{\int_\tau \frac{\varrho'}{R}\exp(-j\beta R)d\tau' + \int_\Sigma \frac{\mathfrak{n}'}{R}\exp(-j\beta R)d\sigma'\right\} \quad (1)$$

$$\mathbf{A} = \frac{1}{4\pi\nu}\left\{\int_\tau \frac{\overline{\varrho_m \mathbf{v}'}}{R}\exp(-j\beta R)d\tau' + \int_\Sigma \frac{\overline{\mathfrak{n}_m \mathbf{v}'}}{R}\exp(-j\beta R)d\sigma'\right\} \quad (2)$$

The volume integrals in (1) and (2) are carried out over all regions where ϱ' and $\overline{\varrho_m \mathbf{v}'}$ are nonvanishing but *not including* charges and currents associated with the simple medium of which account is taken in ξ, ν, and β. The surface integrals are evaluated over all urfaces where $\mathfrak{n}'$ and $\overline{\mathfrak{n}_m \mathbf{v}'}$ are nonvanishing, but they do not include terms due to densities in the simple medium. Integrals corresponding to (3.17) and (3.18) for the potentials due to distributions of current and charge in a cylindrical conductor are

$$\phi = \frac{1}{4\pi\xi}\int_{-h}^{+h} q'\frac{\exp(-j\beta R)}{R}dz' \quad (3)$$

$$\mathbf{A} = \frac{\hat{z}}{4\pi\nu}\int_{-h}^{+h} I_z'\frac{\exp(-j\beta R)}{R}dz' \quad (4)$$

If the notation (III.15.1)

$$\beta = \beta_s - j\alpha_s \quad (5)$$

is introduced together with (III.14.28)

$$\xi = \epsilon_e - j\frac{\sigma_e}{\omega} = \epsilon_e(1 - jh_e) = \epsilon_e\sqrt{1 - h_e^2}\,e^{-j\tan^{-1}h_e} \quad (6)$$

and ν is assumed to be real so that

$$\nu = \nu \quad (7)$$

then

$$\phi = \frac{e^{j\tan^{-1}h_e}}{4\pi\epsilon_e\sqrt{1 - h_e^2}}\int_{-h}^{+h}\frac{q'}{R}e^{-\alpha_s R}e^{-j\beta_s R}dz' \quad (8)$$

$$\mathbf{A} = \frac{\hat{z}}{4\pi\nu}\int_{-h}^{+h}\frac{I_z'}{R}e^{-\alpha_s R}e^{-j\beta_s R}dz' \quad (9)$$

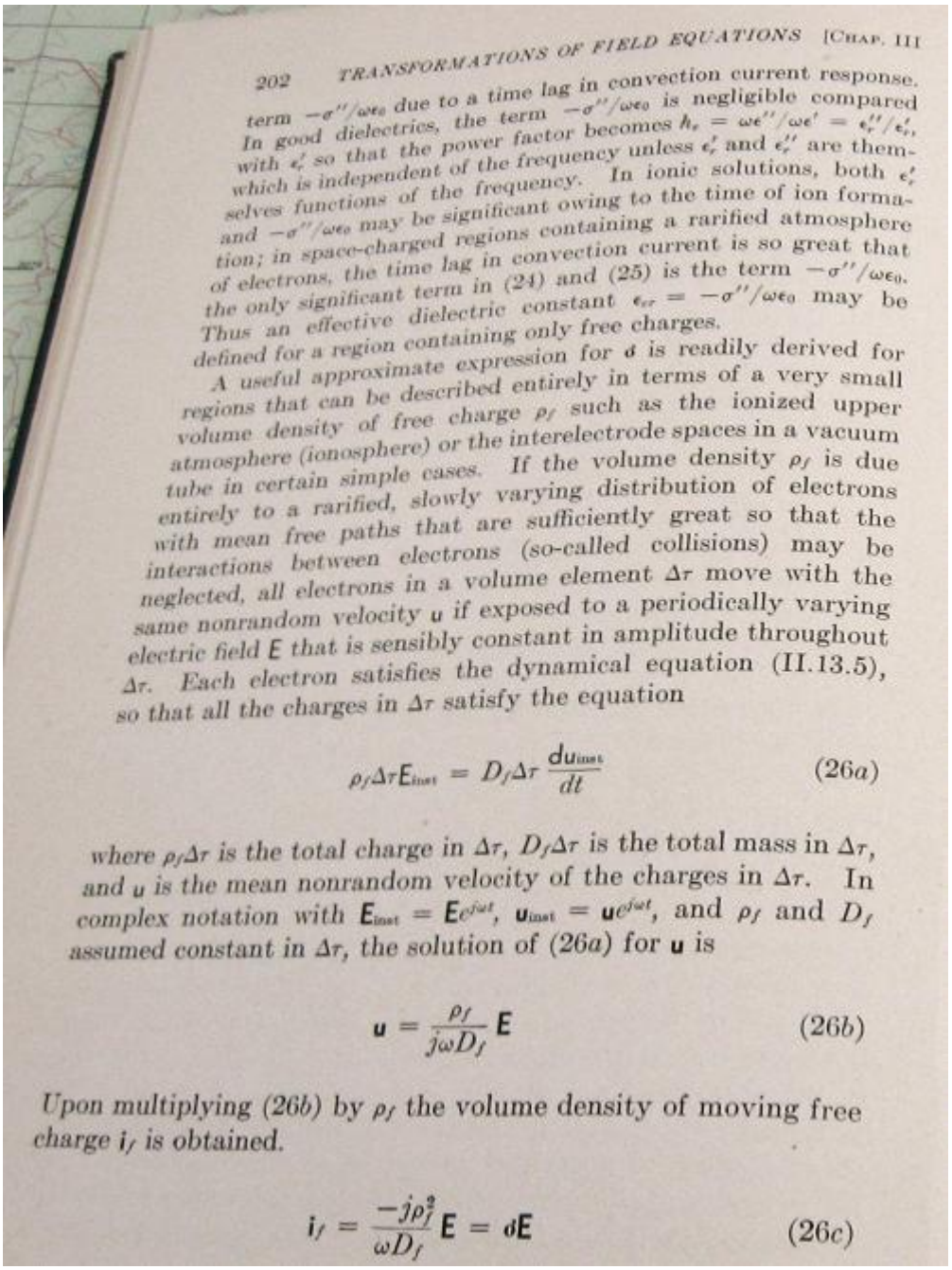
In a poorly conducting simple medium

$$h_e^2 << 1; \qquad \tan^{-1}h_e \doteq h_e \quad (10)$$

and (8) can be simplified accordingly.

The wave picture may be used successfully in interpreting the integral in (8) and (9). The damping term $e^{-\alpha_s R}$ indicates

Here he interprets the Green's propagator in terms of the decay component, fine.



term $-\sigma''/\omega\epsilon_0$ due to a time lag in convection current response. In good dielectrics, the term $-\sigma''/\omega\epsilon_0$ is negligible compared with ϵ_r' so that the power factor becomes $h_e = \omega\epsilon''/\omega\epsilon' = \epsilon_r''/\epsilon_r'$, which is independent of the frequency unless ϵ_r' and ϵ_r'' are themselves functions of the frequency. In ionic solutions, both ϵ_r' and $-\sigma''/\omega\epsilon_0$ may be significant owing to the time of ion formation; in space-charged regions containing a rarified atmosphere of electrons, the time lag in convection current is so great that the only significant term in (24) and (25) is the term $-\sigma''/\omega\epsilon_0$. Thus an effective dielectric constant $\epsilon_{er} = -\sigma''/\omega\epsilon_0$ may be defined for a region containing only free charges.

A useful approximate expression for σ is readily derived for regions that can be described entirely in terms of a very small volume density of free charge ρ_f such as the ionized upper atmosphere (ionosphere) or the interelectrode spaces in a vacuum tube in certain simple cases. If the volume density ρ_f is due entirely to a rarified, slowly varying distribution of electrons with mean free paths that are sufficiently great so that the interactions between electrons (so-called collisions) may be neglected, all electrons in a volume element $\Delta\tau$ move with the same nonrandom velocity $\mathbf{u}$ if exposed to a periodically varying electric field $\mathbf{E}$ that is sensibly constant in amplitude throughout $\Delta\tau$. Each electron satisfies the dynamical equation (II.13.5), so that all the charges in $\Delta\tau$ satisfy the equation

$$\rho_f \Delta\tau \mathbf{E}_{inst} = D_f \Delta\tau \frac{d\mathbf{u}_{inst}}{dt} \tag{26a}$$

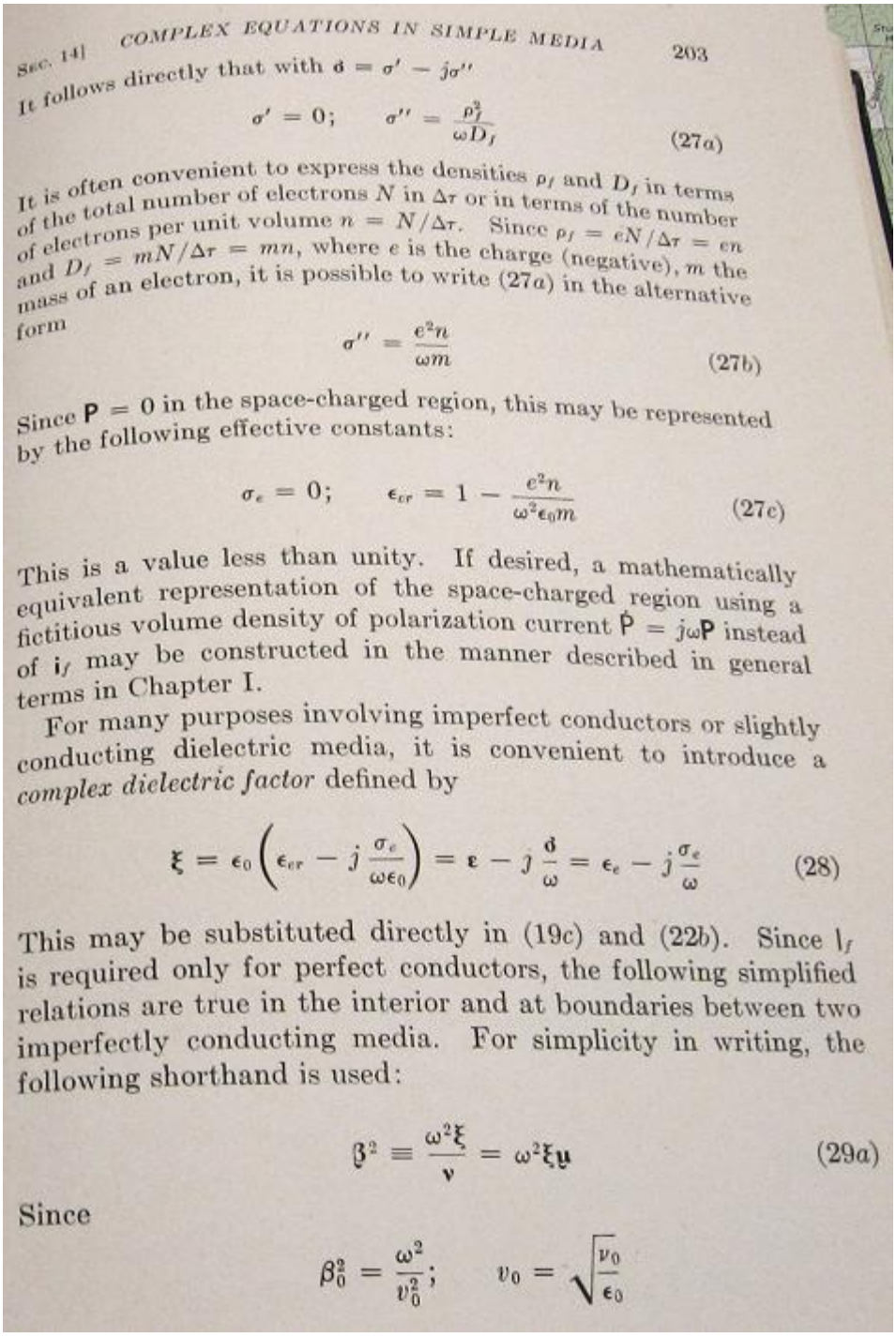
where $\rho_f\Delta\tau$ is the total charge in $\Delta\tau$, $D_f\Delta\tau$ is the total mass in $\Delta\tau$, and $\mathbf{u}$ is the mean nonrandom velocity of the charges in $\Delta\tau$. In complex notation with $\mathbf{E}_{inst} = \mathbf{E}e^{j\omega t}$, $\mathbf{u}_{inst} = \mathbf{u}e^{j\omega t}$, and ρ_f and D_f assumed constant in $\Delta\tau$, the solution of (26a) for $\mathbf{u}$ is

$$\mathbf{u} = \frac{\rho_f}{j\omega D_f}\mathbf{E} \tag{26b}$$

Upon multiplying (26b) by ρ_f the volume density of moving free charge i_f is obtained.

$$i_f = \frac{-j\rho_f^2}{\omega D_f}\mathbf{E} = \sigma\mathbf{E} \tag{26c}$$

Here is the Chapter III section 14 stuff he quoted above. But let's see where it goes. In (28) there is our friend ξ, and his friend β^2 as usual.



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It follows directly that with $\mathfrak{d} = \sigma' - j\sigma''$

$$\sigma' = 0; \qquad \sigma'' = \frac{\rho_f^2}{\omega D_f} \tag{27a}$$

It is often convenient to express the densities ρ_f and D_f in terms of the total number of electrons N in $\Delta\tau$ or in terms of the number of electrons per unit volume $n = N/\Delta\tau$. Since $\rho_f = eN/\Delta\tau = en$ and $D_f = mN/\Delta\tau = mn$, where e is the charge (negative), m the mass of an electron, it is possible to write (27a) in the alternative form

$$\sigma'' = \frac{e^2 n}{\omega m} \tag{27b}$$

Since $\mathbf{P} = 0$ in the space-charged region, this may be represented by the following effective constants:

$$\sigma_e = 0; \qquad \epsilon_{er} = 1 - \frac{e^2 n}{\omega^2 \epsilon_0 m} \tag{27c}$$

This is a value less than unity. If desired, a mathematically equivalent representation of the space-charged region using a fictitious volume density of polarization current $\dot{\mathbf{P}} = j\omega\mathbf{P}$ instead of $\mathbf{i}_f$ may be constructed in the manner described in general terms in Chapter I.

For many purposes involving imperfect conductors or slightly conducting dielectric media, it is convenient to introduce a *complex dielectric factor* defined by

$$\xi = \epsilon_0 \left(\epsilon_{er} - j \frac{\sigma_e}{\omega \epsilon_0} \right) = \epsilon - j \frac{\mathfrak{d}}{\omega} = \epsilon_e - j \frac{\sigma_e}{\omega} \tag{28}$$

This may be substituted directly in (19c) and (22b). Since $\mathbf{i}_f$ is required only for perfect conductors, the following simplified relations are true in the interior and at boundaries between two imperfectly conducting media. For simplicity in writing, the following shorthand is used:

$$\beta^2 \equiv \frac{\omega^2 \xi}{\nu} = \omega^2 \xi \mu \tag{29a}$$

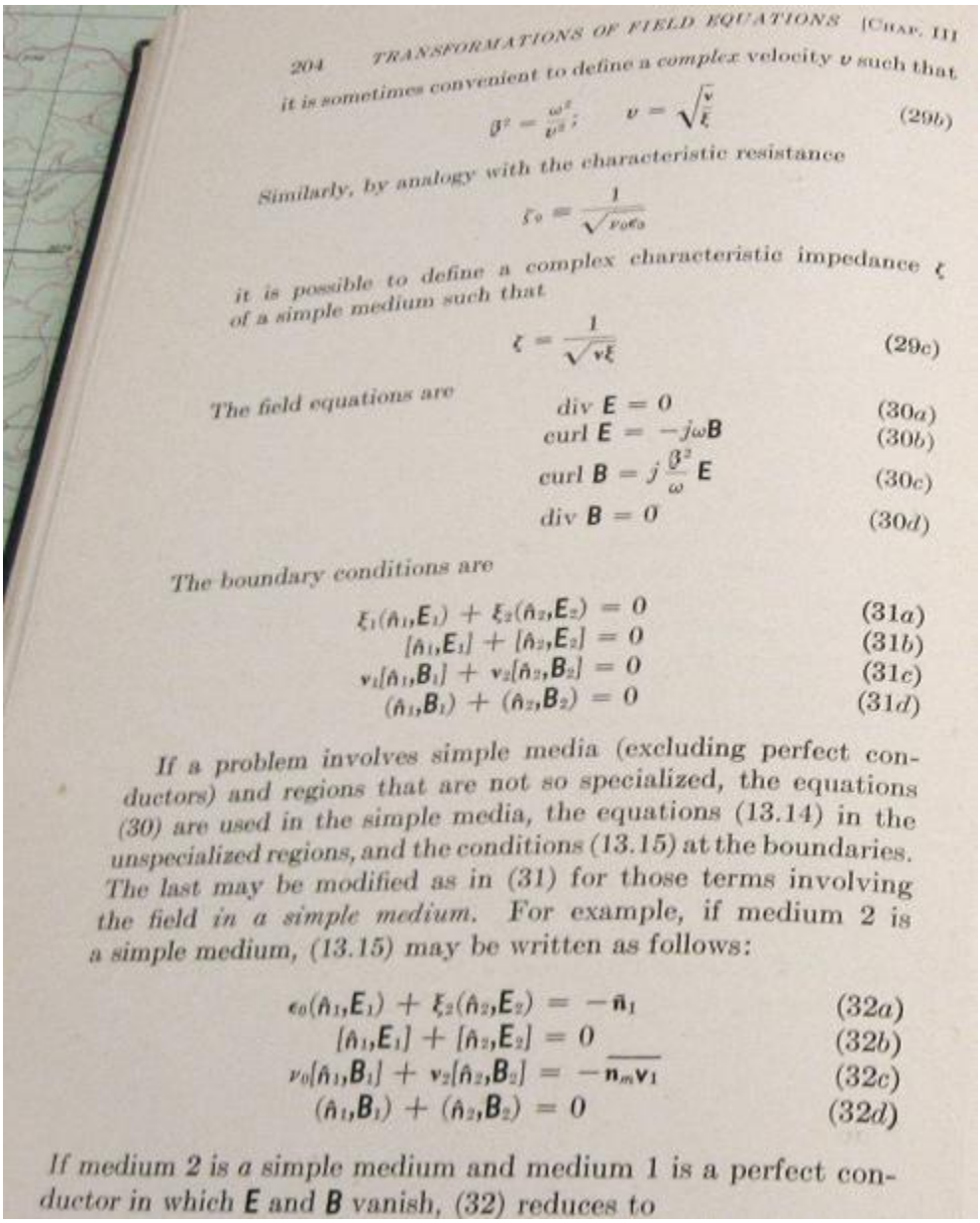
Since

$$\beta_0^2 = \frac{\omega^2}{v_0^2}; \qquad v_0 = \sqrt{\frac{\nu_0}{\epsilon_0}}$$

But now where do we go after this ξ definition? Yes, β^2 appears in the curl B equation because of the displacement current contribution, but I already had that much. The interface boundary conditions are new to me, however. It is hard to see, but (31a) starts off as $\xi_1(\hat{n}_1,\mathbf{E_1})$ which means $\xi_1 E_{1n}$ where E_{1n} is the normal component of E in medium 1. This conflicts ? with my claim that

$$[\varepsilon_1 E_\perp{}^{(1)} - \varepsilon_2 E_\perp{}^{(2)}] = n \quad = \text{surface charge density} \qquad (1.1.22)$$

I need to ponder all these BC's and see if I have something wrong for a conducting medium.



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it is sometimes convenient to define a complex velocity v *such that*

$$\beta^2 = \frac{\omega^2}{v^2}; \qquad v = \sqrt{\frac{\nu}{\xi}} \qquad (29b)$$

Similarly, by analogy with the characteristic resistance

$$\zeta_0 = \frac{1}{\sqrt{\nu_0 \xi_0}}$$

it is possible to define a complex characteristic impedance ζ *of a simple medium such that*

$$\zeta = \frac{1}{\sqrt{\nu \xi}} \qquad (29c)$$

The field equations are

$$\text{div } \mathbf{E} = 0 \qquad (30a)$$
$$\text{curl } \mathbf{E} = -j\omega \mathbf{B} \qquad (30b)$$
$$\text{curl } \mathbf{B} = j\frac{\beta^2}{\omega}\mathbf{E} \qquad (30c)$$
$$\text{div } \mathbf{B} = 0 \qquad (30d)$$

The boundary conditions are

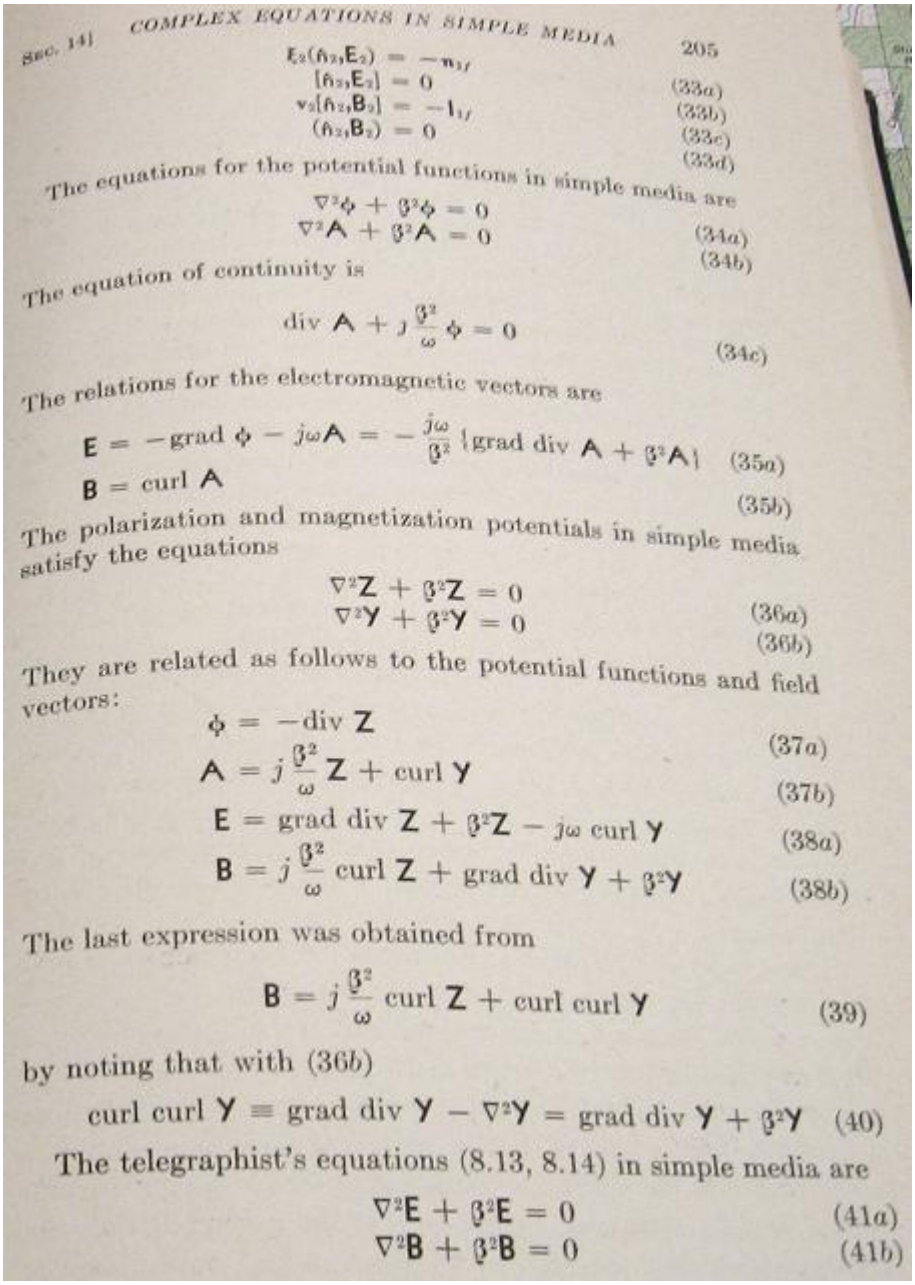
$$\xi_1(\hat{n}_1,\mathbf{E}_1) + \xi_2(\hat{n}_2,\mathbf{E}_2) = 0 \qquad (31a)$$
$$[\hat{n}_1,\mathbf{E}_1] + [\hat{n}_2,\mathbf{E}_2] = 0 \qquad (31b)$$
$$\nu_1[\hat{n}_1,\mathbf{B}_1] + \nu_2[\hat{n}_2,\mathbf{B}_2] = 0 \qquad (31c)$$
$$(\hat{n}_1,\mathbf{B}_1) + (\hat{n}_2,\mathbf{B}_2) = 0 \qquad (31d)$$

If a problem involves simple media (excluding perfect conductors) and regions that are not so specialized, the equations (30) are used in the simple media, the equations (13.14) in the unspecialized regions, and the conditions (13.15) at the boundaries. The last may be modified as in (31) for those terms involving the field in a simple medium. For example, if medium 2 is a simple medium, (13.15) may be written as follows:

$$\epsilon_0(\hat{n}_1,\mathbf{E}_1) + \xi_2(\hat{n}_2,\mathbf{E}_2) = -\bar{n}_1 \qquad (32a)$$
$$[\hat{n}_1,\mathbf{E}_1] + [\hat{n}_2,\mathbf{E}_2] = 0 \qquad (32b)$$
$$\nu_0[\hat{n}_1,\mathbf{B}_1] + \nu_2[\hat{n}_2,\mathbf{B}_2] = -\overline{\mathbf{n}_m \nu_1} \qquad (32c)$$
$$(\hat{n}_1,\mathbf{B}_1) + (\hat{n}_2,\mathbf{B}_2) = 0 \qquad (32d)$$

If medium 2 is a simple medium and medium 1 is a perfect conductor in which $\mathbf{E}$ *and* $\mathbf{B}$ *vanish, (32) reduces to*

More on those BC's and then we have the complex wave equations with 0 driving terms. And then we have the King gauge sitting there. They we are off to other stuff, but he does end up with the E and B field wave equations again with complex parameter.



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$$\hat{n}_2(\hat{n}_2,\mathbf{E}_2) = -\mathbf{n}_{1f}$$
$$[\hat{n}_2,\mathbf{E}_2] = 0 \quad (33a)$$
$$v_2[\hat{n}_2,\mathbf{B}_2] = -\mathbf{l}_{1f} \quad (33b)$$
$$(\hat{n}_2,\mathbf{B}_2) = 0 \quad (33c)$$
$$(33d)$$

The equations for the potential functions in simple media are

$$\nabla^2\phi + \beta^2\phi = 0$$
$$\nabla^2\mathbf{A} + \beta^2\mathbf{A} = 0 \quad (34a)$$
$$(34b)$$

The equation of continuity is

$$\operatorname{div}\mathbf{A} + j\frac{\beta^2}{\omega}\phi = 0 \quad (34c)$$

The relations for the electromagnetic vectors are

$$\mathbf{E} = -\operatorname{grad}\phi - j\omega\mathbf{A} = -\frac{j\omega}{\beta^2}\{\operatorname{grad}\operatorname{div}\mathbf{A} + \beta^2\mathbf{A}\} \quad (35a)$$
$$\mathbf{B} = \operatorname{curl}\mathbf{A} \quad (35b)$$

The polarization and magnetization potentials in simple media satisfy the equations

$$\nabla^2\mathbf{Z} + \beta^2\mathbf{Z} = 0 \quad (36a)$$
$$\nabla^2\mathbf{Y} + \beta^2\mathbf{Y} = 0 \quad (36b)$$

They are related as follows to the potential functions and field vectors:

$$\phi = -\operatorname{div}\mathbf{Z} \quad (37a)$$
$$\mathbf{A} = j\frac{\beta^2}{\omega}\mathbf{Z} + \operatorname{curl}\mathbf{Y} \quad (37b)$$
$$\mathbf{E} = \operatorname{grad}\operatorname{div}\mathbf{Z} + \beta^2\mathbf{Z} - j\omega\operatorname{curl}\mathbf{Y} \quad (38a)$$
$$\mathbf{B} = j\frac{\beta^2}{\omega}\operatorname{curl}\mathbf{Z} + \operatorname{grad}\operatorname{div}\mathbf{Y} + \beta^2\mathbf{Y} \quad (38b)$$

The last expression was obtained from

$$\mathbf{B} = j\frac{\beta^2}{\omega}\operatorname{curl}\mathbf{Z} + \operatorname{curl}\operatorname{curl}\mathbf{Y} \quad (39)$$

by noting that with (36b)

$$\operatorname{curl}\operatorname{curl}\mathbf{Y} \equiv \operatorname{grad}\operatorname{div}\mathbf{Y} - \nabla^2\mathbf{Y} = \operatorname{grad}\operatorname{div}\mathbf{Y} + \beta^2\mathbf{Y} \quad (40)$$

The telegraphist's equations (8.13, 8.14) in simple media are

$$\nabla^2\mathbf{E} + \beta^2\mathbf{E} = 0 \quad (41a)$$
$$\nabla^2\mathbf{B} + \beta^2\mathbf{B} = 0 \quad (41b)$$