Grind through King p 14-18 PhL 10.16.13

I was for a very long time mystified by King's 1/4πξ factor in his φ Helmholtz integral and was hoping to learn something about it by carefully going through his very first example of parallel wires. I did learn that the ξ is essential if you want this little calculation to produce the correct capacitance AND conductance which combine to form a complex capacitance. Maybe it was at this point that I realized this complex C' was not coming from the propagator $e^{-j\beta R}$ factor with β complex. For a long time I thought that was the case. I have now explained King's 1/4πξ factor in lines doc (separate section on that specific topic) and perhaps doing this careful reading helped me get that figured out.

I had hoped to avoid this painful effort, but I am running out of "Plans" to rescue my lines doc. This is going to take me perhaps 3-14 days, I don't know. I suspect after I do all this work, I won't really be closer to solving my problem. But at least I will see the kind of calculations that King is doing in his book. I have these equations all "checked off", but I have no document that shows the derivations! So let's try to "derive our way through" his series of equations. Instead of z going along the transmission line, he uses w. So we need a translation system:

me	King
z	-w
$i_1(z)$	$I_1(w)$
$q_1(\mathbf{x})$	$q_1(z)$
$q(\mathbf{x})$	$q(z)$

Comments on p 11 (23) and (24)

These are the source of all my confusion. King claims these are the correct solutions of the two homogeneous wave equations shown in (17) and (18). Now I am happy with the wave equations which include loss as being applicable in the dielectric where we assume there are no current or charge sources. The real equations are these, using the King gauge

$$(\nabla^2 - \mu\varepsilon\,\partial_t^2 - \mu\sigma\partial_t)\varphi = -\rho/\varepsilon \qquad (1.3.16)$$

$$(\nabla^2 - \mu\varepsilon\,\partial_t^2 - \mu\sigma\partial_t)\,\mathbf{A} = 0 \qquad (1.3.15)$$

which we can write as

$$(\nabla^2 + \beta_c^2)\,\varphi = -\rho/\varepsilon \qquad (1.3.16)$$

$$(\nabla^2 + \beta_c^2)\,\mathbf{A} = 0 \qquad (1.3.15)$$

where β_c is Kings bolded $\boldsymbol{\beta}$. Then *without any explanation whatsoever*, King claims that the solutions of these equations are (23) and (24) where, you might note, we have *complex* 1/4πξ and 1/4πν sitting out front. He never mentions "Green's Function" or anything like that. He has no Appendix on Green's.

Aside: I am going to download his book just to see on search if there is something.

http://archive.org/details/TransmissionLineTheory

I pulled in the PDF, but it has no OCR. I try the djvu and have my usual Firefox java problem. I now try to update Java using the offline method. It is version 7 update 45. called "7u45" as part of file name. I think I have done this all before and it took a long time and got me nowhere in Firefox. It is slow because MSE is studying it pretty hard. It took about 2 minutes, all done it says. And yes, I had done this before and it is still refused by Firefox. If I override I just get a Lizardtech djvu viewer that has no way to download. So that is their copyright thing I guess. So I downloaded the PDF and am now adding an OCR layer and index.

This is a huge weak point in King's whole book for me. That is why I am doing all the work below, just to see if I am missing something.

Derive p 14 (4)

What does he mean by a "balanced line" ? It seems reasonable to me that the currents are equal and opposite, and the charges as well. But why are the potentials equal and opposite? This is something new to me. I think this is true only for the symmetric special case of twin lead in Fig 4.1 that he is now studying. But how exactly do you arrive that (4) in this case? Look at the A_z integral in (30b) which I accept given (24). If you reflect the geometry through a mirror plane between the conductors, you get the same picture back but I has reversed, and everything else stays the same. You are still integrating over both conductors. So that is why A_z changes sign. In fact, this mirror plane is a symmetry plane and at any two mirror points, A_z and φ will be equal and opposite, so this is also true for a point pair on the conductors. Fine!

Derive p 14 (5a)

Even the first equation leaves me at a loss. I need some Gaussian box to apply

$$\text{div } \mathbf{J} = -\partial_t \rho \qquad \Leftrightarrow \qquad -\partial_t \left[\int_V \rho \, dV\right] = \int_S \mathbf{J} \bullet d\mathbf{A} \tag{1.1.16}$$

OK, let the box be a little piece of shrink tube around the conductor of length dz.

$$-\partial_t \left[q_1(z)dz\right] = [J(z+dz)-J(z)]dA \qquad \text{But } J(z) = i(z)/A \text{ so}$$

$$= [i(z+dz)-i(z)] = \partial_z i_1(z)\, dz$$

so then

$$-j\omega\, q_1(z) = \partial_z i_1(z)$$

and in terms of w he would have

$$+j\omega\, q_1(\omega) = \partial_\omega i_1(\omega) \qquad \Rightarrow \qquad \partial_\omega i_1(\omega) - j\omega\, q_1(\omega) = 0$$

$$\Rightarrow \qquad \partial_\omega i(\omega) - j\omega\, q(\omega) = 0 \qquad \text{(5a) verified}$$

OK, this is a good equation to be aware of! It relates longitudinal current i(z) to transverse charge q(z). I may have missed this in my own work??

Derive p 14 (5b)

We have to backtrack a bit here. At bottom p 11 King claims that $\mathbf{A} = A_z\hat{\mathbf{z}}$. How does he arrive at this concept? He is in the transmission line limit p 11 (29) . The footnote I think says the line has to be at least 5 times longer than wire radius a. I think his basic argument is that for thin conductors, the transverse currents will be tiny, and then using p 11 (24) he can neglect the transverse A_x and A_y. I note he states that there is no I_θ , please take note (confirming my claim), and this is his argument for $A_\theta = 0$ exactly. He then confirms that $i_r << i_z$ which is another one of my "things". Doing the line integration, he notes that in the A_r integral over the lines, you expect to get cancelling contributions going around the wire, I agree. This is his argument that $A_r \approx 0$, so this is good stuff to be reading.

So, the upshot is that **p 12 (31)** is just a statement of the King gauge condition where only A_z is showing up in div **A**.

$$\beta^2 = \omega^2 \mu\xi = \omega^2 \mu [\, \varepsilon + \sigma/j\omega] \qquad \text{from lines}$$

$$\text{div}\mathbf{A} = -\mu\varepsilon\, \partial_t\varphi - \mu\sigma\varphi \;. \qquad \Rightarrow \partial_z A_z = -\mu\,(j\omega\varepsilon + \sigma)\varphi = -\mu j\omega(\varepsilon + \sigma/j\omega)\varphi = -\mu j\omega\, [\, \beta^2/\mu\omega^2]\varphi$$

$$= -j\, [\, \beta^2/\omega]\varphi$$

So the King gauge says

$$\partial_z A_z = -j\, [\, \beta^2/\omega]\varphi \qquad \Rightarrow \qquad \partial_z A_z + j\, [\, \beta^2/\omega]\varphi = 0 \qquad \text{// which is p 12 (31)}$$

So this is how his complex β gets into things.

This is just my wave equation for φ,

$$(\nabla^2 + \beta_c^2)\, \varphi(\mathbf{x},\omega) = -(1/\varepsilon)\, \text{grad}\, \rho(\mathbf{x},\omega) \qquad (1.5.1)$$

So change z = -w to get

$$\partial_w A_z - j\, [\, \beta^2/\omega]\varphi = 0 \qquad \text{which is p 14 (5b).}$$

Derive p 14 (6a)

$$\mathbf{B} = \text{curl}\, \mathbf{A} \qquad \mathbf{E} = -\text{grad}\, \varphi - \partial_t \mathbf{A} \;. \qquad (1.3.1)$$

$\Rightarrow \quad E_z = -\partial_z\varphi - \partial_t A_z = -\partial_z\varphi - j\omega A_z$

$\Rightarrow \partial_z\varphi = -E_z - j\omega A_z \quad \Rightarrow \quad \partial_w\varphi = E_z + j\omega A_z$ which is p 14 (6a)

Derive p 14 (6b) and (6c)

Take our two equations so far:

$\partial_w A_z = j[\beta^2/\omega]\varphi$
$\partial_w\varphi = E_z + j\omega A_z$

Apply ∂_w to both

$\partial_w^2 A_z = j[\beta^2/\omega]\partial_w\varphi = j[\beta^2/\omega](E_z + j\omega A_z) = j[\beta^2/\omega]E_z - \beta^2 A_z$
$\partial_w^2\varphi = \partial_w E_z + j\omega\partial_w A_z = \partial_w E_z + j\omega(j[\beta^2/\omega]\varphi) = \partial_w E_z - \beta^2\varphi$

or

$\partial_w^2 A_z + \beta^2 A_z = j[\beta^2/\omega]E_z$ which is p 14 (6c)
$\partial_w^2\varphi + \beta^2\varphi = \partial_w E_z$ which is p 14 (6b)

Recall that I had in chapter 1

$(\nabla^2 + \beta^2)\,\varphi(\mathbf{x},\omega) = -(1/\varepsilon)\rho_T(\mathbf{x},\omega)$ (1.5.3)
$(\nabla^2 + \beta^2)\,\mathbf{A}(\mathbf{x},\omega) = -\mu\mathbf{J}_a(\mathbf{x},\omega)$ (1.5.4)

In my equations you have ∇^2 which includes ∂_z^2 plus other terms, so these are different equation pairs.

Derive p 14 (6d)

At any point on the conductor, $E_z = 0$ because we assume conductor is ideal!!! In that case,

$\partial_w^2 A_z + \beta^2 A_z = 0$
$\partial_w^2\varphi + \beta^2\varphi = 0$ eval at any point on either conductor which is (6d)

Derive p 15 (7) and (8)

Here he derives my V and W functions. He picks specific points Q_1 and Q_2 as shown in Fig 4.1 which are mirror points, so that is where the factors of 2 come from. He uses notation

$\varphi_1(w) \equiv \varphi(x, y=Q_1, z=-w).$

So that is different from the way I defined φ_1 so be careful.

Derive p 15 (9a) and (9b)

The idea here is to use (6b) and (6c) evaluated first at Q_1 then at Q_2 then subtract them. On the left you will then get his V and W, and on the right the E_z differences as shown. This is an easy step, nothing really to derived. I wonder if you could replace the E_{1z} differences with $2E_{1z}$ again based on the general symmetry of things, but he does not do this.

Derive equations p 15 (10a) through (12).

Here King is using twin lead with a << b and is using (23) and (24). This little section has nothing to do with the work derived above. It is a separate little section.

OK, I will use my basic formula

$$\varphi(\mathbf{x},\omega) = \frac{1}{4\pi\varepsilon}\int_{upper} dV'\,\rho_u(\mathbf{x}',\omega)\frac{e^{-j\beta(\omega)R}}{R} + \frac{1}{4\pi\varepsilon}\int_{lower} dV'\,\rho_\ell(\mathbf{x}',\omega)\frac{e^{-j\beta(\omega)R}}{R}$$

$$= \frac{1}{4\pi\varepsilon}\left[\int_{upper} dV'\,\rho_u(\mathbf{x}',\omega)\frac{e^{-j\beta(\omega)R}}{R} - \int_{lower} dV'\,\rho_u(\mathbf{x}',\omega)\frac{e^{-j\beta(\omega)R}}{R}\right]$$

In his notation, I can evaluate this at my $\mathbf{x}_1$ = his $\mathbf{Q}_1'$ on the upper conductor to get

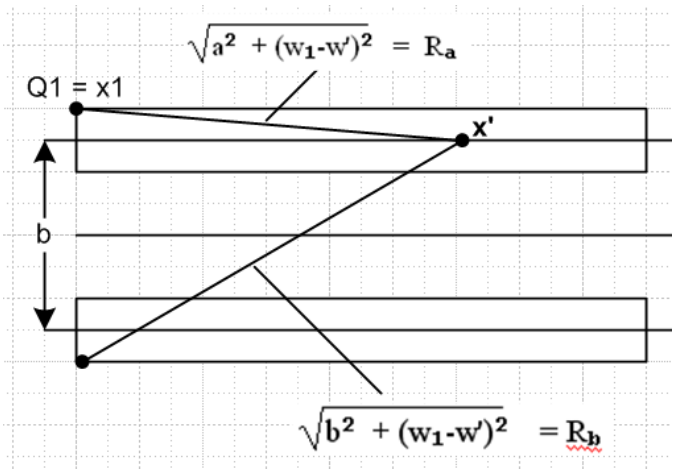
$$\varphi(\mathbf{x}_1,\omega) = \frac{1}{4\pi\varepsilon}\left[\int_{upper} dV'\,\rho_u(\mathbf{x}',\omega)\frac{e^{-j\beta(\omega)R_u}}{R_u} - \int_{lower} dV'\,\rho_u(\mathbf{x}',\omega)\frac{e^{-j\beta(\omega)R_\ell}}{R_\ell}\right]$$

$$R_u = |\mathbf{Q}_1 - \mathbf{x}'| = \sqrt{(x_1-x')^2 + (y_1-y')^2 + (w_1-w')^2}$$
$$R_\ell = |\mathbf{Q}_2 - \mathbf{x}'| = \sqrt{(x_2-x')^2 + (y_2-y')^2 + (w_1-w')^2}$$

He now assumes a << b so the conductors are super thin wires AND he takes the integration point $Q_1' = \mathbf{x}$ to be at the exact center of the upper conductor so that $R_1^2 = (w_1-w)^2 + a^2$ meaning the transverse distance is a. For the other conductor you get b as transverse distance assuming thin wires, so then

$$R_1 = |\mathbf{Q}_1 - \mathbf{x}'| = \sqrt{a^2 + (w_1-w')^2} = R_a$$
$$R_2 = |\mathbf{Q}_2 - \mathbf{x}'| = \sqrt{b^2 + (w_1-w')^2} = R_b$$ which is (12)

Here is my picture



This means the propagators depend only on integration variable w' (or z') so we then write

$$\varphi(\mathbf{x}_1,\omega) = \frac{1}{4\pi\varepsilon} *$$

$$[\int_{upper} dx'dy'dz' \, \rho_u(\mathbf{x}',\omega)\frac{e^{-j\beta(\omega)R_u}}{R_u} - \int_{lower} dx'dy'dz' \, \rho_u(\mathbf{x}',\omega)\frac{e^{-j\beta(\omega)R_\ell}}{R_\ell}]$$

or

$$\varphi(\mathbf{x}_1,\omega) = \frac{1}{4\pi\varepsilon} *$$

$$[\int dz' \{ \int_{upper} dx'dy' \, \rho_u(\mathbf{x}',\omega)\frac{e^{-j\beta(\omega)R_u}}{R_u} - \int_{lower} dx'dy' \, \rho_u(\mathbf{x}',\omega)\frac{e^{-j\beta(\omega)R_\ell}}{R_\ell}] \}$$

or

$$\varphi(\mathbf{x}_1,\omega) = \frac{1}{4\pi\varepsilon} \{ \int dz' \{ \int dx'dy' \, \rho_u(\mathbf{x}',\omega) [\frac{e^{-j\beta(\omega)R_u}}{R_u} - \frac{e^{-j\beta(\omega)R_\ell}}{R_\ell}] \}$$

or

$$\varphi(\mathbf{x}_1,\omega) = \frac{1}{4\pi\varepsilon} [\int dz' [\frac{e^{-j\beta(\omega)R_u}}{R_u} - \frac{e^{-j\beta(\omega)R_\ell}}{R_\ell}] \{ \int dx'dy' \, \rho_u(\mathbf{x}',\omega) \}]$$

In the above set of steps, we assumed the upper and lower transverse integrals are the exact same since the wires are identical. One could flesh out this detail with more notation.

Now you can do the transverse integral to get $q_u(z)$ so then

$$\varphi(\mathbf{x}_1,\omega) = \frac{1}{4\pi\varepsilon} [\int dz' [\frac{e^{-j\beta(\omega)R_u}}{R_u} - \frac{e^{-j\beta(\omega)R_\ell}}{R_\ell}] \{ q_u(z')$$

We convert this to his notation,

$$\varphi(\mathbf{Q_1},\omega) = \frac{1}{4\pi\varepsilon} \int dw'\, q(w') \left[\frac{e^{-j\beta(\omega)Ra}}{R_a} - \frac{e^{-j\beta(\omega)Rb}}{R_b} \right]$$

$$\varphi(\mathbf{Q_1},\omega) = \frac{1}{4\pi\varepsilon} \int dw'\, q(w')\, P_L(w,w')$$

where

$$P_L(w,w') = \frac{e^{-j\beta(\omega)Ra}}{R_a} - \frac{e^{-j\beta(\omega)Rb}}{R_b}$$ which is (11)

Now the final step is that we write

$$V(w) = \varphi(\mathbf{Q_1},\omega) - \varphi(\mathbf{Q_2},\omega) = 2\,\varphi(\mathbf{Q_1},\omega)$$

and then our final result is

$$V(w) = \frac{1}{2\pi\varepsilon} \int dw'\, q(w')\, P_L(w,w')$$ which is (10a)

If we now use (24) instead of (23) we get (10b) so no need to do that detail, all the same!

Derive equations p 15 (13a) through (14b).

The power series expansions shown in (13a) and (13b) are exactly what I did. For him, w' is the integration variable and w is the observation point. So his next step is to derive the two derivatives.

From (5a) continuity we had

$$\partial_\omega i(\omega) - j\omega\, q(\omega) = 0 \quad \Rightarrow \quad \partial_\omega^2 i(\omega) = j\omega\, \partial_\omega q(\omega)$$

so that

$$\partial_\omega q(\omega) = (1/j\omega)\, \partial_\omega^2 i(\omega)$$ which is (14a)

$$\partial_\omega i(\omega) = j\omega\, q(\omega)$$ which is (14b)

Derive equations p 16 (15a) through (21), and also p 17 (26) and (27)

We have from above

$$V(w) = \frac{1}{2\pi\varepsilon} \int dw'\, q(w')\, P_L(w,w')$$

$$= \frac{1}{2\pi\varepsilon} \int dw' \left[q(w) + (1/j\omega)\, \partial_\omega^2 i(\omega)(w'-w) \right] P_L(w,w')$$

$$= \frac{1}{2\pi\varepsilon}\ q(w) \int dw'\ P_L(w,w') + \frac{1}{2\pi\varepsilon}\ (1/j\omega)\ \partial_\omega^2 i(\omega) \int dw'(w'-w)]\ P_L(w,w')$$

$$= \frac{1}{2\pi\varepsilon}\ q(w)\ k_0(w) + \frac{1}{2\pi\varepsilon}\ (1/j\omega)\ \partial_\omega^2 i(\omega)\ k_1(w)/\beta$$

where he just defines these integrals

$$k_0(w) = \int dw'\ P_L(w,w') = \int dw' \left[\frac{e^{-j\beta(\omega)R_a}}{R_a} - \frac{e^{-j\beta(\omega)R_b}}{R_b} \right]$$

$$k_1(w)/\beta = \int dw'(w'-w)]\ P_L(w,w')\ \int dw'(w'-w)] \left[\frac{e^{-j\beta(\omega)R_a}}{R_a} - \frac{e^{-j\beta(\omega)R_b}}{R_b} \right]$$

He then does an add and subtract gimmick to rewrite these integrals as shown with F(a) and F(b). He is then going to throw out these corrections, probably I might have just done that from the start. With these terms thrown out (in the transmission line limit!) Then you are left with the first term integrals, and i guess the 1/R difference one gives the famous 2 ln(b/2) result.

$$\int dw' \left[1/R_a - 1/R_b \right] = \int dw' \left[1/\sqrt{a^2 + (w_1-w')^2} - 1/\sqrt{b^2 + (w_1-w')^2} \right]$$

$$= \int dw' \left[1/\sqrt{a^2 + w'^2} - 1/\sqrt{b^2 + w'^2} \right]$$

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Int(1/sqrt(a^2+z^2) - 1/sqrt(b^2+z^2),z=-infinity..infinity);
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$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{a^2+z^2}} - \frac{1}{\sqrt{b^2+z^2}}\, dz$$

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int(1/sqrt(a^2+z^2) - 1/sqrt(b^2+z^2),z=-infinity..infinity);
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$$-\ln(a^2) + \ln(b^2)$$

$= 2 \ln(b/a)$. which is (19)

So we have shown that

$k_0(w) = 2\ln(b/a)$ + small correction term which is (26)

The other integral is

$$\int dw' \left[1/R_a - 1/R_b \right](w-w') = \int dw' \left[1/\sqrt{a^2 + w'^2} - 1/\sqrt{b^2 + w'^2} \right]\ w' = 0$$ which is (20)

because integral of an odd integrand. This we have

$k_1(w)/\beta = 0 +$ small correction term which is (27)

Then we end up with

$$V(w) = \frac{1}{2\pi\varepsilon}\, q(w)\, k_0(w) + \frac{1}{2\pi\varepsilon}\, (1/j\omega)\, \partial_\omega^2 i(\omega)\, k_1(w)/\beta$$

$$\approx \frac{1}{2\pi\varepsilon}\, q(w)\, 2\ln(b/a) + \frac{1}{2\pi\varepsilon}\, (1/j\omega)\, \partial_\omega^2 i(\omega)\, 0$$

$$\approx \frac{1}{2\pi\varepsilon}\, q(w)\, 2\ln(b/a)$$

So I guess this says that capacitance per unit length of twin lead is this

$C = Q/V = 2\pi\varepsilon/2\ln(b/a)$

which agrees with my currently numbered equation (6.4.4). good.

Comments on equations p 16 (21) through p 17 (25)

Equations (21) through (25) are all just to show that the correction terms can be neglected. Since this is not critical to me right now (I already accepted it long ago), I will not derive these equations, and they then get no red checkmarks. The rough idea is that $e^x - 1 \approx 0$ as per (22).

Derive equations p 17 (28) through (30b)

Oh boy. Now I realize that his fancy formulas (23) and (24) have the complex ν and ε versions, so really he has

$$V(w) = \frac{1}{2\pi\xi}\, q(w)\, 2\ln(b/a) = \frac{1}{\pi\xi}\, q(w)\, \ln(b/a)$$

So much has been loaded into those two starting formulas! Now again King wanders off the reservation, and I will have to do a rescue. First, use Q = C'V to say

$1/C' = V/Q = \ln(b/a)/(\pi\xi)$ $C' = \pi\xi/\ln(b/a)$

where C' is a complex capacitance. We are then used to saying $Y = G + j/X_C$ which is this:

$Y = G + j\omega C = j\omega C' = j\omega\pi\xi/\ln(b/a) = j\omega\pi\,[\varepsilon + \sigma/j\omega] / \ln(b/a)$

so that

$G = \sigma\pi / \ln(b/a)$

$C = \varepsilon\pi / \ln(b/a)$ which is p 17 (30b) but not the L part

Meanwhile, note from above that

$Y = j\omega C' = j\omega Q/V \quad \Rightarrow \quad V = j\omega Q/Y$ which is p 17 (28)

The "rest of" (28) is continuity (5a) which said

$$\partial_{\omega} i(\omega) - j\omega\, q(\omega) = 0 \qquad (5a)$$

so then we get

$V = (1/Y)\, \partial_{\omega} i(\omega)$ which is the rest of 17 (28)

This is one of the two "transmission line equations" which will be restated together below.

Now where does (19) come from? I got this from my little picture, but I have lost it! It is in temp1 right now and there I show that

$W(z) = L_e\, i(z)$ which is p 17 (29)

Now from (15b) previous page we have

$W(z) = (1/2\pi\nu)\, I_z\, k_0 = (1/\pi\nu) I_z \ln(b/a) = \mu/\pi\, I_z \ln(b/a)$

and comparing the last two we get

$L_e = (\mu/\pi) \ln(b/a)$ which is p 17 (30b) the L_e part

which is the last of (30b).

Derive p 17 (31)

For me, I have

$$\beta_c^2 = \mu\varepsilon\omega^2 - j\omega\mu\sigma = \omega^2\mu\,(\varepsilon - j\sigma/\omega) = \omega^2\mu\,\xi \qquad \xi \equiv \varepsilon - j\sigma/\omega \quad . \qquad (1.5.2)$$

where ξ is already complex. King has $\boldsymbol{\beta^2 = \omega^2\mu\xi}$ which is fine, his page 9 (10). Now from above,

$L_e = (\mu/\pi) \ln(b/a) \qquad Y = j\omega\pi\xi/\ln(b/a)$

Solve these to get

$\mu = \pi L_e/\ln(b/a)$ $\xi = Y\ln(b/a)/(j\omega\pi)$

Then we get

$\beta^2 = \omega^2\mu\xi = \omega^2(\pi L_e/\ln(b/a))(Y\ln(b/a)/(j\omega\pi)) = \omega(L_e)(Y/(j)) = -j\omega L_e Y$

Now from our reactance discussion, we have $Z_L = j(\omega L) = jX_L$ so in this case. $Z_{Le} = j\omega L_e$ and then

$\beta^2 = -j\omega L_e Y = -Z_{Le} Y$ which is p 17 (31)

Derive p 17 (32)

$\partial_w\varphi = E_z + j\omega A_z$ (6a)

$V = \varphi_1 - \varphi_2$ (7) // recall φ_1 means evaluation on conductor 1

$W = A_{z1} - A_{z2}$ (8) // recall φ_1 means evaluation on conductor 1

We want to find an expression for $\partial_w V - j\omega W$ so write

$\partial_w V - j\omega W = (\partial_w\varphi_1 - \partial_w\varphi_2) - j\omega(A_{z1} - A_{z2})$

$= [\partial_w\varphi_1 - j\omega A_{z1}] - [\partial_w\varphi_2 - j\omega A_{z2}]$

$= E_{z1} - E_{z2}$ which is p 17 (32)

Derive p 17 (33a) and (33b) and p 18 (33c)

The quantity z^i_1 is, by my definition, the *surface* impedance of conductor 1 at "point 1" which I guess is still our Q_1 of Fig 4.1. I introduce this in (C.2.1). In the current situation, since the two lines are identical we are going to have $z^i_1 = z^i_2$ and we can define z^i to be the sum of these and then $z^i_1 = 1/2\ z^i$ which is all fine. Thus, I am happy with all three of these equations.

Question: When is Z the same as Z_S ?

We have these two definitions:

$Z = V/I$ total impedance of the conductor per unit length

$Z_s(\theta) = E_z^s(\theta)/I$ surface impedance at some point on the surface

If Z_s varies around the fat conductor in θ, then $E_z(\theta)$ is varying with θ. How then can you have equal potentials on a cross section surface ring at constant z? Recall that

$\mathbf{E} = -\text{grad}\,\varphi - \partial_t \mathbf{A} \qquad \Rightarrow \qquad E_z(\theta) = -\partial_z \varphi(\theta) - j\omega A_z(\theta) = \text{complex!}$

so that

$-\partial_z \varphi(\theta) = E_z(\theta) + j\omega A_z(\theta)$

So if you want the potential difference between two aligned points on a conductor surface,

$V = \varphi_1 - \varphi_2 = \int dz\, \partial_z \varphi = \int dz\, [E_z(\theta) + j\omega A_z(\theta)]$

you will find that this integral is independent of θ, even though $E_z(\theta)$ is a function of θ! (no E_θ assumed) I was forgetting about the A_z term. So we then have, for a piece of wire from z1 to z2,

$Z = V/I = (1/I) \int dz\, [E_z(\theta) + j\omega A_z(\theta)]$

and Z in general is complex. We associate A_z with inductance and E_z with resistance. So

$Z = V/I = (1/I) \int dz\, [J_z(\theta)/\sigma + j\omega A_z(\theta)]$.

Now consider a very short piece of wire of length dz as our object of interest. Then

$Z = (1/I)\, [E_z(\theta) + j\omega A_z(\theta)]\, dz$

$Z_s = (1/I)\, E_z(\theta)\, dz$

The condition for these to be the same is that $\omega A_z(\theta) \approx 0$. But when $\omega > 0$, this is not zero, so my conclusion is that "at AC" you never have $Z = Z_s$. This is only true "at DC". So my conclusion is that King is only talking about surface impedance!

Now, what is A_z inside a round wire? I omitted potentials from my round wire analysis, maybe that is something I should add! I am going on hold right now to answer this question in a different doc. // OK I got started on that, and it is too big a job to digress on right now, so I am back here with all the Z's being surface impedances.

The answer to my question: They are the same at DC, otherwise they are different, and I think this is true no matter what you assume about your wire.

Derive p 18 (34)

From lines doc I have this large ω limit for surface impedance,

$$Z_s(\omega) \approx \approx \frac{1}{\sigma(2\pi a)\delta}\ (1+j) \qquad \delta << 16a\ . \qquad (2.3.16)$$

$$\delta \equiv \sqrt{2/\omega\mu\sigma}\ = \text{skin depth} \qquad // \ \omega\mu\sigma = 2/\delta^2 \qquad (2.1.20)$$

The factor is then

$$\frac{1}{\sigma(2\pi a)\delta} = \frac{1}{\sigma(2\pi a)\sqrt{2/\omega\mu\sigma}} = [\frac{\omega\mu\sigma}{2\sigma^2(2\pi a)^2}]^{1/2} = [\frac{\omega\mu}{2\sigma(2\pi a)^2}]^{1/2} = \frac{1}{2\pi a}\ [\frac{\omega\mu}{2\sigma}]^{1/2}$$

so I get

$$Z_s(\omega) \approx \approx (1+j)\frac{1}{2\pi a}\sqrt{\frac{\omega\mu}{2\sigma}} \qquad \text{which is p 18 (34)}$$

My inequality says

$$\delta << 16a \quad => \quad a >> \delta/16 = \sqrt{2/\omega\mu\sigma}\ /16 = \sqrt{1/\omega\mu\sigma}\ /(16/\sqrt{2}\ )$$

or

$a\sqrt{\omega\mu\sigma} >> 16/\sqrt{2}$ = 11.313 and King takes this to be 10, fine! which is p 18 (34) else

It is all just ballpark anyway.

Derive p 18 (35a) and (35b)

We had

$\partial_w V - j\omega W = E_{z1} - E_{z2} = I(z_1+z_2) = Iz$ which is p 17 (32)

$W(z) = L_e\, i(z)$ which is p 17 (29)

which says

$\partial_w V - j\omega L_e\, i(z) = Iz$

$\partial_w V = I[\, z + j\omega L_e]$ which is (35a)

Equation (35b) is just a restatement of (28)

$V = (1/Y)\,\partial_\omega i(\omega)$

so now we have this pair of transmission line equations

$\partial_\omega i(\omega) = YV \qquad Y = G + j\omega C = j\omega C'$

$\partial_w V = ZI$ $\quad Z = z + j\omega L_e$

which in terms of z = -w becomes

$-\partial_z I(z) = YV(z)$ $\quad Y = G + j\omega C = j\omega C'$
$-\partial_z V(z) = ZI(z)$ $\quad Z = z + j\omega L_e$

I read out the rest of King Sec 4. I notice he has Sec numbers at the top of each page so maybe could refer to an equation as Sec 4 (13a).