Capacitor with non constant V ? 9.14.14

I got nowhere with this. I was trying to physically model the notion of an "average capacitance" if the surfaces are not really at constant potential. In Appendix S I never had any need for an average C. But I did use this notion:

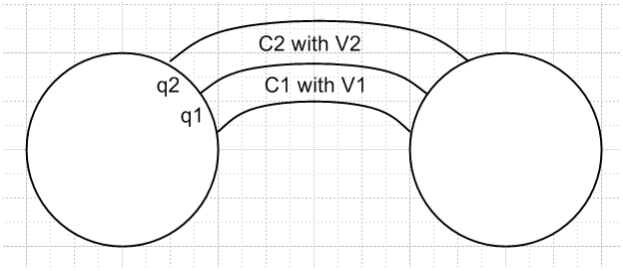
$$\frac{1}{C'(\mathbf{x}_1,\mathbf{x}_2)} = \frac{V(\mathbf{x}_1,\mathbf{x}_2)}{q(z)} \qquad (S.3)$$

and then

$$< \frac{1}{C'(\mathbf{x}_1,\mathbf{x}_2)} >_{c1,c2} = < V(\mathbf{x}_1,\mathbf{x}_2) >_{c1,c2} / q(z)$$

and the double average on the right makes complete sense and then defines what is on the left. It was I think a valid question to ask! Reviewed 10.8.14.

Plan A. Suppose we took our two cylinders and following the E field lines, partitioned the dielectric space into a set of small capacitors, each of which has a particular voltage across it:



Perhaps we parameterize things with an angle θ around the left conductor. Then for a particular one of these small capacitors,

$$dq(\theta) = [C(\theta)d\theta]\, V(\theta)$$

A tiny sliver capacitor has capacitance $C(\theta)d\theta$ which we assume we could calculate from the geometry. We regard $V(\theta)$ as a specified function. Then the total charge on the left conductor is

$$q = \int dq(\theta) = \int d\theta\, C(\theta)\, V(\theta)$$

We might also talk about a mean voltage V as

$$V_{\mathbf{av}} = (1/2\pi a)\int d\theta\, V(\theta)$$

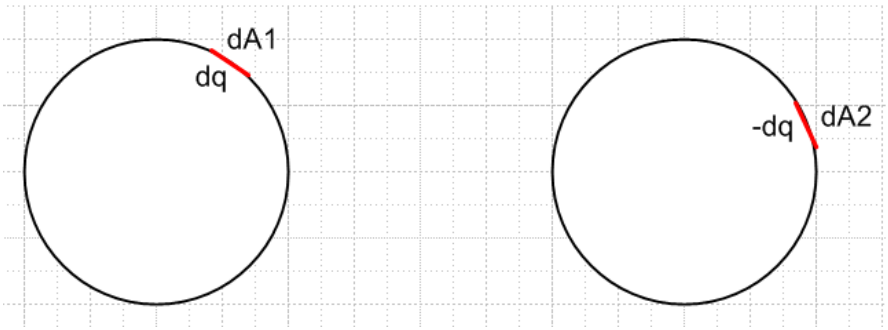
The average capacitance would have to be

$$C_{av} = (1/2\pi a) \int d\theta\, C(\theta)$$

Notice that we can NOT then say that

$$q = C_{av}\, V_{av}$$

Plan B. Use this different picture



A differential capacitor consists of two tiny metal patches of area dA_1 and dA_2 (patches are shown in red). The left patch is at $\mathbf{x}_1$ and the right patch is at $\mathbf{x}_2$. The voltage between these patches is $V(\mathbf{x}_1, \mathbf{x}_2)$. We could then write

$$dq = [C(\mathbf{x}_1,\mathbf{x}_2)dA_1dA_2]\, V(\mathbf{x}_1,\mathbf{x}_2) \qquad dC = C(\mathbf{x}_1,\mathbf{x}_2)dA_1dA_2$$