Low Freq Discussion PhL 9.25.14

During Sept 25- Oct 7 2014 this was my main dialog doc for the low-ω problems with infinite B, and also regarding superposition. I am now happy with all these issues, though they are unfortunate. Section 3 mentions a superposition paradox I was unable to resolve but which I don't think greatly affects lines doc. Reviewed on 10.8.14 and "all accounted for".

Status 9.15.14: Lines doc is once again stabilized in one piece, now at 567 pages. I added an entire new chapter to deal with the painful Low Frequency Problem. I have managed to blame the failure of the theory at low ω on the fact that the e^{-jkz} ansatz is no good there, but there are some residual discomforts with this way out which are lingering and refuse to go away, hence this "discussion" doc.

Part of the Chapter 7 effort is to show WHY you get uniform J_z when you start directly at ω = 0, though you don't get this result as a limit. In the ω = 0 case, Helmholtz becomes Laplace, the $J_m(\beta' r)$ functions become simple r^m functions, and the two BC's sort of collapse and say $E_r = E_\theta = 0$ at the surface. The function n(θ) still exists, but is completely cut off from the problem in the BC and so the η_m no longer enter the problem. Furthermore, you assert that B = finite, and that tells you curl E = 0 and that is what finally kills off the $m \neq 0$ moments of $E_z(r,m)$. I think I should study this ω = 0 situation a bit in search of a clue to why the limit ω→0 fails.

In my ω = 0 proof, I have to fudge a little bit by saying that G is extremely small and that $k = -j\sqrt{RG} \approx 0$ and that is what lets me descend from a 3D to a 2D problem.

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1. Redo the Chapter 7.3 Uniform Jz proof where we start with ω = 0, but stay in 3D.

Here I show that if you *start* with ω = 0 in Appendix D, you get $a_m = K_m = 0$ and everything simply vanishes, which is in fact what does happen with an ∞ transmission line. But you get no idea at all here about the *approach* to ω→ 0.

Here I try to "start off" with ω = 0 but k ≠0. The two BC's still "kill off" n(θ) from the problem. I guess I would have this:

$\nabla^2_{3D}\mathbf{E}(r,\theta) = 0$ $\quad \text{div}_{3D}\,\mathbf{E}(r,\theta) = 0$ $\quad E_r(r=a,\theta) = 0$ $\quad E_\theta(r=a,\theta) = 0$ (7.3.6)

$\nabla^2_{3D}\mathbf{E}(r,m) = 0$ $\quad \text{div}_{3D}\,\mathbf{E}(r,m) = 0$ $\quad E_r(r=a,m) = 0$ $\quad E_\theta(r=m,\theta) = 0$ (7.3.7)

I would then confiscate the (D.1.20) equations and they would then be these:

The Three Helmholtz Equations and the div E = 0 equation (in partial waves) (D.1.20)

$[\nabla^2\mathbf{E}]_z\, E_z = 0$:

$[r^2\partial_r^2 + r\,\partial_r - m^2 + r^2(-k^2)]\, E_z(r,m) = 0$ *(D.1.15)*

$[\nabla^2\mathbf{E}]_r\, E_r = 0$:

$[r^2\partial_r^2 + r\partial_r - (m^2+1) + r^2(-k^2)]\, E_r(r,m) - 2jm\, E_\theta(r,m) = 0$ *(D.1.17)*

$[\nabla^2\mathbf{E}]_\theta\, E_\theta = 0$:

$[r^2\partial_r^2 + r\partial_r - (m^2+1) + r^2(-k^2)]\, E_\theta(r,m) + 2jmE_r(r,m) = 0$ *(D.1.18)*

$\text{div}\,\mathbf{E} = 0$:

$\partial_r\,[r\,E_r(r,m)] + jmE_\theta(r,m) - jk\,r\,E_z(r,m) = 0$ *(D.1.19)*

where k is still in the picture, but I have set $\beta^2 = 0$. I have then replaced $\beta'^2 = \beta^2 - k^2$ with $\beta'^2 = -k^2$ and I guess the idea then is that $\beta' = \pm jk$, and I will just choose $\beta' = jk$ for the moment. If I read through Appendix D, it seems that I would then be led to this "first summary" for the E fields, where a_m and K_m are not yet determined:

First summary of the E field solutions (D.2.21)

$E_z(r,m) = -j\,(\beta'/k)\,\frac{K_m}{2}\,J_m(x)$ $\qquad x = \beta' r$ *(D.1.27)*

$E_r(r,m) = a_m\,x^{-1}\,J_m(x) + \frac{K_m}{2}\,J_{m+1}(x)$ $\qquad \beta'^2 = -k^2 \quad \beta' = jk$ *(D.2.11)*

$jE_\theta(r,m) = -a_m\,x^{-1}\,J_m(x) + \left(\frac{K_m}{2} + \frac{a_m}{m}\right) J_{m+1}(x)$ *(D.2.15)*

I would then impose the boundary conditions which *now* say

$E_r(r=a,m) = 0$

$E_\theta(r=a,m) = 0$

I have done this before somewhere, I think it kills off all the coefficients [yes!], but let's do it again. I can just copy the equations from appendix D setting $N_m = 0$ since that arose from $E_r(r=a,m) = (j\omega/\sigma)\,N_m$

$$a_m\ x_a^{-1}\ J_m(x_a) + \frac{K_m}{2}\ J_{m+1}(x_a)\ = 0 \qquad (1) \quad // \text{ 0 was } (j\omega/\sigma)\ N_m$$

$$-\ a_m\ x_a^{-1}\ J_m(x_a) + \left(\frac{K_m}{2}\ + \frac{a_m}{m}\right)\ J_{m+1}(x_a)\ = 0\ . \qquad (2)$$

Addition and subtraction of these equations gives two new equations,

$$\frac{K_m}{2}\ J_{m+1}(x_a)\ +\ \left(\frac{K_m}{2}\ + \frac{a_m}{m}\right)\ J_{m+1}(x_a)\ =\ 0 \qquad (3)$$

$$2\ a_m\ x_a^{-1}\ J_m(x_a) - \frac{a_m}{m}\ J_{m+1}(x_a)\ = 0\ . \qquad (4)$$

I now switch back to manual control. For $m \neq 0$ equation (4) clearly says $a_m = 0$, though it is unclear what it says for $m = 0$. Then given $a_m \neq 0$ for $m \neq 0$, equation (5) says $K_m = 0$ as well. This then says that all partial wave fields vanish for $m \neq 0$.

So what about $m = 0$? It was at this point that I wrote "Redo App D solutions for m = 0". Rewrite (1) and (2) above for $m = 0$ (formally),

$$a_0\ x_a^{-1}\ J_0(x_a) + \frac{K_0}{2}\ J_1(x_a)\ = 0 \qquad (1)$$

$$-\ a_0\ x_a^{-1}\ J_0(x_a) + \left(\frac{K_0}{2}\ + \frac{a_0}{0}\right)\ J_1(x_a)\ = 0\ . \qquad (2)$$

The upshot of the "Redo" doc (which is now stated below D.2.21) is this

$$a_0 = 0 \text{ and also } \left(\frac{K_0}{2} + \frac{a_0}{0}\right)\ = 0,$$

Therefore the above equations read

$$\frac{K_0}{2}\ J_1(x_a)\ = 0 \qquad (1)$$

$$0 = 0\ . \qquad (2)$$

The equation says $K_0 = 0$!

Conclusion: If we start off with $\omega = 0$, the entire E field of the "first summary box" vanishes! Now recall the "second summary box"

Second summary of the E field solutions : $\qquad R_{dc} = \frac{1}{\sigma\pi a^2} \qquad \beta'^2 = \beta^2 - k^2 \qquad (D.2.33)$

$$E_z(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\ (a\beta')\, f_m \qquad f_m = \left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right] \qquad x = \beta' r$$

$$E_r(r,m) = (j/4)\,\eta_m\, I\, R_{dc}\ (ak)\, g_m \qquad g_m = \left[\frac{J_{m+1}(x)}{J_{m+1}(x_a)} + \frac{J_{m-1}(x)}{J_{m-1}(x_a)}\right] \qquad x_a = \beta' a$$

$$E_\theta(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\ (ak)\ h_m \qquad h_m = \left[\frac{J_{m+1}(x)}{J_{m+1}(x_a)} - \frac{J_{m-1}(x)}{J_{m-1}(x_a)}\right]$$

where $I = 2\pi\omega\,(a/k)\,N_0$. But if we start off with $\omega = 0$, we are starting with $I = 0$, and that then gives a consistent result that the entire E field vanishes. So I have learned nothing new here, but the fact that everything vanishes seems correct.

I tried to show that J_z = uniform if you start with $\omega = 0$, but I got nowhere! If I were to back off and have ω be small and then finally take $\omega \to 0$, then $n(\theta)$ gets into the machine and I then get the anomalous result that I always get (that I originally got for ω very small). [agreed]

In the above effort, I set $\omega = 0$ from the start, but I maintained k as a free parameter and this led to a non-useful solution since $I = 0$. For $G = 0$ we intuitively expect to have $k = 0$ since there is no loss at $\omega = 0$. But just the fact that $I = 0$ ruins everything for $\omega = 0$, and that is why I had to fake it in Chapter 7 in my derivation.

I still suppose the total current and the eddy current could both go to zero in such a way as to maintain the asymmetry of J_z, I have never ruled that out.

2. The Infinite B field Problem and how to explain it away

But the infinite B field problem has no face-saving explanation like this $I = 0$ business! I have fully written this up in Chap 7. Part of the proof of infinite B fields assumes that the $k(\omega)$ expression is valid for small ω The whole idea that we have $k^2 \sim \mathbf{zy}$ is based on the TL equations! If the TL equations have extra terms, then

$$\exp[\, -k\, \{-j\sqrt{\mathbf{zy}}\,\}\, z\,]$$

is NOT a solution. So the whole basis of Appendix D then collapses and then the infinite B fields are not really a problem. For a given ω, then there IS no object I have been calling $k(\omega)$ that you interpret as "k" in e^{-jkz} . [Note added: $e^{-jk(\omega)z}$ is still a *homogeneous* solution, but there is also some particular solution that I don't know about. Then solution is of the form $V(z) = V_p(z) + A\, e^{-jkz} + B\, e^{+jkz}$. I guess this still rules out the ansatz that everything just goes as e^{+jkz}.]

The idea is that due to the e^{-jkz} ansatz right at the start, you cannot use *anything* from later Appendix D results in the limit $\omega \to 0$. This then explains away the two "signs of trouble".

3. The Most General Form Issue (Superposition) and persistence of asymmetric J_z

This is a slightly different issue, also mentioned in Chapter 7, Section 7.2. I claim right now that whatever the solution is to the E_z Helmholtz equation, it *can* be expressed in this manner

$$E_z(r,m,z) = \int dk\, d_m(k,\omega)\, e^{-jkz}\, J_m(\beta' r) \qquad (*)$$

because these are the relevant atomic forms and I am in m-space. This certainly allows for a non e^{-jkz} overall z dependence of the solution. This pathway is not fully documented in Chapter 7, but see Part C of "show that linear..." doc. I have just reviewed that doc section. Here is what I do there:

- I write Helm and div E equations with ∂_z still showing.
- I study all these equations and conclude among other things that

$$E_r(r,m,z)^{(3)} = \int dk\, e^{-jkz}\, F_m(r)$$

where

$$[r^2\partial_r^2 + 3r\partial_r + (1-m^2) + \beta'^2 r^2]\, F_m(r) = 2j\, r\, d_m(k,\omega)\, k\, J_m(\beta' r)$$

- I recognize this as an equation inside Appendix D in which $d_m(k,\omega)$ is called C_{zm} related to K_m.

- I conclude that the solution to the above equation is

$$F_m(r) = a_m\, x^{-1}\, J_m(\beta' r) + \frac{K_m}{2}\, J_{m+1}(\beta' r)$$

where a_m and K_m are TBD.

- This leads to the following, where a_m and K_m are to be determined and are not as given in App D.

First summary of the E field solutions (D.2.21)

$$E_z(r,m,z)^{(3)} = \int dk\, e^{-jkz}\, [-j\,(\beta'/k)\,\frac{K_m}{2}\, J_m(x)] \qquad x = \beta' r \qquad \textit{(D.1.27)}$$

$$E_r(r,m,z)^{(3)} = \int dk\, e^{-jkz}\, [a_m\, x^{-1}\, J_m(x) + \frac{K_m}{2}\, J_{m+1}(x)] \qquad \beta'^2 = \beta^2 - k^2 \qquad \textit{(D.2.11)}$$

$$jE_\theta(r,m,z)^{(3)} = \int dk\, e^{-jkz}\, [-a_m\, x^{-1}\, J_m(x) + (\frac{K_m}{2} + \frac{a_m}{m})\, J_{m+1}(x)] \qquad \textit{(D.2.15)}$$

Without going further, notice that this is nothing more than a linear combination of the App D solutions over a range of k. I am just adding a bunch of solutions together. But the solutions I am adding up all assume that e^{-jkz} is valid for each value of k! If that ansatz e^{-jkz} is invalid for low ω, then I am adding up things that are NOT solutions for low ω, and thus the above integrals are NOT solutions for low ω.

- I then go on to fiddle with the boundary conditions. I conclude that

$$a_m = (j\omega/2\sigma)\, 2m\, N_m \frac{1}{J_{m-1}(x_a)}\, r_k$$

$$\frac{K_m}{2} = (j\omega/2\sigma)\, N_m \left[\frac{1}{J_{m+1}(x_a)} - \frac{1}{J_{m-1}(x_a)}\right] r_k \qquad r_k = C(k,\omega) = \text{alternate name}$$

where r_k is the fourier transform of r(z) which appears in

$$E_r(r=a-\varepsilon,\theta,z)^{(3)} = (j\omega/\sigma)\, n(\theta,z) = (j\omega/\sigma)\, n(\theta) r(z)$$

This leads to (sub in the new coefficients)

$$E_z(r,m,z)^{(3)} = \int dk\, e^{-jkz}\, (1/4)\, \eta_m\, 2\pi\omega\, (a/k)\, N_0\, r_k\, R_{dc}\, (a\beta')\, f_m \qquad f_m = \left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right]$$

$$E_r(r,m,z)^{(3)} = \int dk\, e^{-jkz}\, (j/4)\, \eta_m\, 2\pi\omega\, (a/k)\, N_0\, r_k\, R_{dc}\, (ak)\, g_m \qquad g_m = \left[\frac{J_{m+1}(x)}{J_{m+1}(x_a)} + \frac{J_{m-1}(x)}{J_{m-1}(x_a)}\right]$$

$$E_\theta(r,m,z)^{(3)} = \int dk\, e^{-jkz}\, (1/4)\, \eta_m\, 2\pi\omega\, (a/k)\, N_0\, r_k\, R_{dc}\, (ak)\, h_m \qquad h_m = \left[\frac{J_{m+1}(x)}{J_{m+1}(x_a)} - \frac{J_{m-1}(x)}{J_{m-1}(x_a)}\right]$$

and then specifically,

$$d_m(k,\omega) = (1/2j)(\beta'/k)\, K_m^{new} = (1/2j)(\beta'/k)\, 2\, (j\omega/2\sigma)\, N_m \left[\frac{1}{J_{m+1}(x_a)} - \frac{1}{J_{m-1}(x_a)}\right] r_k$$

$$= (1/2)(\beta'/k)(\omega/\sigma)\, N_m \left[\frac{1}{J_{m+1}(x_a)} - \frac{1}{J_{m-1}(x_a)}\right] r_k$$

in the original Smythian form.

- Then in Part E I use the above expansions to look at a ratio

$$\frac{E_z(r,m,z)}{E_z(r,0,z)} = \eta_m \frac{\int dk\; C(k,\omega)\, (\beta'/k) f_m(\sqrt{\beta^2-k^2}\, r)\, e^{-jkz}}{\int dk\; C(k,\omega)\, (\beta'/k) f_0(\sqrt{\beta^2-k^2}\, r)\, e^{-jkz}} \qquad (**) \qquad f_m = \left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right]$$

I then consider this at $\omega \rightarrow 0$ where it says

$$\frac{E_z(r,m,z)}{E_z(r,0,z)} = \eta_m \frac{\int dk\; C(k,0)\, (jk/k) f_m(jkr)\, e^{-jkz}}{\int dk\; C(k,0)\, (jk/k) f_0(jkr)\, e^{-jkz}} \qquad (**) \qquad f_m = \left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right]$$

I then "bemoan" the fact that no choice for C(k,0) can make this ratio be 0 and thus "superposition" cannot rescue the J_z asymmetry.

Comments on the above. Since the single-k solutions are not valid for low ω, the superposition of single-k solutions cannot be expected to be valid either, since it is the superposition of invalid solutions! The issue of E_z asym in the single-k solution just shines through into the superposed k solution, and I don't see this as being any kind of contradiction, it is in fact what I would expect from superposing invalid single-k solutions.

More Comments and The Paradox:

This superposition paradox was never resolved, and I decided to just give up on it.

(1) E_z in the box above would be a most-general Smythian form for E_z, where $r_k = C(k,\omega)$ is the coefficient. This is the same as equation (*) above. So it MUST be possible to write the E_z field in this form, and there MUST be some coefficient C(k,ω) that gives the exact solution for all ω for E_z.

(2) Given that magical C(k,ω) that works, the E_r and E_θ fields MUST then have the forms shown in the lower two lines of the above box. Otherwise the fields don't satisfy the Helmholtz E equation, the div E = 0 equation, and the two boundary conditions.

(3) As $\omega \rightarrow 0$ (for G = 0) we expect the current to decrease, and we expect the voltage drop along the line to go away, so that there is then no z dependence at all (G = 0 only!) This would seem to imply that as we get to the DC limit, we have ω = 0 and k = 0 and maybe $\lim_{\omega \rightarrow 0} C(k,\omega) = \delta(k)f(k)$ for some f. This would then pull out the k = 0 contribution from the integrals, and then we have the single-k case with k = 0. This of course has J_z asymmetry going into the limit, and it also has infinite B fields in the limit.

The above three steps amount to a **paradox** which I don't know how to solve!

Possible escapes:

(1) the I = 0 escape perhaps explain the J_z asym, but not the infinite B fields

(2) perhaps the Smythian form I quote is not general enough due to some math detail I have overlooked. Maybe the spectrum is "mixed"? What about negative k? Are the e^{-jkz} functions really complete in the z direction?

(3) the boundary conditions really are a complete mystery. Maybe something about them causes the Smythian form not to work.

(4) Maybe the solution does not have a z Fourier transform, something does not converge.

(5) Maybe something really is wrong with my boundary conditions or with Debye currents.

(6) You would think that you would be superposing at the same time on the interior and exterior problems. To do that, you would think that single-k interior and exterior solutions in isolation ought to work out before you do the inside + outside superposition. But I know the single-k does not work because of the T(z) in the TL equations. This then leads to the earlier statement that superposing bad solutions cannot give a good solution.

(7) Maybe each E_i field component has a separate coefficient function in a manner somehow not accounted for in my work.

How I shall leave things: I don't have a resolution of this paradox. But I know that you have a complicated situation with some non-travelling wave for both the interior and exterior problems, and there will be a tricky matching of boundary conditions whose details I have never looked at (and I don't want to look at them).

I think once again the time has come to "punt" the ball. I have done 570 pages of stuff that is the best I can do on the general problem, and this little limit is just beyond my abilities right now. I have suggested how it might be done, which is a big messy problem.

4. Pro and Con Arguments regarding Anomalies

Pro arguer says that the superposition should be accurate and does not use the e^{-jkz} ansatz.
Con arguer says that the superposition is not accurate as the problem solution.

Now, the various equations appearing in the analysis all have ∂_z unreplaced, it is true, *but* then when you posit the superposed form $\int dk\, d_m(k,\omega)\, e^{-jkz}\, J_m(\beta' r)$, then ∂_z once again becomes -jk inside the integral. So it is true that I am never making the ansatz that e^{-jkz} for a single-k solution, but once you superpose, that fact reappears in effect for each "term" in the superposition.

So here is the "pro" argument. We have only used valid equations like Helmholtz and divE = 0 and the boundary conditions to obtain our ratio above. We have not assumed any e^{-jkz} "ansatz", we have just used the valid Smythian most general form. We did assume a factored form in the CP BC:

$$E_r(r=a-\varepsilon,\theta,z)^{(3)} = (j\omega/\sigma)\, n(\theta,z) = (j\omega/\sigma)\, n(\theta)\, r(z)$$

With this superposed solution we showed that J_z is asymmetric. The pro argument says that this superposed solution should be valid at all ω because the equations it came from are valid for all ω and the ansatz e^{-jkz} was never "assumed".

The con arguer then says:

" OK fine, your valid superposed solution gives asym J_z at ω = 0. But what does that superposed solution have to do with the transmission line problem? In that problem, there is external action in the dielectric that is a traveling wave with $e^{-jk(\omega)z}$ (at least for large ω), and this has to be matched at the boundary with a conductor-internal solution which is running at that same value k(ω), otherwise the boundary

cannot match. But the superposed solution you propose only does this matching if we set C(k,0) = δ(k - k(ω)) and in that case, we are back to the original argument that the single-k solution is invalid for small ω."

Con arguer continues: Just inside the boundary r = a-ε the superposition says this

$$E_z(a-\varepsilon,m,z) = \int dk\, e^{-jkz}\, (1/4)\, \eta_m\, 2\pi\omega\, (a/k)\, N_0\, r_k\, R_{dc}\ (a\beta')\, f_m(x_a)$$

but on the outside of the boundary we get something of this form

$$E_z(a+\varepsilon,m,z) = e^{-jk(\omega)z} * [\text{ stuff not dependent on z }]$$

My confidence level for k(ω) in the dielectric is increasing over time! I think that it is extremely reasonable very close to DC, for example. This is really a "third boundary condition" for Appendix D.
[But see below...]

The *reason* I even make the ansatz at the start of App D is that I *think* the *dielectric field* has this form with k = k(ω), and I anticipate matching the boundary conditions on z behavior.

Now if T(z) is strong at low ω, then in the dielectric we *no longer have* a solution of the form

$$E_z(a+\varepsilon,m,z) = e^{-jk(\omega)z} * [\text{ stuff not dependent on z }]$$

This then means that we won't be able to boundary-match an internal field which has the form e^{-jkz}. Maybe there is nothing wrong with the App D ansatz *a priori,* it is just that we won't be able to match it, so we won't then get a solution to the total problem. The problem then lies not in the conductor but in the dielectric. With the ansatz as is in App D, the solutions really are valid solutions even at small ω. It's just that they are not solutions which relate to the transmission line at low ω. They are valid solutions, but those valid solutions are not part of the transmission line problem's solution except at large ω.

At low ω, the solution in the dielectric has some z dependence which *we don't know* due to T(z) being large at low ω. Since the solution is NOT $e^{-jk(\omega)z}$, we can no longer interpret Im(k) as the loss decay coefficient and all that stuff. That must only be valid for low-loss which means only valid for ω above some ω_0 value.

What about interpreting $Z_0(\omega)$ at low ω? The expression is derived in (4.12.18) and that derivation is definitely based on the $e^{j(\omega t-kz)}$ idea for both V(z) and i(z). So if this idea goes away due to T(z). then this $Z_0(\omega)$ expression is also meaningless at now ω, despite my taking these low ω limits.

On the other hand, I kind of like what $e^{-jk(\omega)z}$ predicts for loss at DC. How can it predict the right value for the loss coefficient at low ω, and at the same time be "invalid at low ω" ?

Realization: Even if T(z) is large, the TL equations *still have* those e^{-jkz} solutions as homogeneous solutions, it's just that they also have a particular solution, so the solution is then

$$V(z) = V_p(z) + A\, e^{-jkz} + B\, e^{+jkz}$$

So I guess have not said this right and have not considered the implication. // But still, you cannot just set $V_p(z)$ to zero. It does not have an overall constant like A and B. So it is then incorrect to say that V(z) can be chosen to have a simple e^{-jkz} dependence. The implication then is that the E fields inside the round wire don't really have e^{-jkz} simple dependence at low ω.

Third Party Observation: I have listened to the pro and con arguers. Both arguers say that the Appendix D solution, either as a single k, or as a k superposition, does generate a valid solution which satisfies all four of Maxwell's Equations and which satisfies div E = 0 and meets the two BC's. Both arguers would agree that this solution has an infinite B field as ω→ 0, so both would have to admit that something is wrong with the solution in this limit, but is still "physical" very close to this limit.

It is now 2PM, I have run out of pre-Torrey time, so will resume this next week, it is now Thurs 9/25.

Torrey Maintenance Trip Happened Right Here.

Resuming on 9/29/14.

5. Rederive the infinite B field expressions directly from the full B field expressions.

Suppose this e^{-jkz} ansatz really is the cause of the low ω problems. How does it create that infinite B field, for example? What is the physical mechanism?

Task: Let's repeat the Chapter 7 B→∞ development using (D.4.13) directly. I just did this in a separate doc (only for m > 0), and I got the same results as (7.4.2). In doing this, I really got no new insight as to why this is happening, it is just that you have an overall 1/k factor in both B_r and B_θ . I think my method of App 7 is clearer for this calculation of infinite B, so I will maintain it , but I added a comment.

In "Infinite B fields directly from (D.4.13).doc" lower part I rederive (7.4.2) directly from (D.9.39) both for m > 0 and m = 0.

6. Verify the B Helmholtz Equation

Recall that I have fully vetted the full B field results using Maple inasmuch as I verified the four Maxwell equations in Section D.5. I did not show that the B fields satisfy the **Helm equation for B** ! Would that be an easy thing to do?? I have the three B fields sitting in a Maple file. Let's go try that right now. The file is called "B field verify 9_20.mws".

I cannot use the fancy Maple diffops because they assume θ space, but I am in m space. I have shown however in m space that div **B** = 0, so I do know that

$$\nabla^2 \mathbf{B} \equiv \text{grad}(\text{diverge } \mathbf{B}) - \text{curl}\,(\text{curl } \mathbf{B}) = -\,\text{curl}\,(\text{curl } \mathbf{B})$$

I already have expressions for curl B components. So perhaps I could do this last curl operation. OK, I have done this at the end of the file, and it does show that the B Helm is OK, assuming I did the m-space curl components correctly. See end of the mws file for how this works, no need to copy here.

7. Facts for the G = 0 Case in the small-x limit of things:

Here I review all the small-ω anomalies for G = 0.

(1) for small ω, I assume that both β and k are very small and this makes β' be very small.
(I know β is very small since $\sim\sqrt{\omega}$ so no assumption there.)
(I assume nothing about the power with which k approaches zero, if it even does)
(2) I in fact assume that β' << 1 (really β'a << 1 but I ignore the a)
(3) item 2 then justifies my small-x expressions for the E and B field Bessel functions
(4) since β' << 1, I treat β' ~ x as a smallness parameter for expansions
(5) Consider curl **B** = j (β^2/ω) **E** which is the exact curl B Maxwell equation. I can write

$$\text{curl } \mathbf{B} = j\,(\beta^2/\omega)\,\mathbf{E} = j\,(k^2/\omega)\,\mathbf{E} + j\,(\beta'^2/\omega)\,\mathbf{E}$$

$$= j\,(k^2/\omega)\,\mathbf{E} + \beta'^2\,[j\,(1/\omega)\,\mathbf{E}]$$

If I am keeping only lowest terms in the E and B fields for x << 1, then I should keep only such lowest terms in the equation curl **B** = j (β^2/ω) **E**. I can therefore ignore the β'^2 term above and write

$$\text{curl } \mathbf{B} = j\,(k^2/\omega)\,\mathbf{E}\ .$$

(6) Using the small-x leading expansion terms for the B and E fields, I have shown in Maple that the above equation is exactly satisfied for *any* value of k. Of course I have already assumed that k is small, so it is valid then for any small value of k. [just another of many checks!]

(7) My B field values for small x are these:

$$B_z(r,m) = (j/2)\,\eta_m CVR_{dc}\,(r/a)^m\,(m+1)$$
$$B_r(r,m) = (1/4)\,\eta_m CVR_{dc}\,(r/a)^{m-1}\,(1/ak)[\,-2m(m+1) + k^2(a^2-r^2)]$$
$$B_\theta(r,m) = (j/4)\,\eta_m CVR_{dc}\,(r/a)^{m-1}\,(1/ak)\,[\,-2m(m+1) + k^2(r^2+a^2)] \qquad m > 0$$

Notice the 1/k sitting in the last two expressions. When I compute curl **B** , these 1/k factors no longer appear due to multiplying factors of k in the curl formula. So although the B fields themselves contain 1/k factors, the curlB components *do not* contain 1/k factors.

(8) My E field values for small x are these:

$$E_z(r,m) = (1/4)\,\eta_m\,CV\,R_{dc}\,(\omega/k)\ \ (r/a)^m\,(m+1)\ (2)$$
$$E_r(r,m) = (j/4)\,\eta_m\,CV\,R_{dc}\,(\omega a)\ [(r/a)^{m+1} + (r/a)^{m-1}]$$
$$E_\theta(r,m) = (1/4)\,\eta_m\ CV\,R_{dc}\,(\omega a)\ [r/a)^{m+1} - (r/a)^{m-1}]$$

Thus, the right hand side of the curl **B** = j (k^2/ω) **E** equations are these expressions

$RHS_z(r,m) = j\,(1/4)\,\eta_m\,CV\,R_{dc}\,(k)\;\;(r/a)^m\,(m+1)$ (2)
$RHS_r(r,m) = j\,(j/4)\,\eta_m\,CV\,R_{dc}\,(k^2a)\;\;[(r/a)^{m+1} + (r/a)^{m-1}]$
$RHS_\theta(r,m) = j\,(1/4)\,\eta_m\;CV\,R_{dc}\,(k^2a)\;\;[r/a)^{m+1} - (r/a)^{m-1}]$

None of these expressions contain 1/k factors.

(9) If I now think of k → 0 in some manner as ω → 0, I have shown these facts :

- some **B** components diverge as 1/k (which seems unphysical)
- the curl **B** components do not diverge
- the RHS expressions also do not diverge
- Thus, both sides of curl **B** = j (k^2/ω) **E** are "finite" as k → 0.

(10) Note that the notion that $k \rightarrow \sqrt{\omega}$ was not used anywhere in the above items!

(11) If I now add the assumption that $k \rightarrow \sqrt{\omega}$, then you can see that as ω→ 0, all three E field components vanish in the limit ω→ 0. Since (k^2/ω) is then just a constant, we see that the right hand side of curl **B** = j (k^2/ω) **E** goes to 0 as ω → 0. In this case, we then know that each of the curl B components will also go to 0 as ω → 0.

(12) We thus end up with a strange situation: All the E fields go to 0, but some of the B fields go infinite.

Detail for Item 7. Start with:

$B_z(r,m) = (j/2)\,\eta_m CV R_{dc}\,(r/a)^m\,(m+1)$
$B_r(r,m) = (1/4)\,\eta_m CV R_{dc}\,(r/a)^{m-1}\,(1/ak)[\,-2m(m+1) + k^2(a^2-r^2)]$
$B_\theta(r,m) = (j/4)\,\eta_m CV R_{dc}\,(r/a)^{m-1}\,(1/ak)\,[\,-2m(m+1) + k^2(r^2+a^2)]$ $\qquad m > 0$

In order to compute curl B, we would do this

$[\text{curl } B]_r = [r^{-1}jmB_z + jkB_\theta]$
$[\text{curl } B]_\theta = [-jkB_r - \partial_r B_z]$
$[\text{curl } B]_z = [r^{-1}\partial_r(rB_\theta) - r^{-1}jmB_r]$, *(D.4.7)*

Let's just study each curl component *qualitatively*.

$[\text{curl } B]_r = [r^{-1}jmB_z + jkB_\theta]$

Here the jmB_z term is finite, while the jkB_θ term is also finite because the jk factor cancels our problem divergence factor (1/ak). Next, consider

$[\text{curl } B]_\theta = [-jkB_r - \partial_r B_z]$.

The same thing happens here: second term is finite, while first term then in also finite! Finally,

$$[\text{curl } B]_z = r^{-1}[\partial_r(rB_\theta) - jmB_r]$$

This one is less obvious because now the 1/k terms have to exactly cancel. I think I can maintain only the problematic terms, so write

$$B_r(r,m) = (r/a)^{m-1}\,(1/ak)[-2m(m+1)]$$
$$B_\theta(r,m) = j\,(r/a)^{m-1}\,(1/ak)\,[-2m(m+1)]$$

Then

$$\partial_r(rB_\theta) = j\,(1/ak)[-2m(m+1)]\partial_r[r(r/a)^{m-1}] = (1/ak)[-2m(m+1)]\partial_r[(r/a)a(r/a)^{m-1}]$$

$$= j\,(1/k)[-2m(m+1)]\partial_r[(r/a)^m] = (1/k)[-2m(m+1)]m(r/a)^{m-1}\,(1/a)$$

$$= j\,(1/ka)[-2m(m+1)]m(r/a)^{m-1}$$

Meanwhile,

$$-jmB_r = -jm\,(r/a)^{m-1}\,(1/ak)[-2m(m+1)]$$

So these divergent-only terms give

$$r\,[\text{curl } B]_z = [\partial_r(rB_\theta) - jmB_r]$$

$$= j\,(1/ka)[-2m(m+1)]m(r/a)^{m-1} - jm\,(r/a)^{m-1}\,(1/ak)[-2m(m+1)]$$

$$= j\,(1/ka)[-2m^2(m+1)](r/a)^{m-1} + j\,(r/a)^{m-1}\,(1/ak)[2m^2(m+1)]$$

$$= j\,(1/ka)[-2m^2(m+1)](r/a)^{m-1}\,[1 - 1]$$

$$= 0$$

So we have at least an interesting result:

Fact: Although the B fields diverge, curl B remains finite! (G = 0 case)

8. Now, what corresponding thing happens in the G > 0 Case

Due to the fancier charge pump BC, all E and B fields will be "larger" by a factor ξ_2/ε_2 [not quite an accurate statement since k changes, but OK for here] where

$$\xi_2/\varepsilon_2 = (\varepsilon_2 - j\sigma_2/\omega)/\varepsilon_2 = 1 - j(\sigma_2/\varepsilon_2)(1/\omega) \quad \rightarrow \quad -j(\sigma_2/\varepsilon_2)(1/\omega)$$

It happens that as $\omega \rightarrow 0$, this ratio becomes very large. Still, this factor is just a constant for a given small value of ω .

(a) Since we are increasing the E and B fields by the same constant, we certainly have curl $\mathbf{B} = j\,(k^2/\omega)\,\mathbf{E}$ still being valid. Recall that I verified this in Maple for the small-x E and B fields in the G = 0 case, and to get to the G> 0 case, we just add the factor shown above to both E and B fields. So there is then no doubt about the fact that curl $\mathbf{B} = j\,(k^2/\omega)\,\mathbf{E}$ is valid.

[The above paragraph is no longer valid since it is not really just a scale change. What happens is that E does scale up by ξ_2/ε_2, but $k \rightarrow k'$ on the right as well and also inside the E_i components. In Ch 7.4 I compute the low-ω B_i directly from the low-ω E_i, and I do this for $G \geq 0$, so the infinite B conclusions are not affected by the above paragraph.]

(b) Now, in the G > 0 case we know curl B will diverge as $\omega \rightarrow 0$ simply due to this new $-j(\sigma_2/\varepsilon_2)(1/\omega)$ factor. In fact, it will diverge as $1/\omega$ due to this factor. So we can no longer say that "although B goes infinite, curl B stays finite" as we did for G = 0. Instead we have both B and curl B going infinite in the G > 0 case. In fact some B fields go as $(1/\omega)*(1/k)$ divergent, whereas curl B is only $(1/\omega)$ divergent.

My conclusion is that everything I have said in Chapter 7 is OK. [agreed]

<u>Rereading all of Chapter 7 on 10.5.14:</u>

7.1 review of App D. OK, but I do comment on the Smythian form concept.

7.2 First sign of trouble. I think this section is just fine. "something is wrong with $\omega \rightarrow 0$"

The superposition section is OK, leave it in, it leads to the reflection scenario mention.

7.3 My proof that J_z is uniform, I sort of like it, despite wobbly assumptions. Keep it!

7.4 Infinite B fields. It reads just fine, I like it, right to the point.

7.5 What is the trouble. OK, it is fine.

This whole Chapter 7 is OK. I really don't want to mention my residual paradox above, I want the reader to feel that the T(z) theory explains the anomalies, and the reader can then do the full bore solution if he wants. I feel this chapter does "due diligence" in dealing with the small ω anomolies.

Rereading Section D.11

(a) it is OK, there is repetition with Chapter 7, but I think repetition is OK.