Quickie : Verify that the low-ω E fields satisfy the two BC's.
Quickie : Make sure curl components are correct 10.5.14

These are just a few "checks" on my theory, both OK. The more checks the better. Always looking for something wrong, it goes on forever.

Here are the E fields

$$E_z(r,m) = (1/2)\,\eta_m\,CV\,R_{dc}\,(\omega/k)\,(r/a)^{|m|}\,(|m|+1) \qquad (7.4.1)$$
$$E_r(r,m) = (j/4)\,\eta_m\,CV\,R_{dc}\,(\omega a)\,[(r/a)^{|m|+1} + (r/a)^{|m|-1}]$$
$$E_\theta(r,m) = (1/4)\,\eta_m\,CV\,R_{dc}\,(\omega a)\,[(r/a)^{|m|+1} - (r/a)^{|m|-1}]$$

$$E_z(r,0) = CV\,R_{dc}\,(\omega/k)$$
$$E_r(r,0) = (j/2)\,CV\,R_{dc}\,(\omega r)$$
$$E_\theta(r,0) = 0 \qquad \text{// low } \omega \text{ E fields}$$

Here are the E fields evaluated at r = a:

$$E_z(a,m) = (1/2)\,\eta_m\,CV\,R_{dc}\,(\omega/k)\,(|m|+1) \qquad (7.4.1)$$
$$E_r(a,m) = (j/4)\,\eta_m\,CV\,R_{dc}\,(\omega a)\,(2)$$
$$E_\theta(a,m) = 0$$

$$E_z(a,0) = CV\,R_{dc}\,(\omega/k)$$
$$E_r(a,0) = (j/2)\,CV\,R_{dc}\,(\omega a)$$
$$E_\theta(a,0) = 0 \qquad \text{// low } \omega \text{ E fields}$$

The E_θ BC is obviously OK.

The other BC says,

$$E_r(r=a,m) = (j\omega/\sigma)\,N_m$$

But from (D.1.8) I know that

$$N_0 = (1/2\pi a)\,q(0) = \langle n(\theta)\rangle \qquad \text{// } q(0) = CV \qquad (D.1.8)$$

or

$$N_0 = CV/(2\pi a)$$

so

$$N_m = \eta_m\,CV/(2\pi a)$$

so this BC then requires that

$$E_r(r=a,m) = (j\omega/\sigma)\,N_m = (j\omega/\sigma)\,\eta_m\,CV/(2\pi a) = j\omega\,\eta_m\,CV\,(1/2)\,a\,(1/\sigma\pi a^2)$$

$= j\omega\, \eta_m\, CV\, (1/2)\, a\, R_{dc} = (j/2)\, \eta_m\, CV\, (\omega a)\, R_{dc}$

and this does in fact agree with the above box.

Quickie : Make sure my cylindrical curl equation is right.

In my diffops doc I state it this way

$$\text{curl } \mathbf{F} = \hat{\mathbf{r}}\,[\, r^{-1}\partial_\theta F_z - \partial_z F_\theta] + \hat{\boldsymbol{\theta}}\,[\partial_z F_r - \partial_r F_z] + \hat{\mathbf{z}}\,[\, r^{-1}\partial_r(rF_\theta) - r^{-1}\partial_\theta F_r\,]$$

Source #1: M&F state it this way which I already quote in lines doc

$$\text{curl } \mathbf{A} = \mathbf{a}_r\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \mathbf{a}_\phi\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \mathbf{a}_z\left(\frac{1}{r}\frac{\partial}{\partial r}rA_\phi - \frac{1}{r}\frac{\partial A_r}{\partial \phi}\right)$$

So this source agrees with me.

Source #2: M&S page 12 have it in matrix form. I read their thing as saying this

$$r\,\text{curl } E = \hat{r}\,[\,\partial_\theta E_z - \partial_z(E_\varphi r)] - \hat{\theta}\, r\,[\,\partial_r E_z - \partial_z E_r\,] + \hat{z}\,[\,\partial_r(E_\theta r) - \partial_\theta E_r\,]$$

so then

$$\text{curl } E = \hat{r}\,[\, r^{-1}\partial_\theta E_z - \partial_z(E_\varphi)] + \hat{\theta}\,[-\partial_r E_z + \partial_z E_r\,] + \hat{z}\,[\, r^{-1}\partial_r(E_\theta r) - r^{-1}\partial_\theta E_r\,]$$

OK OK OK

So this source agrees with me.

Source #3. Smythe page 60, also agrees with diffops above.

Source #4: A web pdf

$$\nabla \times \boldsymbol{u} = \hat{\boldsymbol{r}}\left(\frac{1}{r}\frac{\partial u_z}{\partial \phi} - \frac{\partial u_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right) + \hat{\boldsymbol{z}}\frac{1}{r}\left[\frac{\partial (r u_\phi)}{\partial r} - \frac{\partial u_r}{\partial \phi}\right].$$

which also agrees. That is enough.

Now using $e^{jm\theta}$ and e^{-jkz} I can say

$$\text{curl } \mathbf{F} = \hat{\mathbf{r}}\,[\, r^{-1}\partial_\theta F_z - \partial_z F_\theta] + \hat{\boldsymbol{\theta}}\,[\partial_z F_r - \partial_r F_z] + \hat{\mathbf{z}}\,[\, r^{-1}\partial_r(rF_\theta) - r^{-1}\partial_\theta F_r\,]$$

$$\text{curl } \mathbf{F} = \hat{\mathbf{r}}\,[\, r^{-1}jmF_z + jkF_\theta] + \hat{\boldsymbol{\theta}}\,[-jkF_r - \partial_r F_z] + \hat{\mathbf{z}}\,[\, r^{-1}\partial_r(rF_\theta) - r^{-1}jmF_r\,]$$

Source #5 : lines doc

$$\text{curl } \mathbf{E} = \hat{\mathbf{r}}\,[\, r^{-1}\partial_\theta E_z - \partial_z E_\theta] + \hat{\boldsymbol{\theta}}\,[\partial_z E_r - \partial_r E_z] + \hat{\mathbf{z}}\,[\, r^{-1}\partial_r(rE_\theta) - r^{-1}\partial_\theta E_r\,] \qquad \text{(D.4.2)}$$

$$-j\omega\mathbf{B}(r,m) \;=\; \hat{\mathbf{r}}\,[\, r^{-1}jmE_z + jkE_\theta] + \hat{\boldsymbol{\theta}}\,[-jkE_r - \partial_r E_z] + \hat{\mathbf{z}}\,[\, r^{-1}\partial_r(rE_\theta) - r^{-1}jmE_r\,] \qquad \text{(D.4.6)}$$

Source #6. My Chapter 7:

Enter the Bi expressions

```
Br := (j/omega)*((j*m/r)*Ez + j*k*Eth);
```

$$Br := \frac{j\left(\frac{j\,m\,Ez}{r} + j\,k\,Eth\right)}{\omega}$$

```
Bth := (j/omega)*(-j*k*Er - Diff(Ez,r));
```

$$Bth := \frac{j\left(-j\,k\,Er - \left(\frac{\partial}{\partial r}Ez\right)\right)}{\omega}$$

```
Bz := (j/omega)*(1/r)*( Diff(r*Eth,r) - j*m*Er);
```

$$Bz := \frac{j\left(\left(\frac{\partial}{\partial r}r\,Eth\right) - j\,m\,Er\right)}{\omega\,r}$$

Conclusion: I think I can rule out any possible error I might have made with the correct expressions for the curl components.