Review of the Averaging Repair Stuff PhL 8.19.14

This "repair" rescues the TL equations in the case of closely spaced conductors where $Z_s(\theta)$ varies and gets replaced by an average Z_s and so on. I have a new question whose answer I don't know:

Does this averaging repair somehow make the TL equations valid at very low ω?

To find out, I will here do a little running dialog about the steps of Chapter 4. I may have done this before, but I was not thinking of this new question.

In Chapter 4 I go first through the "V stuff" and I end up with (4.4.6)

$$V(z) = q(z)\frac{1}{4\pi\xi_d}\ \{\ \int_{C_1} dx_1'\, dy_1'\, \alpha_1(x_1',y_1')\, \ln(s_{21}^2/s_{11}^2) - \int_{C_2} dx_2'\, dy_2'\, \alpha_2(x_2',y_2')\, \ln(s_{22}^2/s_{12}^2)\ \}$$

where in theory this should be $V(\mathbf{x_1},\mathbf{x_2})$ if I have to give up the idea that φ = constant on each conductor's cross section. The integral stuff here is K, which then becomes really $K(\mathbf{x_1},\mathbf{x_2})$. We then end up with

$$\frac{1}{C'}(\mathbf{x_1},\mathbf{x_2}) = \frac{V(\mathbf{x_1},\mathbf{x_2})}{q(z)}$$

$$= \frac{1}{4\pi\xi_d}\ \{\ \int_{C_1} dx_1'\, dy_1'\, \alpha_1(x_1',y_1')\, \ln(s_{21}^2/s_{11}^2) - \int_{C_2} dx_2'\, dy_2'\, \alpha_2(x_2',y_2')\, \ln(s_{22}^2/s_{12}^2)\ \}$$

$$= \frac{1}{4\pi\xi_d}\ K(\mathbf{x_1},\mathbf{x_2}) \qquad \text{and} \qquad V(\mathbf{x_1},\mathbf{x_2}) = q(z)\frac{1}{4\pi\xi_d}\ K(\mathbf{x_1},\mathbf{x_2}) \qquad (4.4.7)$$

Now what does the above even mean? Better, think of this

$$C(\mathbf{x_1},\mathbf{x_2}) = 4\pi\varepsilon_d / K(\mathbf{x_1},\mathbf{x_2})\ .$$

How can the capacitance between two conductors depend on the touch points? The quantity q(z) seems well defined. But then $q(z) = C\, V(\mathbf{x_1},\mathbf{x_2})$. I guess you would just define

$$C(\mathbf{x_1},\mathbf{x_2}) \equiv V(\mathbf{x_1},\mathbf{x_2}) / q(z)$$

Interpretation: Imagine two TL conductors. Each cross section is at some *constant* V_i and there is then some constant $V = V_1 - V_2$. If we were to set $V = V(\mathbf{x_1},\mathbf{x_2})$ for a fixed specific choice of the two points $\mathbf{x_1}$ and $\mathbf{x_2}$, then $C(\mathbf{x_1},\mathbf{x_2})$ would be the capacitance of that capacitor. Now the left conductor always has q(z) and the right has - q(z). The capacitance then varies with your choice of x_1 and x_2. I think you could then define

$$C = \langle C(\mathbf{x_1},\mathbf{x_2}) \rangle = \langle V(\mathbf{x_1},\mathbf{x_2}) \rangle / q(z)$$

and that would then define some kind of "mean capacitance"

OK, that seems OK. I then also get

$$L_e(\mathbf{x}_1,\mathbf{x}_2) \equiv (\mu_d/4\pi)\, K(\mathbf{x}_1,\mathbf{x}_2) \ . \qquad (4.4.11)$$

I guess the flux through the loop varies depending on where you put the loop.

At this point we have my two VERY long examples in Sections 4.5 and 4.6, two digressions in the theory flow, but I wanted to get them presented before the messy A_z part of things began. So we then jump way down to the start of Section 4.7.

Now comes an important issue. I am going to say the $\mathbf{A} = A_z\,\hat{\mathbf{z}}$ so that $\mathbf{A}_t = 0$. Let's go look at Appendix M on this just a bit.

Ouch! I just found a major lines doc error! In Appendix M I refer to App D.11 saying $\beta_d = (\omega - j\omega_c)/v_d$ where ω_c is some "cutoff" frequency related to losses. But I have gotten rid of that entire theory, so the reader will find nothing in D.11 on this subject!

Maybe I have to rewrite Appendix M using k instead of β_d . I am talking about the vector potential in the dielectric, but everything is really going at e^{-jkz} (in dielectric and in conductors). My comments are all OK in the lossless limit where $k = \beta_d$. But then I roll out this ω_c thing and I don't think I can justify it any more. Where did that cutoff come from?? What older doc talks about that? I search for fc in all of lines and I get 17 hits. I find an "old Section D.11 doc". There, in section (b) I derive my "simple loss model" which says the decay constant is $\alpha = [\, L(G + j\omega C) + C(R + j\omega L)]\,(v_d/2)$. This assumes that the wave is traveling at the dielectric speed v_d, so this would be a very low-loss model. I get $\alpha = (R_{dc}C)(v_d/2)$ as an approximation for low ω (!), where this model probably does not apply. Then I seem to just define $\omega_c = v_d\alpha = (R_{dc}C)(v_d^2/2)$. Well, I am writing $\beta_d = \beta_{d0} - j\alpha$ where $\beta_{d0} = (\omega/v_d)$ and then as $\omega \to 0$, I get $-j\alpha = (-j)(R_{dc}C)(v_d/2) = (-j)(1/v_d)(R_{dc}C)(v_d^2/2) = (-j)(1/v_d)\,\omega_c$ so then

$$\beta_d = \beta_{d0} - j\alpha = (\omega/v_d) + (-j)(1/v_d)\,\omega_c = [\,\omega - j\omega_c]/v_d$$

and so that was my "cutoff" idea. But this is all completely wrong in light of the k(ω) model of things which I now use to handle loss. What really happens for small ω is this,

$$v_\varphi = \omega/k_r = \omega \,/\, [\sqrt{RC/2}\;\omega^{1/2}] = \sqrt{\frac{2}{RC}}\;\omega^{1/2} \qquad \text{phase velocity}$$

$$\lambda = 2\pi/k_r = 2\pi \,/\, [\sqrt{RC/2}\;\omega^{1/2}] = 2\pi\sqrt{\frac{2}{RC}}\;\omega^{-1/2} \qquad \text{wavelength}$$

$$\Delta z = 1/(-k_i) = 1/k_r = \sqrt{\frac{2}{RC}}\;\omega^{-1/2} \qquad \text{decay distance}$$

$$\alpha = -k_i = 1/\Delta z = \sqrt{RC/2}\;\;\omega^{1/2} \ .$$

Then we have

$k = k_r + jk_i = \sqrt{RC/2}\ \omega^{1/2}\ (1\text{-}j)$

and this does NOT behave as some $k = [\ \omega - j\omega_c]/v_d$ with a cutoff. In fact, as $\omega \rightarrow 0$ we get $k \rightarrow 0$. Well, I realized this before releasing lines doc in Aug, but I failed to update Appendix M!!!

I will try right now to rewrite Appendix M in a separate doc.

// I got hung up in App M because it refers to a now-gone loss model with ω_c . This led in turn to a problem in Ch 4, so this doc here is going on hold for a while! It is the issue of β_d versus k in various locations.

[**Status on 9.10.14:** Since writing the above, I have redone Appendix Q so the R,L.G,C are modeled instead of being taken as constants, but the critical low ω limits stay the same! I have also rewritten Appendix M so it no longer refers to the "cutoff model" mentioned above. So I could come back now and continue on this discussion of the averaging fix. Here is the conflict I now face:

1) suppose averaging rescues the transmission line equations so k(ω) applies at all ω.

2) then how do I explain the various low-ω "anomalies" of my theory. (asym of J_z and the curlE thing)]

I have said in App D11 that the TL equations are what fail at low ω. But maybe it is the ansatz e^{-jkz} which is the real failure! At low frequencies, the ansatz leads to contradictions, and so it must not be a good assumption at low ω. You would probably have to solve the REAL problem (exterior/interior in full) to find out why.