The DC Limit of the Appendix D electric fields PhL 3.19.14

Review 5.15.14.

In **Section 1** I consider the loss effect on β_d and find that $\beta_d \rightarrow -j\,(\omega_c/v_d)$ instead of 0. This seems important since β_d is embedded in Appendix D analysis.

In **Section 2** however I show that despite this effect, you still have $\beta'^2 = -j\omega\mu\sigma$ as $\omega \rightarrow 0$ unless you want to talk about things way down at $\omega = 10^{-9}$ Hz, which I don't.

Section 3 is then a very brief review of the basic asymmetry paradox as $\omega \rightarrow 0$.

Section 4 is a nothing as far as I can tell.

Section 5 is the realization that $Z_0 = \infty$ as $\omega = 0$, with nice ladder picture, and this is perhaps the first time I realize this "escape hatch" to my paradox. Date is 3/20/14 on this section.

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1. Power Loss and the limit of β_d as $\omega \rightarrow 0$.

[When the loss term is included, one finds that $\beta_d \rightarrow$ constant at low ω, not $\beta_d \rightarrow 0$. This constant becomes significant at some frequency f_c which is 7 KHz for Belden 8281. $\beta_d = A + B\omega$]

In Appendix D we assume $e^{-j\beta_d z}$ as the z-dependence of all quantities. If we want to include "loss", we have to *reinterpret* β_d in the following manner

$$\beta_d \rightarrow (\omega/v_d) \;-\; j\;[(R_{dc1} + R_{dc2})C\,]\,(v_d/2) \qquad\qquad \beta_{d0} = (\omega/v_d)$$

$$\text{1/m} \qquad\quad \text{sec/m}^2 \qquad\quad \text{m/sec}$$

Here R and C are the effective network values and it might take some effor oops

The details of the imaginary part are not so important compared to the fact that it is "some constant" which does not depend on ω. I may find that this imaginary part has an error in it, but for now just note that it is a constant independent of ω. After reinterpretation we have

$$\beta_d = (1/v_d)\,[\,\omega - j\,(v_d^2/2)\,(R_{dc}\,C)\,] \qquad = (\omega/v_d) - j\alpha \qquad \alpha = R_{dc}\,C\,(v_d/2)$$

$$= (1/v_d)\,[\,2\pi f - j\,(v_d^2/2)\,(R_{dc}\,C)\,]$$

$$= (2\pi/v_d)\,[f - j\,(v_d^2/4\pi)\,(R_{dc}\,C)\,] \qquad f_c = (v_d^2/4\pi)\,(R_{dc}C)$$

$$\equiv (2\pi/v_d)\,[f - j\,f_c\,] \qquad \omega_c = (v_d^2/2)\,(R_{dc}C) = v_d\alpha$$

$= (1/v_d)(\omega - j\omega_c)$. $= \beta_{d0} - j\alpha$ $\beta_{d0} = \omega/v_d$ $\alpha = R_{dc}\, C\, (v_d/2)$

Note that we get the expected decay with parameter α,

$$e^{-j\beta_d z} = e^{-j\beta_{d0} z}\, e^{-j(-j\alpha) z} = e^{-j\beta_{d0} z}\, e^{-\alpha z}$$

We see that there is a certain "time constant" τ_c associated with the transmission line which is

$f_c = \tau_c^{-1} = R_{dc}\, C\, v_d^2 / 4\pi$ $\text{sec/m}^2 * \text{m}^2/\text{sec}^2 = 1/\text{sec}$

Here is some data for Belden 8281 from a pdf I just downloaded,

Electrical Characteristics (Overall)

Nom. Characteristic Impedance:

Impedance (Ohm)
75

Nom. Inductance:

Inductance (µH/m)
0.429811

Nom. Capacitance Conductor to Shield:

Capacitance (pF/m)
68.901

Nominal Velocity of Propagation:

VP (%)
66

Nominal Delay:

Delay (ns/m)
5.05274

Nom. Conductor DC Resistance:

DCR @ 20°C (Ohm/km)
32.4819

Nominal Outer Shield DC Resistance:

DCR @ 20°C (Ohm/km)
3.6091

Nom. Attenuation:

Freq. (MHz)	Attenuation (dB/100m)
1	0.9843
3.6	1.6405
10.0	2.6248
71.5	6.8901
135	9.843
270	14.1083
360	16.7331
540	20.6703
720	24.2794
750	24.9356
1000	30.1852

It says that

$R_{dc} = (32.5 + 3.6)\ \Omega/\text{km} = 36.1 \times 10^{-3}$ ohms/m

$C = 69$ pF/m $= 69 \times 10^{-12}$ farad/m

$v_d = 0.66c = 0.66 \times 3 \times 10^8$ m/sec $= 2 \times 10^8$ m/sec

so therefore

$f_c = \tau_c^{-1} = R_{dc}\, C\, v_d^2/(4\pi)$ $\omega_c = R_{dc}\, C\, v_d^2/2$

and Maple to the rescue:

```
Rdc := 36.1e-3;
```

$$Rdc := .0361$$

```
C := 69e-12;
```

$$C := .69\ 10^{-10}$$

```
vd := 0.66 * 3e8;
```

$$vd := .198\ 10^{9}$$

```
fc := Rdc*C*vd^2/(4*evalf(Pi));
```

$$fc := 7770.998213$$

So basically $f_c \approx 7.7$ KHz. So go back to

$$\beta_d = (2\pi/v_d)\,[f - j\,f_c] \qquad f_c = \tau_c^{-1} = R_{dc}\,C\,v_d^2/(4\pi)\,.$$

If we are operating substantially above this frequency f_c, perhaps $f > 10\,f_c$, then we can neglect f_c and we then have

$$\beta_d \approx (2\pi/v_d)\,[f] = (\omega/v_d) = \beta_{d0}$$

which is our "traditional" meaning for this symbol β_d . But we now have a specific frequency which in a ball park sense tells us when we have to pay attention to the second term!

In particular, if we are talking about $\omega \to 0$ we find that

$$\beta_d \to (2\pi/v_d)[-j\,f_c] = -j\,(\omega_c/v_d) = \text{some constant}$$

In earlier work, I assumed that $\beta_d = (\omega/v_d) \to 0$ as $\omega \to 0$ and this was the cause of much of my trouble!

2. The limit of β' as ω → 0

[I conclude that $-j\omega\mu\sigma$ is good for any ω, but find a few peculiar things at *ultra* low ω like 10^{-9} Hz. I conclude here that basically $\beta'^2 \approx -j\omega\mu\sigma$ unless you go way down to 10^{-9} Hz !]

First, the Helmholtz parameter inside the conductor is given by

$$\beta^2 = \mu\varepsilon\omega^2 - j\omega\mu\sigma = \omega^2\mu\,(\varepsilon - j\sigma/\omega) = \omega^2\mu\,\xi \qquad \xi \equiv \varepsilon - j\sigma/\omega \qquad \text{parameter } \beta \qquad (1.5.1)$$

Keeping even the ε term (which we normally neglect), we compute

$$\beta'^2 = \beta^2 - \beta_d^2 \approx \mu\varepsilon\omega^2 - j\omega\mu\sigma - [(1/v_d)(\omega - j\omega_c)]^2$$

$$= \mu\varepsilon\omega^2 - j\omega\mu\sigma - (1/v_d^2)\,(\omega - j\omega_c)^2$$

$$= \mu\varepsilon\omega^2 - j\omega\mu\sigma - (\omega^2/v_d^2) + (\omega_c^2/v_d^2) + (2j\omega_c\omega/v_d^2)$$

If we go to very low but finite ω, we can drop both ω^2 terms and we get

$$\beta'^2 = -j\omega\mu\sigma + (2j\omega_c\omega/v_d^2) + (\omega_c^2/v_d^2) \quad = -j\omega\mu\sigma + (2j\alpha\omega/v_d) + \alpha^2$$

$$= [-j\mu\sigma + 2j\omega_c/v_d^2]\ \omega \ + \ (\omega_c^2/v_d^2) \qquad\qquad \textit{very} \text{ low } \omega \ > 0$$

t1 t2 t3

Pause: What are the relative sizes of terms here?

$$\mu\sigma = \mu\varepsilon\sigma/\varepsilon \ = (1/v_d^2)(\sigma/\varepsilon)$$

so in abs value,

$$t1/t2 = (1/v_d^2)(\sigma/\varepsilon) \ / \ 2\omega_c/v_d^2 \ = \ (\sigma/2\omega_c\varepsilon)$$

For our Belden cable example this ratio is

```
wc := evalf(2*Pi*fc);
```

$$wc := 48826.62181$$

```
sigma := 5.81e7;
```

$$\sigma := .581\ 10^{8}$$

```
epsilon := 2.3* 8.85e-12;
```

$$\varepsilon := .20355\ 10^{-10}$$

```
t1_over_t2 := sigma/(2*wc*epsilon),
```

$$t1_over_t2 := .2922929580\ 10^{14}$$

which I think I compute elsewhere in terms of something being 10^{18} Hz. So obviously we can drop the second term and have this for the low ω form of β'^2 (that is, the β^2 contribution is just $-j\mu\sigma\ \omega$)

$$\beta'^2 = \ -j\mu\sigma\ \omega + \ (\omega_c^2/v_d^2) \quad = -j\mu\sigma\ \omega + \ \alpha^2$$

Now you might ask at what frequency the two terms become equal in mag,

$$\mu\sigma\omega = \omega_c^2/v_d^2 \quad = \ \alpha^2$$

$$\omega\mu\varepsilon\sigma/\varepsilon = \omega_c^2/v_d^2$$

$$\omega\ \sigma/\varepsilon = \omega_c^2$$

$$\omega = \omega_c^2/(\sigma/\varepsilon)$$

I know from above (2.2.3) that for $\varepsilon = \varepsilon_0$,

$(\sigma/2\pi\varepsilon) = 1.04 \times 10^{18}$ Hz

And in our example case we have

$(\sigma/\varepsilon) = 0.3 \times 10^{19}$

so we then get equality of the terms above when

$$\omega = \omega_c^2/(\sigma/\varepsilon) = \omega_c^2 / [0.3 \times 10^{19}] = (48{,}836)^2 / [0.3 \times 10^{19}] = 0.8 \times 10^{-9}$$

which is a VERY low frequency indeed. Lower in fact that I would have expected. This last fact makes my VERY dubious that I am on the right track here. What this says is that for any reasonable low frequency like 1/10 Hz, we are going to have $\beta'^2 = -j\mu\sigma\,\omega$ with no correction to speak of.

[This says I can forget about that loss correction stuff where $\beta_d \rightarrow$ constant instead of 0.]

Resume: Finally, we go to $\omega \rightarrow 0$ to get [not sure what this is about]

$$\beta'^2 = + (\omega_c^2/v_d^2) = + [R_{dc}\, C\, v_d^2/2]^2/v_d^2 = [R_{dc}^2 C^2 v_d^2/4] = \alpha^2$$

$\beta'(\omega=0) = R_{dc} C\, v_d/2 = \alpha$ $\quad$ sec/m^2 * m/sec = 1/m = correct

The claim is that this is a "small number" because R_{dc} is supposedly small. Recall that

$$\omega_c = R_{dc}\, C\, v_d^2/2 = [R_{dc} C\, v_d/2]\, v_d = -j\,\beta'_0\, v_d$$

so

$$\beta'_0 = j\,(\omega_c/v_d) = j\,k_c = j(2\pi/\lambda_c)$$

Now λ_c is the wavelength at 8 KHz for our 8281 cable which is pretty large.

```
bp0 := I*(2*Pi*fc/vd): evalf(%);
                  .0002465991000 I
Lc := 2*Pi/abs(bp0): evalf(%);
                    25479.35214
```

Here you see that λ_c = 25 km and β'_0 = .00025 which does seem "small". Certainly λ_c is a lot larger than the transverse line dimensions, so we expect $J_m(\beta'_0 r)$ to be far in the small argument limit!!

3. What happens if we try $\omega \rightarrow 0$ on the Appendix D $E_z(r,m)$ expression in (D.2.33)?

[5.14.14: I see the Paradox here! As ω gets *very* small, it seems likely that φ = constant is valid for the two cylinders, and then the capacitor problem is valid, the η_m are valid. The limit $E_z(r,m) \neq 0$ for $m \neq 0$ is

correct and you end up at ω= 0 with J_z asymmetry! The only way out is the $Z_0 = \infty$ escape hatch! The problem is that n(θ) has the same shape at all ω including ω = 0. The Paradox still lives!]

That formula says

$$E_z(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\ (a\beta')\, f_m \qquad\qquad f_m = \left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right]$$

Looking at "very low freq... v 2" we get from our Maple program, for ω → 0,

$$f_m = (2/a\beta'_0)\,(m+1)(r/a)^m \qquad m > 0$$
$$f_0 = (4/a\beta'_0) \qquad m = 0$$

We then get

$$E_z(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\ (a\beta'_0)\, f_m(\omega=0)$$

$$= (1/4)\,\eta_m\, I\, R_{dc}\ (a\beta'_0)\,(2/a\beta'_0)\,(m+1)(r/a)^m$$

$$= (1/2)\,\eta_m\, I\, R_{dc}(m+1)(r/a)^m$$

and this is the Problem Child limit that we don't want!! We need to re-derive the field equations, and that is done in the next section.

4. New application of the boundary conditions **[** this leads nowhere **]**

Here are the two boundary conditions taken from below (D.2.27) :

$$a_m\, x_a^{-1}\, J_m(x_a) + \frac{K_m}{2}\, J_{m+1}(x_a) = (j\omega/\sigma)\,\eta_m N_0 \qquad (1)$$

$$-a_m\, x_a^{-1}\, J_m(x_a) + \left(\frac{K_m}{2} + \frac{a_m}{m}\right)\, J_{m+1}(x_a) = 0\ . \qquad (2)$$

If we now use the leading small x term $J_m(x) = (x/2)^m / m!$, we can write the small x_a limit of the two boundary conditions as:

$$a_m/2\ (\beta' a/2)^{m-1} / m! + \frac{K_m}{2}\ (\beta' a/2)^{m+1} / (m+1)! = (j\omega/\sigma)\, N_m \qquad (1) \text{ low } \omega$$

$$-a_m/2\ (\beta' a/2)^{m-1} / m! + \left(\frac{K_m}{2} + \frac{a_m}{m}\right)\ (\beta' a/2)^{m+1} / (m+1)! = 0\ . \qquad (2) \text{ low } \omega$$

Our purpose in writing this out is merely to show that, as ω → 0, the two left sides go to certain constant values, whereas the RHS of the first equation goes to 0. So very close to this limit, we write

$$a_m/2\ (\beta'_0 a/2)^{m-1}/m! + \frac{K_m}{2}\ (\beta'_0 a/2)^{m+1}/(m+1)! = (j\omega/\sigma)\,\eta_m N_0 \qquad (1)\ \text{low}\ \omega$$

$$-a_m/2\ (\beta'_0 a/2)^{m-1}/m! + \left(\frac{K_m}{2} + \frac{a_m}{m}\right)(\beta'_0 a/2)^{m+1}/(m+1)! = 0\ . \qquad (2)\ \text{low}\ \omega$$

$$N_0 = (\beta_d/2\pi\omega a)\,I = I/(2\pi a v_d) = \text{a constant} \qquad (D.2.31)$$

BUT here is a fly in the ointment. The last line should really say

$$N_0 = (\beta_d/2\pi\omega a)\,I = [(1/v_d)(\omega-\omega_c)]\,(I/2\pi\omega a) = (1/v_d)\,(I/2\pi a)\,(1-\omega_c/\omega)$$

and then

$$(j\omega/\sigma)\,\eta_m N_0 = (j\omega/\sigma)\,\eta_m\,(1/v_d)\,(I/2\pi a)\,(1-\omega_c/\omega) = (j/\sigma)\,\eta_m\,(1/v_d)\,(I/2\pi a)\,(\omega-\omega_c)$$

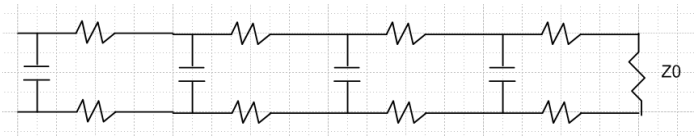
$$\rightarrow\ (j/\sigma)\,\eta_m\,(1/v_d)\,(I/2\pi a)\,(-\omega_c)$$

and then it does NOT vanish after all !! Ouch, tower of cards collapses again!

5. Pick up the ball and run a little more on March 20, 2014

[This is a good use of the network picture, it supports the idea that $Z_0 = \sqrt{R/j\omega C} \rightarrow \infty$]

Let's keep this picture in mind, where we have no conductance G and we are so low in ω that we ignore L.



The Z_0 is given at low ω by

$$Z_0 = \sqrt{R/j\omega C}$$

and yes, as $\omega \rightarrow 0$ it goes infinite. But let's keep ω small but non-zero. Then

$$I = V/Z_0 = V\sqrt{C/R}\,\sqrt{j\omega}$$

so this shows how current I is being shut down as Z_0 is increasing in the above picture. Here then are our fields with this current so expressed: (from very low freq v 2)

$$E_z(r,m) = (1/4)\,\eta_m\,I\,R_{dc}\,(a\beta)\,(2/a\beta)\,(m+1)(r/a)^m = (1/2)\,\eta_m\,I\,R_{dc}\,(m+1)(r/a)^m$$
$$E_r(r,m) = (j/4)\,\eta_m\,I\,R_{dc}\,(a\beta_d)\,g_m = (j/4)\,\eta_m\,I\,R_{dc}\,(-aj\alpha)\,(1/ar)\,(r^2+a^2)(r/a)^m$$
$$E_\theta(r,m) = (1/4)\,\eta_m\,I\,R_{dc}\,(a\beta_d)\,h_m = (1/4)\,\eta_m\,I\,R_{dc}(-aj\alpha)\,(1/ar)\,(r^2-a^2)(r/a)^m$$

Then for m = 0 we get instead, as $\omega \rightarrow 0$,

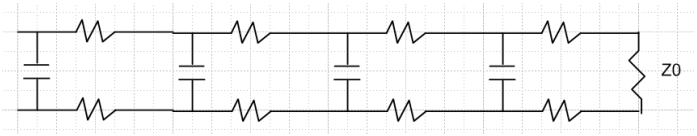
$E_z(r,0) = (1/4)\, I\, R_{dc}(a\beta)\, f_0 = (1/4)\, I\, R_{dc}(a\beta)\, (4/a\beta) = I\, R_{dc}$
$E_r(r,0) = (j/4)\, I\, R_{dc}\, (a\beta_d)\, g_0 = (j/4)\, I\, R_{dc}\, (a\beta_d) 2(r/a) = (j/4)\, I\, R_{dc}(-aj\alpha)2(r/a)$
$E_\theta(r,0) = (1/4)\, I\, R_{dc}\, (a\beta_d)\, h_0 = (1/4)\, I\, R_{dc}\, (a\beta_d)\, 2(r/a) = (1/4)\, I\, R_{dc}(-aj\alpha)2(r/a)$

Which I then write as

$E_z(r,m) = (1/2)\, \eta_m\, V\sqrt{C/R}\, \sqrt{j\omega}\, R_{dc}\, (m+1)(r/a)^m$
$E_r(r,m) = (j/4)\, \eta_m\, V\sqrt{C/R}\, \sqrt{j\omega}\, R_{dc}\, (-aj\alpha)\, (1/ar)\, (r^2 + a^2)(r/a)^m$
$E_\theta(r,m) = (1/4)\, \eta_m\, V\sqrt{C/R}\, \sqrt{j\omega}\, R_{dc}(-aj\alpha)\, (1/ar)\, (r^2 - a^2)(r/a)^m$

$E_z(r,0) = V\sqrt{C/R}\, \sqrt{j\omega}\, R_{dc}$
$E_r(r,0) = (j/4)\, V\sqrt{C/R}\, \sqrt{j\omega}\, R_{dc}(-aj\alpha)2(r/a)$
$E_\theta(r,0) = (1/4)\, V\sqrt{C/R}\, \sqrt{j\omega}\, R_{dc}(-aj\alpha)2(r/a)$

Now as we take the limit $\omega \rightarrow 0$, ALL these fields go to 0 !! This is what our physical experimental setup absolutely required! This limit involves I = 0, so there is nothing happening whatsoever at DC. You don't have a slowly decaying voltage going down the line. In this limit there is no voltage drop on any of these resistors in the picture,



so you basically have "one giant long capacitor" which is charged to V = 6 volts, and nothing happens. This is the ultimate DC limit of my theory!