Including Loss in the Appendix D Model PhL 3.17.14

Reviewed 5.14.14.

The main item here is a derivation of the decay parameter α for small ω:

$\beta_d \rightarrow (\omega/v_d) - j\ [(R_{dc1} + R_{dc2})C\ ]\ (v_d/2) = (\omega/v_d) - j\alpha$ $\alpha = (R_{dc1} + R_{dc2})C\ ]\ (v_d/2)$

This is then the replacement you should make in App D wherever β_d appears "solo". I then use this in other docs.

The second item here was done wrong, trying to get β'^2 for low ω. I repaired things and it now says $\beta'^2 \rightarrow \alpha^2$ which is a constant and agrees with elsewhere, but only for super super low ω is this relevant as I show in "conclusions on low freq".

Here we look at the power in a wave, and at its rate of loss per distance down the line.

We need to know how much power is in the wave in the first place. Maybe get that from L and C based on field energy storage.

$U_L = (1/2)L\ I^2$ L = per unit length U_L = energy/unit length

$U_C = (1/2)CV^2$

If the wave travels at v_D roughly speaking, then

P_{wave} = U(joules/m) * v_d (m/sec) = joules/sec = watts

Then we have

$P_{wave} = [(1/2)L\ I^2 + (1/2)CV^2\ ]\ v_d$

Since the current is uniform, I think we can argue that $K_L = K$ without using any King theory. Then let's just use our standard forms (ignoring L_i for the moment)

$C = 4\pi\varepsilon/K$ $L_e = \frac{\mu}{4\pi} K$

then we have so far

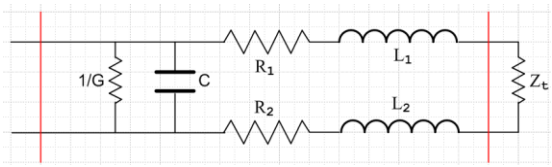
$P_{wave} = [(1/2)\frac{\mu}{4\pi} K\ I^2 + (1/2)\ 4\pi\varepsilon/K\ V^2\ ]\ v_d$

Now write this for the rate of power loss going down the line,

$$dP_{wave}/dz = [\frac{\mu}{4\pi} K I\, dI/dz + 4\pi\varepsilon/K\, V\, dV/dz\,]\, v_d$$

Now I can see that $dV/dz = -IR_{dc}$ (low ω) in each conductor, so perhaps $dV/dz = -I(R_{dc1} + R_{dc2})$.

But I don't see how I can decrease! Current in a resistor does not decrease with length. But this seems to conflict with $I = \exp(j\omega t - j\beta_d)$ if β_d has a complex part. Is this the AC vs DC issue? Well think of



Current is shunted through C, so even without G, I can reduce to do this loss through C. How much current goes through C? $Z_C = 1/(j\omega C)$ and So $dI(z) = -V(z)/Z_c = -V(z)\, j\omega C dz$ so

$$dI/dz = -j\omega C\, V(z) \qquad \text{where C is per unit length}$$

OK, then I get

$$dP_{wave}/dz = -[\frac{\mu}{4\pi} K I\, j\omega C\, V(z) + 4\pi\varepsilon/K\, V\, I(R_{dc1} + R_{dc2})\,]\, v_d$$

$$= -I\, V\, [\frac{\mu}{4\pi} K\, j\omega C + 4\pi\varepsilon/K\ (R_{dc1} + R_{dc2})\,]\, v_d$$

$$= -I\, V\, [\frac{\mu}{4\pi} K\, j\omega C + C\ (R_{dc1} + R_{dc2})\,]\, v_d$$

$$= -I\, V\, C\, [\frac{\mu}{4\pi} K\, j\omega + (R_{dc1} + R_{dc2})\,]\, v_d$$

$$= -P_{wave}\, C\, [\frac{\mu}{4\pi} K\, j\omega + (R_{dc1} + R_{dc2})\,]\, v_d$$

So as $\omega \rightarrow 0$ this says (in this limit, there is in fact no dI/dz so first term is 0)

$$dP_{wave}/dz = -P_{wave}\, [(R_{dc1} + R_{dc2})C\,]\, v_d$$

For units, we have to show that [...] $v_d = 1/m$, but $RC = sec/m^2 * m/sec = 1/m$ so OK.

Now go back to

$$I(z) = I_0 \exp(-jkz)$$
$$V(z) = V_0 \exp(-jkz)$$

$$P(z) = P_0 \exp(-2jkz) = P_0 \exp(-2j[k_R + jk_I]z) = P_0 \exp(-2jk_R z) \exp(+2k_I z)$$

$$|P(z)| = P_0 \exp(+2k_I z)$$

$$d|P(z)|/dz = 2k_I P_0$$

Compare this to

$$dP_{wave}/dz = - P_{wave} [(R_{dc1} + R_{dc2})C] v_d$$

and we conclude that

$$2k_I = - [(R_{dc1} + R_{dc2})C] v_d \qquad \text{// units} = (\text{sec}/\text{m}^2)*(\text{m}/\text{sec}) = 1/\text{m}, \text{ correct}$$

This is the first time (I think) that I have ever attempted this estimate of loss for low ω.

The implication is that the wave is really doing this:

$$\exp(-jkz) \qquad \text{where} \qquad k \approx \beta_d + jk_I = \beta_d - j [(R_{dc1} + R_{dc2})C] (v_d/2)$$

This means that in Appendix D right from the start we should replace

$$\beta_d \rightarrow \beta_d - j [(R_{dc1} + R_{dc2})C] (v_d/2) \qquad \text{[this is the main point here]}$$

or

$$\beta_d \rightarrow (\omega/v_d) - j [(R_{dc1} + R_{dc2})C] (v_d/2) \qquad [\beta_d \rightarrow (\omega/v_d) - j\alpha \text{ , note def of } \alpha]$$

at least for very low ω, since this is what the line is really doing !!! Now we always think of the imaginary loss part as being small compared to the real β_d part. BUT, this is not true as ω → 0 as this last equation makes clear!!! As ω → 0, we have "all loss and no wave". I think this might change the low ω limit of my Appendix D fields and perhaps resolve that conundrum. Notice that

$$\beta_d^2 \rightarrow [(\omega/v_d) - j [(R_{dc1} + R_{dc2})C] (v_d/2)]^2$$

$$= (\omega/v_d)^2 - [(R_{dc1} + R_{dc2})C]^2 (v_d/2)^2 - 2j [(R_{dc1} + R_{dc2})C] (v_d/2) (\omega/v_d)$$

$$[= (\omega/v_d)^2 - \alpha^2 - 2j\alpha (\omega/v_d)]$$

$$\beta'^2 = \beta^2 - \beta_d^2$$

$$\rightarrow \quad -j\omega\mu\sigma - (\omega/v_d)^2 + [(R_{dc1} + R_{dc2})C]^2 (v_d/2)^2 + 2j [(R_{dc1} + R_{dc2})C] (v_d/2) (\omega/v_d)$$

$$[= -j\omega\mu\sigma - (\omega/v_d)^2 + \alpha^2 + 2j\alpha(\omega/v_d)]$$

Now we take ω → 0 we get

$$\beta'^2 \rightarrow [(R_{dc1} + R_{dc2})C]^2 (v_d/2)^2 \quad [= \alpha^2]$$

whereas it used to go to 0 ! If resistances are low, perhaps we have $[(R_{dc1}+ R_{dc2})C] (v_d/2) << 1$ so we can then approximate

[I had errors above, fixed them on 5.15.14, new conclusion is just]

$\beta'^2 \rightarrow \quad [(R_{dc1}+ R_{dc2})C]^2 (v_d/2)^2 \qquad [= \alpha^2] \qquad C = 4\pi\varepsilon/K$

[Hold the phone here: this disagrees with what I say in my 3.20.14 "Conclusions on low freq" doc. OK, I had errors above and they are now fixed and things agree. The above limit is for $\omega < 10^{-9}$ Hz !]]

OK, then what happens to this

$$E_z(r,m) = (1/4)\, \eta_m\, I\, R_{dc}\, (a\beta') \left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right]$$

$$f_m = \left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right]$$

Notice that small R_{dc} implies small β' but also small δ . But I still don't see a nice way to claim that the fields $E_z(r,m) \rightarrow 0$ as $\omega \rightarrow 0$. I know that $x \rightarrow 0$ does not do it, and $x \rightarrow \infty$ involves this

10.7(ii) $z \rightarrow \infty$

When ν is fixed and $z \rightarrow \infty$,

10.7.8

$$J_\nu(z) = \sqrt{2/(\pi z)}\left(\cos\left(z - \tfrac{1}{2}\nu\pi - \tfrac{1}{4}\pi\right) + e^{|\Im z|}\, o(1)\right),$$

This is a very poorly defined limit and I cannot believe it would apply.

[happily I stopped here. This stuff is done more correctly in "conclusions on low freq limit ".]