**Low Freq Status** PhL 5.16.14

One of my many painful "reviews" of the low-ω asymmetry problem, this one May 2014. Below I argue that averaging in Ch 4 makes everything OK for small ω, but Appendix S later shows this is not true. Below I also develop the idea from div E = 0 that $J_z$ tracks n(θ) and *that* is where the aysm is coming from. I now think it all has to do with the $e^{-jkz}$ ansatz. (10.8.14)

Yesterday I finished reviewing all the docs in the low freq folder. I am still unhappy about the fact that asymmetry of $J_z$ seems to persist as ω→0 despite the escape hatch of I→0. Maybe I can come up with "something new" after doing all that reviewing.

**Question #1**: Which parts of "the theory" are valid, and which are not, at low ω?

At very low ω, one thinks that φ = constant would be pretty solid, though I cannot prove how high ω could go. Certainly in the ω→0 limit φ = constant is good. And we know for sure that $A_z \neq$ constant at low ω. If we look at Chapter 4, I think it is all valid through Sec 4.9. It is not until Sec 4.10 that W(z) appears. I updated lines doc just now to interpret $L_e = \langle L_e(\mathbf{x_1},\mathbf{x_2}) \rangle$ which rescues $W(z) = L_e\ i(z)$

So now we start marching down Section 4.11. I will quote equations which are not valid at low ω:

$$\partial_z A_z(\mathbf{x}) = -\,j\,(\beta^2/\omega)\,\varphi(\mathbf{x})\ . \qquad \text{maybe?} \qquad (4.11.3)$$

I am just not sure whether we can drop the rest of div A. Now we follow down to

$$E_z(\mathbf{x_1}) - E_z(\mathbf{x_2}) = -\,\partial_z V - j\omega W \qquad (4.11.6a)$$

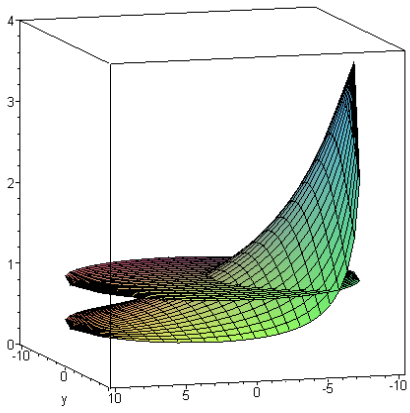
$$\partial_z W = -\,j\,(\beta^2/\omega)\,V\ . \qquad (4.11.6b)$$

But I rescue these equations using my averaging trick, so that ought be good even at low ω where I know for example that W ≠ constant on the conductors! So maybe my averaging trick rescues everything at low $A_z$ !! I get right down to the classic line equations, and again averaging fixes everything!!! You just have to interpret ALL quantities like C, $L_e$, K, $K_L$, $Z_0$ as involving these averages. That is nice because the "network model" does this same thing implicitly! For example, R would be the current averaged resistance. [ maybe compute this somewhere in an example ***** ]

**Fact**: My entire theory is rescued by the current-averaging process and therefore applies at low ω !

WRONG! Averaging does not remove the need to maintain the transverse terms like $\partial_x A_x$ . Appendix S shows how this works now with T(z). So there really IS a low-ω assumption that averaging does not make go away! (10.8.14)

**Question #2:** In the eddy current approach, $J_z$ asymmetry must go away as ω→0. If I have an infinitely long transmission line, as ω → 0 it is true that $Z_0 \to \infty$ and this $I = V/Z_0 \to 0$. But suppose we look for some very small ω where there is then some very small current I and very large $Z_0$ . How do I explain that for such a small ω, where eddy currents are very small, that I get this huge asymmetry:




The problem is that my theory causes $J_z$ to track n(θ), and n(θ) does not depend on ω. I address this issue in Section 6.5 (d) using the divE = 0 equation. Here is the logic flow concerning this tracking:

$\partial_r (r\, E_r(r,\theta)) + \partial_\theta E_\theta(r,\theta) + r\, \partial_z E_z(r,\theta) = 0$ .

$\partial_r (r\, E_r(r,\theta)) + r\, \partial_z E_z(r,\theta) \approx 0$ // near r = a " near surface"

$\partial_r (r\, E_r(r,\theta)) - j\beta_d\, r\, E_z(r,\theta) \approx 0$ // using $\partial_z \rightarrow -j\beta_d$, see (D.1.16) "wave"

$E_z(r,\theta) \approx (1/j\beta_d)\,(1/r)\,\partial_r (r\, E_r(r,\theta))$ . // near r = a (6.5.17)

$E_z(r) = E_z(a)\,\dfrac{J_0(\beta r)}{J_0(\beta a)}$ . // From Appendix D *(2.2.29)*

Up to this point I have assumed nothing about ω. I continue,

$E_r(r,\theta) \approx E_r(a,\theta)\sqrt{\dfrac{a}{r}}\; e^{(r-a)/\delta}\; e^{j(r-a)/\delta}$ // assumption of large ω (6.5.18)

$E_r(a,\theta) = (j\omega/\sigma)\, n(\theta)$ the cpbc

$E_z(a,\theta) = (1/ja\,\beta_d)\,[(1+j)a/\delta\,]\,(j\omega/\sigma)\, n(\theta)$ tracking result (6.5.20)

I have no real support for my assumption in red. But suppose instead I use Appendix D,

$E_r(r,m) = (j/4)\,\eta_m\, I\, R_{dc}\; (a\beta_d)\, g_m$

I could now look at this for both large ω and for small ω. I have never computed the large ω limit of $g_m$ but I could easily do that and I should do that! DONE. And my red assumption above is vindicated!

Now suppose instead we do the low ω limits. Then from "very low freq v2" I quote

$$E_z(r,m) = (1/2)\,\eta_m\, I\, R_{dc}\,(m+1)(r/a)^m$$
$$E_z(r,0) = I\, R_{dc}$$

$$E_r(r,m) = (j/4)\,\eta_m\, I\, R_{dc}\,(a\beta_d)\,[(r/a)^{m+1} + (r/a)^{m-1}]$$
$$E_r(r,0) = (j/2)\, I\, R_{dc}\,(r\beta_d)$$

$$E_\theta(r,0) = (1/4)\,\eta_m\, I\, R_{dc}\,(a\beta_d)\,[(r/a)^{m+1} - (r/a)^{m-1}]$$
$$E_\theta(r,0) = (j/2)\, I\, R_{dc}\,(r\beta_d) \qquad (*)$$

Now I think things will be different! The div E equation stated above is this in m space,

$$E_z(r,m) \approx (1/j\beta_d)\,(1/r)\,\partial_r\,(r\,E_r(r,m))$$

where for $r \approx a$ we see the cancelling minus sign that sort of removes $E_\theta$.

But now I have explicit forms for $E_z$ so why not just use them. In fact, let's back up and quote these results from "very low freq 2"

$$f_m = (2/a\beta)\,(m+1)(r/a)^m \qquad m > 0$$
$$f_0 = (4/a\beta) \qquad m = 0 \qquad\qquad \beta = e^{3\pi j/4}\,\sqrt{\sigma\mu}\;\;\omega^{1/2}$$

$$g_m = (1/ar)\,(r^2 + a^2)(r/a)^m = (r/a)^{m+1} + (r/a)^{m-1} \qquad m > 0$$
$$g_0 = 2(r/a) \qquad m = 0$$

$$h_m = (1/ar)\,(r^2 - a^2)(r/a)^m = (r/a)^{m+1} - (r/a)^{m-1} \qquad m > 0$$
$$h_0 = 2(r/a) \qquad m = 0$$

Recall from (6.5.13) that

$$E_z(r,\theta) = I\, R_{dc}\,(\beta a/4)\,[\; f_0(r) + 2\,\Sigma_{m=1}^{\infty}\,(-1)^m\, e^{-m|\xi_1|}\, f_m(r)\cos(m\theta)\;]$$

So we can just install our results to get

$$E_z(r,\theta) = I\, R_{dc}\,(\beta a/4)\,[\;(4/a\beta) + 2\,\Sigma_{m=1}^{\infty}\,(-1)^m\, e^{-m|\xi_1|}\,(2/a\beta)\,(m+1)(r/a)^m\cos(m\theta)\;]$$

$$= I\, R_{dc}\;\;[\;1 + \Sigma_{m=1}^{\infty}\,(-1)^m\, e^{-m|\xi_1|}\,(m+1)(r/a)^m\cos(m\theta)\;]$$

and now we do NOT get a simple image of $n(\theta)$, though we do get asymmetry still. Do I have a plot of this already somewhere? Well his is in fact just what I show above from "very low freq 1"

**Question #2.5.** Look at the equations (*). As $\omega \to 0$, we get $E_r = E_\theta = 0$. So if these fields are zero, how can there be some violently varying $E_z$ field?

Answer: We have from (*) that $E_r = \omega\ e_r$ where $e_r$ is not small. Then we get

$$E_z(r,\theta) \approx (1/j\beta_d)\,(1/r)\,\partial_r\,(r\,E_r(r,\theta)) = (1/\omega) * \text{constant} * (1/r)\,\partial_r\,(r\,\omega\,e_r(r,\theta))$$

and the two ω factors cancel each other resulting in $E_z$ still being influenced by $E_r$ at full strength.

**A Fast Path to Asymmetry at low ω**

1. I believe that my Appendix D is valid for all ω, as long as you use the correct thing for β and $\beta_d$ which go into β'.

2. What I have done above is this: I have evaluated the Appendix D electric fields at small β', making no assumptions about the nature of β' other than to assume that β'a << 1 for low ω. This then gave

$$E_z(r,\theta) = I\,R_{dc}\ \left[\ 1 + \Sigma_{m=1}^{\infty}\,(-1)^m\,e^{-m|\xi_1|}\,(m+1)(r/a)^m\cos(m\theta)\ \right]$$

which exhibits strong $J_z$ asymmetry!

**Question #3**. Is it really true that β'a << 1 for low ω ? In the skin effect regime we have β'a >> 1 .

Note: Assuming β'a << 1, we find the second line saying $E_z(r,0) = I\,R_{dc}$ . This says that the non-θ dependent term in $E_z(r,\theta)$ is $I\,R_{dc}$ which is correct.

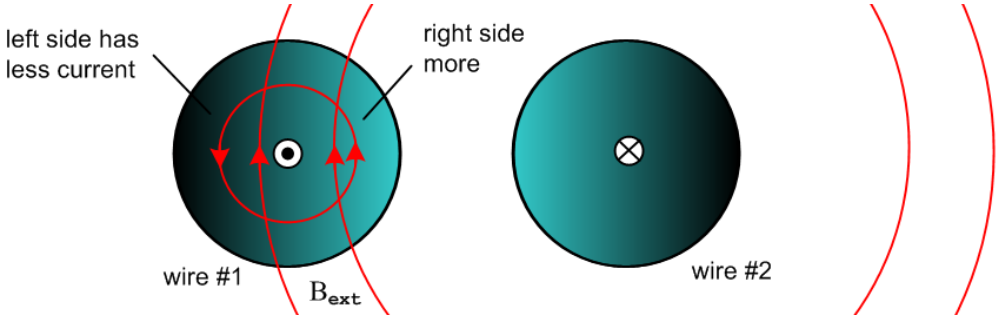
**Comment**: Regardless of the answer to Question #3, we have this result from Appendix D:

$$E_z(r,m) = (1/4)\,\eta_m\,I\,R_{dc}\ (a\,\beta')\,f_m(r\,\beta', a\,\beta')$$

$$E_z(r,0) = (1/4)\,I\,R_{dc}\ (a\,\beta')\,f_0(r\,\beta', a\,\beta')$$

The only way to avoid asymmetry as $\omega \to 0$ is to have the first line vanish and not the second line. But this just doesn't happen, assuming the $\eta_m$ don't somehow vanish.

**Eddy Current Argument**. If we look at our basic picture



then we see where B is strong and weak and that causes the asymmetry. As $\omega \rightarrow 0$, we get weaker B on both sides and then it just goes away. *But* the zeroth order current remains!! The eddy current contribution maintains its asymmetry all the way down to $\omega = 0$, but the amount of eddy current drops off relative to the base current, so you get a gradual diminishing of the asymmetry in this world!!