Low Frequency Limit of My Theory PhL started notes 3.10.14

Review on 5.15.14.

Section 1 I don't like much, not practical and things are wrong. Just not helpful.

Section 2 is a massive web search on proximity effect and a library trip. At the time, I thought there might be a DC proximity effect, but now I am sure there is no such thing despite ambiguous statements of Paul (an author) and various websites. This is now embodied in my Appendix P on eddy currents.

Section 3 is a short abortive attempt but it got me thinking about eddy currents.

In **Section 4** I give my argument for why J_z is uniform in the two cylinders at DC. This does however bring up a topic which later becomes the radial Hall effect of Appendix N. I go on to do some eddy current theory discussion and that is now in Appendix P. Then I wander off on things that just don't seem useful so I ignore them in retrospect.

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Mar 10, 2014

last update 3.12.14

1. Statement of the Problem and my Initial Response

I have studied the Appendix D E and B fields as ω → 0 and results for m ≠ 0 are all weird. Here is the problem: at normal RF line frequencies, I *expect* to have an asymmetric J_z inside conductors. A quick argument in favor of this is that the E and B fields on such a line are, we feel, strong right between the conductors and weak elsewhere. Here we think of shooting a wave down the line and the currents J_z are just induced by the field pattern. As the charge pattern "dances down the surfaces" I expect this creates a radial current J_r to feed the charge pattern. I know the B field pattern and I could use the curl B = J crude rule to show there must be J_z inside. I expect this J_z to be large near the "gap" and small far from the

"gap". So I am happy to have an asymmetric E_z in this situation, I *want* to have asymmetry in fact. This means that E_z varies around the perimeter. ok

Now what happens in the above "description" as we lower ω? The η_m stay the same. The skin depth increases and becomes larger than the conductors. I expect the E outside pattern to stay the same since it is basically caused by the charge density which I claim stays the same. So imagine that the E pattern stays the same and consider

curl **E** = -jω**B**

so

B = 1/(-jω) curl **E** this next section I think is wrong, just skip it

This would SEEM to determine **B** in the dielectric just from the **E** pattern. Away from surfaces, we expect **E** and curl **E** to be finite. So this DOES seem to imply that **B**→∞ as ω→0. [but that is wrong since in fact curl E is proportional to ω even though E is constant in ω.]

So let us reconsider the idea that **E** does not change with ω. Let's just suppose we have some FIXED driving potential V. [ok] The line has some known capacitance that does not vary with ω we shall say, [ok] so fixed V means fixed q on each conductor [ok]. I suspect somehow that in order to maintain this fixed V, we are going to have I → ∞. [wrong] OK, here is how that might work: you see from above that **B** gets larger without limit. I think we can ignore Debye surface currents, so I think assuming all μ the same that B passes right through the surface. Once you have the pattern of B just inside the surface, that must somehow determine I. I don't know offhand exactly how that works, but I can see that if B → ∞, then probably I → ∞ and that is then the explanation! [but B not → ω, see below] Probably the Z_0 goes to 0 ohms or something like that.

There is a vector Helm equation for **B** inside the wire:

$$(\nabla^2+\beta^2)\mathbf{E} = 0 \qquad (\nabla^2+\beta^2)\mathbf{B} = 0 \qquad \text{// region 3} \qquad (1.5.27)$$

where β is the usual inside conductor quantity. so maybe we have this problem:

$$(\nabla_{2d}^2+\beta'^2)\mathbf{B}(r,\theta;\omega) = 0 \qquad \mathbf{B}(a,\theta;\omega) = \text{prescribed} \qquad \text{// vector Helm!}$$

We solve this somehow, then the inside current is given by

curl **H** = $\partial_t\mathbf{D} + \mathbf{J} \approx \mathbf{J}$ inside

[But Appendix D has already solved the Helm equation for **B** so I should be able to use its results]

(a) The first Set of Steps **[** this section does not seem useful, and is wrong **]**

Program for Computing things for two cylinders transmission line. [WRONG]

1a. Compute the potential which is the solution of this system.

$$\varphi(x,y,z) = \frac{1}{4\pi\varepsilon_d}\, q(z)\, \varphi_t(x,y) \qquad \text{// non-conducting dielectric to make simple}$$

$$[\nabla_t^2 + (\beta_d^2 - k_\varphi^2)]\, \varphi(x,y) = 0 \qquad \varphi_t(C_1) = V_1(z) \qquad \varphi_t(C_2) = V_2(z) \qquad (5.1.8)$$

where for low loss we think $(\beta_d^2 - k_\varphi^2) \approx 0$ so this is just the Laplace capacitor problem. [ok]

1b. Then compute the **E** field, then the surface charge, then the moments η_m. I have done this completely in bipolar doc. This assumes some potential $V(z) = V_0 \exp(-j\beta_d z)$. Thus, we have a fixed potential scale from the start, and the **E** field will be proportional to V_0. [ok]

2. Compute the **B** field everywhere in the dielectric from **B** = 1/(-jω) curl **E** [ok]. Note that for fixed V_0, the **E** field does not change with ω and so **B** *blows up* as $\omega \to 0$. [wrong]

3. We know the **B** field on a conductor surface so we have a Helmholtz Dirichlet Problem:

$$(\nabla_{2d}^2 + \beta'^2)\, \mathbf{B}(r,\theta;\omega) = 0 \qquad \mathbf{B}(a,\theta;\omega) = \text{prescribed}$$

Solve this (in theory at least) for **B**(r,θ; ω) everywhere inside the conductor. [ok, in theory. In practice, Appendix D already solved this problem I think in terms of those moments]

4. Compute the current distribution inside the conductor using

$$\mu\mathbf{J} = \text{curl } \mathbf{B} \quad \text{[ok]}$$

5. Compute I in the conductor by integrating J_z obtained in item 4. [ok]

6. As $\omega \to 0$, we expect from step 2 that $\mathbf{B} \to \infty$ in scale, and thus $\mathbf{J} \to \infty$ and $I \to \infty$, again all this being for fixed V_0. [wrong because B does not go to ∞]

Question: But how can $I \to \infty$ if the conductors have resistance? [good question]

[Note: I am only writing $I \to \infty$ because my error above says $B \to \infty$]

Remember that the line has a load which matches its impedance and probably $Z_0 \to 0$ but I am not sure. In any event, as I increases, eventually you get a huge loss and V_0 cannot be maintained very far in z, and then you don't really have a useful wave. [ok] Here is what happens: as $\omega \to 0$, the transmission line terminated in a short is just a resistor [maybe], and V(z) decreases linearly from V_0 at the source end to 0 at the termination end. At each point along the way, it is still a capacitor, and there is still surface charge q that linearly drops off as you go down the line! [ok I think] This DC charge is NOT explained by the Hall effect for each conductor being in the other conductors B field. That creates a tiny charge that is neutral in each conductor and looks like +.....- -.....+. [ok] It is the capacitor charge that is still there, but it drops off linearly as you go down the line. [ok] It has to be there because you have V_o being + on one side and - on the other. So this seems to me to be a reasonable limit of the situation as $\omega \to 0$. Somehow my six-point plan shown above does not get to this limit. Right at the driving end we ought to

have V_0 in full and thus E in full and thus $\mathbf{B} = 1/(-j\omega)$ curl $\mathbf{E}$ as being very large. But it can't really be large because I is limited! So something is missing in the above model . [one thing missing is that I need to actually compute curl **E** and see what it looks like]

One thing I know is missing is this notion in Chap 5 transverse problem:

$$[\nabla_t^2 + (\beta_d^2 - k_\varphi^2)]\, \varphi_t(x,y) = 0 \qquad (5.1.8)$$

$$[\partial_z^2 + k_\varphi^2]\, q(z) = 0 \ . \qquad q(z,t) = q(0)\, e^{j(\omega t - k_\varphi z)} \qquad (5.1.9)$$

Something MUST happen to the transverse equation when k_φ gets a part $-j\kappa$ so q(z,t) decays. Ignoring the dielectric loss we have $\beta_d^2 = (1/v_d)^2\, \omega^2$. I don't know how to deal with this right now, but my guess is that the loss stuff will in fact reduce φ and reduce E in the dielectric, so B does not then get so big after all as we move to the limit. [an interesting "way out" idea from the perceived ∞B problem. This is a little "spin off question" that I started (scratch paper) to pursue in a 1D example and I found that yes φ does seem to change. But I stopped going down that path because I think my confusion still exists even with very low loss lines.]

Mar 11, 2014 [here I seem to repair the B→∞ problem]

As you go to higher ω, where skin effect sets in, same current has to go through sheath, so J_z is larger and you would think that I^2R losses would increase, not decrease. [web sites confirm this fact and it is an engineering concern.] But in medium ω, we don't have a skin effect and I is already "fully spread out" in the conductor, ignoring partial wave contributions. I have never tried to compute the loss stuff including the complex part of k_φ because I know this is not easy to do in terms of potentials. Pozar spends a lot of time on this subject I recall, I should eventually do something more than I have already done. [spinoff question] You would think that as ω decreases below mid range, the I^2R loss would stay the same. It is true that λ gets longer, so loss per wave then increases. But $k_\varphi z$ involves loss per meter, and you would think that then does not change for such low ω getting lower.

Now, once current is already spread out for lowish ω in the conductor, you would not think loss increases per meter as you go to still lower ω. [seems reasonable] So my k_φ then does not get worse in loss sense, so the φ solution does not change, and the electrostatic problem stays the same, so we carry out steps 1a and 1b and 2 above and we end up with that [wrong] problem of B → ∞ as ω→ 0 and then I→∞, so something is wrong with this set of steps. So something is wrong just with my first two steps!

Let's look at curl E in detail in cylindricals: [its about time] This computation is IN THE DIELECTRIC and we will quickly go close to one of the conductors:

$$\text{curl}\ \mathbf{E} = \hat{\mathbf{r}}\,[\, r^{-1}\partial_\theta E_z - \partial_z E_\theta] + \hat{\boldsymbol{\theta}}\,[\partial_z E_r - \partial_r E_z] + \hat{\mathbf{z}}\,[\, r^{-1}\partial_r(rE_\theta) - r^{-1}\partial_\theta E_r\,]$$

and install $\partial_z = -j\beta_d$

$$\text{curl}\ \mathbf{E} = \hat{\mathbf{r}}\,[\, r^{-1}\partial_\theta E_z + j\beta_d E_\theta] + \hat{\boldsymbol{\theta}}\,[-j\beta_d E_r - \partial_r E_z] + \hat{\mathbf{z}}\,[\, r^{-1}\partial_r(rE_\theta) - r^{-1}\partial_\theta E_r\,]$$

Now use the exact equation $\mathbf{B} = 1/(-j\omega)$ curl $\mathbf{E}$ to obtain these three exact equations:

$-j\omega B_r = r^{-1}\partial_\theta E_z + j\beta_d E_\theta$
$-j\omega B_\theta = -j\beta_d E_r - \partial_r E_z$
$-j\omega B_z = r^{-1}\partial_r(rE_\theta) - r^{-1}\partial_\theta E_r$ // in the dielectric just above surface

where we are using cyl coords of left wires say. To simplify things, set $E_z = 0$ relative to transverse components in the dielectric, [justification? At boundary, E_z(dielectric) = E_z(conductor) and we expect for a very good conductor to have E_z(conductor) ≈ 0, though of course not exactly so.]

$-j\omega B_r = j\beta_d E_\theta = j(\omega/v_d)\, E_\theta$
$-j\omega B_\theta = -j\beta_d E_r = -j(\omega/v_d)\, E_r$
$-j\omega B_z = r^{-1}\partial_r(rE_\theta) - r^{-1}\partial_\theta E_r$ // in the dielectric just above surface

curl $\mathbf{E} = \hat{\mathbf{r}}\,[j\beta_d E_\theta] + \hat{\boldsymbol{\theta}}\,[-j\beta_d E_r] + \hat{\mathbf{z}}\,[\, r^{-1}\partial_r(rE_\theta) - r^{-1}\partial_\theta E_r\,]$

Now let's go "close to the left wire" in which region we expect E_θ = small, so get

$-j\omega B_r = 0$ // note NO NORMAL B COMPONENT at surface
$-j\omega B_\theta = -j(\omega/v_d)\, E_r \Rightarrow B_\theta = (\omega/v_d)E_r$
$-j\omega B_z = - r^{-1}\partial_\theta E_r$ // in the dielectric just above surface

curl $\mathbf{E} = \hat{\boldsymbol{\theta}}\,[-j\beta_d E_r] + \hat{\mathbf{z}}\,[- r^{-1}\partial_\theta E_r\,]$ $\mathbf{B} = 1/(-j\omega)$ curl $\mathbf{E}$ close to wire

I feel that the $\hat{\boldsymbol{\theta}}$ is the main term here (no reason given other than intuition of picture, maybe expect small radial E_r at low ω), so

$\mathbf{B} = 1/(-j\omega)$ curl $\mathbf{E} = 1/(-j\omega)[\,[-j\beta_d E_r]\,\hat{\boldsymbol{\theta}} = (\beta_d/\omega)\, E_r\hat{\boldsymbol{\theta}} = (1/v_d)\, E_r\, \hat{\boldsymbol{\theta}}$

So my conclusion that $B_\theta \to \infty$ is then completely 100% wrong! [hurray!] Basically

curl $\mathbf{E} = \hat{\boldsymbol{\theta}}\,[-j\beta_d E_r] = \hat{\boldsymbol{\theta}}\,[-j(\omega/v_d)\, E_r]$

and when you set curl $\mathbf{E} = -j\omega\mathbf{B}$ the ω factors cancel. [β_d is just the wave number v = ω/k in a low loss situation]

(b) The second Set of Steps

So let's make a **new set of steps!** [focus is on **B** so can do $\mu\mathbf{J}$ = curl **B,** but intractable]

1a. Compute the potential which is the solution of this system.

$$\varphi(x,y,z) = \frac{1}{4\pi\varepsilon_d}\, q(z)\, \varphi_t(x,y) \qquad \text{// non-conducting dielectric to make simple}$$

$$[\,\nabla_t^2 + (\beta_d^2 - k_\varphi^2)]\,\varphi(x,y) = 0 \qquad \varphi_t(C_1) = V_1(z) \qquad \varphi_t(C_2) = V_2(z) \qquad (5.1.8)$$

where for low loss we think $(\beta_d^2 - k_\varphi^2) \approx 0$ so this is just the Laplace capacitor problem.

1b. Then compute the **E** field, then the surface charge, then the moments η_m. I have done this completely in bipolar doc. This assumes some potential $V(z) = V_0 \exp(-j\beta_d z)$. Thus, we have a fixed potential scale from the start, and the **E** field will be proportional to V_0.

1c. Then compute curl **E** and find that near a conductor (we ignore the z component for now)

$$\text{curl } \mathbf{E} = \hat{\boldsymbol{\theta}}\,[-j(\omega/v_d)E_r] + \hat{\mathbf{z}}\,[-\,r^{-1}\partial_\theta E_r\,] \qquad [\,\text{curl } \mathbf{E}\,]_t = (-j\omega)\,(1/v_d)\,E_r\,\hat{\boldsymbol{\theta}}$$

2. Compute the **B** field everywhere in the dielectric from $\mathbf{B} = 1/(-j\omega)$ curl **E** . The result is

$$B_\theta = 1/(-j\omega)\,[\,\text{curl } \mathbf{E}\,]_\theta = (1/v_d)\,E_r \qquad \text{// other B components} \approx 0$$

Notice that B_θ does not vary with ω as $\omega \to 0$. But it does vary with θ since E_r varies with θ.
[This B_θ result at the surface is exact to the extent that we have set $\partial_r E_z = 0$ inside the conductor.]

3. We know the **B** field on a conductor surface so we have a Helmholtz Dirichlet Problem:

$$(\nabla_{2d}^2 + \beta'^2)\,\mathbf{B}(r,\theta;\,\omega) = 0 \qquad \mathbf{B}(a,\theta;\,\omega) = \text{prescribed}$$

Solve this (in theory at least) for **B**(r,θ; ω) everywhere inside the conductor. [but remember this is a vector Helmholtz so B_θ and B_r get entangled as shown in (D.1.20) . Since I claim to have already solved this Helm equation in Appendix D, there should be no reason to do it again!]

4. Compute the current distribution inside the conductor using [ok, the plan at least seems good]

$$\mu\mathbf{J} = \text{curl } \mathbf{B}$$

5. Compute I in the conductor by integrating J_z obtained in item 4.

So, with this new set of steps, as $\omega \to 0$, we go deep into the electrostatic limit, and all fields become constant (independent of ω) values as shown above. So I have repaired the $B = \infty$ catastrophe at low ω.

(c) Physical Experiment Description as ω → 0

So imagine our test transmission line with two round conductors driven by $V_0 = 6$ volts and we just keep lowering ω and look at the piece of line close to the driving end (it is a long line). Step 1a tells us there is a certain E_r near the left conductor surface that is proportional to V_0. I must believe that the charge does

not "run away down the line" and we have our usual q(z) at very low ω. As ω → 0, this charge becomes static and q(z) right next to the driving end is, from Q = CV, equal to q(z) = C V_0 and you could in theory measure the charge, positive on left, negative on right. As we move down the line at DC, V(z) drops off linearly due to the IR drop, and E_r and B_θ decrease along the line, and everything seems OK (finally!).

[I am still uncomfortable that maybe that charge will "run away" at DC, but we continue on]

Question: Is J_z symmetric or asymmetric in this limit inside the left wire? [the Big Question]

Assuming closely spaced fat conductors, I know that $E_r(\theta)$ has strong θ variation. This

$$B_\theta(\theta) = (1/v_d)\, E_r(\theta)$$

has strong θ variations just above the surface of the left conductor. This is perhaps a new realization for me? Well, here we are not talking about the direction of B, but its magnitude going around the conductor, or at least the magnitude of the B_θ component. From our formula above where we set $E_\theta \approx 0$ just above the surface, we see that curl **E** = $\hat{\boldsymbol{\theta}}\,[-j(\omega/v_d)E_r] + \hat{\mathbf{z}}\,[-r^{-1}\partial_\theta E_r]$ so that **B** = 1/(-jω) curl **E** has no $\hat{\mathbf{r}}$ component, so we end up with B being *exactly tangent to the surface* and $B_n = 0$, which is a long discussion I have elsewhere. [seems right]

So, we have the boundary value problem of step 3 above,

$$(\nabla_{2d}^2+\beta'^2)\,\mathbf{B}(r,\theta;\omega) = 0 \qquad \mathbf{B}(a,\theta;\omega) = \text{prescribed} = [(1/v_d)\,E_r(\theta)]\,\hat{\boldsymbol{\theta}}$$

The solution to this problem would seem to be some B(r,θ) inside the wire which is a *strong function of θ*, but right now I don't know the exact solution [see App D]. Then for step 4 we get

$$\mu\mathbf{J} = \text{curl}\,\mathbf{B} = \hat{\mathbf{r}}\,[r^{-1}\partial_\theta B_z + j\beta_d B_\theta] + \hat{\boldsymbol{\theta}}\,[-j\beta_d B_r - \partial_r B_z] + \hat{\mathbf{z}}\,[r^{-1}\partial_r(rB_\theta) - r^{-1}\partial_\theta B_r]$$

and of special interest to me then is

$$\mu J_z = r^{-1}\partial_r(rB_\theta) - r^{-1}\partial_\theta B_r$$

I am guessing the second term is small and then we have

$$\mu J_z = r^{-1}\partial_r(rB_\theta) = \partial_r B_\theta + (1/r)\,B_\theta$$

I feel strongly that B_θ varies a lot going around the perimeter region, say, but I don't really know what the first term is doing without having a full solution, so I cannot simply claim that J_z is non-uniform. However, I do know this fact: If I *assume* that J_z is uniform at DC in both wires, I find that the B field is not normal to the conductor surface -- I have plotted this now many times. But the analysis above says there is no $\hat{\mathbf{r}}$ component of **B** , where I did assume $E_z = 0$ as for a perfect conductor.

[Note added 3/12/14 as other red notes: At this point, I seem to conclude that J_z is non-uniform at any frequency ω, including DC, since ω does not appear in the solution shown above. This is in direct conflict

with what I have always assumed, that J_z is uniform at DC. This seems to agree with my calculation of E_z as shown in Very Low Frequency Limit of the Appendix D fields.doc where the m≠0 components do not vanish at DC.]

Comment: My answer to the question of whether the DC current is uniform or non-uniform seems to swing back and forth sometimes on an hourly basis. Currently (no pun), since I have a smooth description of the limit $\omega \to 0$, I am arguing for non-uniform, but I have never found support from another source on this conjecture. My Appendix D stuff also argues for non-uniform.

2. Search the web for evidence of non-uniform J_z at DC.

Let's try another search effort, perhaps with the search handle "proximity effect". Here is a first offering:

http://www.atlascables.com/cable-construction.html



Rope–Lay Stranding

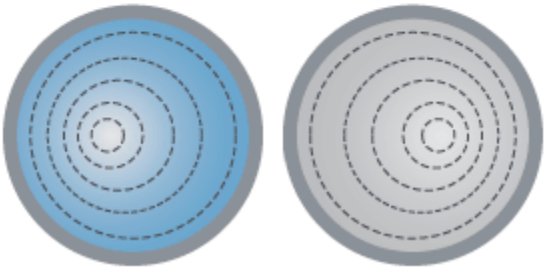
- Larger cross section than simple concentric standed cables, giving better bass resolution
- Twisted to avoid proximity effects and maintain flexibility

5.5 The proximity effect

When cables are positioned side by side (in parallel pairs, or 'figure of eight' speaker cables), the going and return currents flowing in opposite directions repel each other because of the associated magnetic fields. The current flow therefore becomes non-uniform across the cross-section of each conductor. This increases the conductor's resistance. An alternative and more effective method of cable construction consists of twisting the two conductors together so that they are not parallel to each other and therefore do not suffer from the proximity effect.

The Proximity Effect

The currents in the two conductors are flowing in opposite directions



5.6 Conductor resistance

Cable power loss is directly proportional to conductor resistance. Power loss relates to the amount of electrical energy converted into heat as the audio signal travels along the cable.

This is one of those "monster audio speaker cable" companies and no support is offered for their claim or their picture, but at least someone is making the claim!!! There are a lot of hits, and most of them claim this is only an AC effect. They claim that you induce eddy currents in the other wire and that does it.

(a) the book of Clayton Paul

But here is a Google book which seems to make claims without regard to frequency:

http://books.google.com/books?id=3a7z8TzxaDMC&pg=PT150&lpg=PT150&dq=current+wire+%22proximity+effect%22+DC+-skin+-rotating&source=bl&ots=OZlFOOnRIj&sig=eqa0X1Kmi-VbcKE3eqmMKFy8nEY&hl=en&sa=X&ei=kiMfU4rIJpXqoATyoILYAg&ved=0CEUQ6AEwBA#v=onepage&q=current%20wire%20%22proximity%20effect%22%20DC%20-skin%20-rotating&f=false

[I now have this Paul book which has a strange title: Inductance, Loops and Partial.]
[this guy died on 6/27/12 so I won't be doing any email.]

Inductance: Loop and Partial By Clayton R. Paul

4.6 USE OF FILAMENTARY CURRENTS AND CURRENT REDISTRIBUTION DUE TO THE PROXIMITY EFFECT

Throughout this chapter and in Chapter 2 we have assumed that the (dc) current I in a wire was *uniformly distributed* over the wire cross section. Hence, we were able to represent the wire current as a filament of current on the axis of the wire, thereby simplifying the computations.

If there are no other currents in close proximity, this will be the case. However, if another current is within a few radii of this wire, the current over the wire cross section will not be distributed uniformly but will tend to be concentrated toward the side facing the other wire. This phenomenon is called the *proximity effect*. If that is the case, the previous results for the **B** field in Chapter 2 as well as the inductances associated with the wire (both external and internal inductances) in this chapter will be only approximately correct and will become less so the closer the wires are spaced. Typically, the proximity effect does not substantially alter the results that were obtained by assuming that the current is uniformly distributed over the wire cross section if the

Notice his *specific mention* of DC in the first sentence. Other authors always claim AC only. But later he says

Throughout this book we generally concentrate on line currents, denoted as I, whose units are amperes. Considering a differential length of these currents as a small cylinder of length dl and cross-sectional area ds with current density distributed uniformly over the cross section (as will be the case for dc currents [3]), $J\,ds = I$, so that $J\,dv = J\,ds\,dl = I\,dl$. In this case the

and

redistribution, the proximity effect, in Section 4.6). In the case of dc currents, the total current I carried by the wire will also be *uniformly distributed* over the wire cross section with a current density over the cross section of

$$J = \frac{I}{\pi r_w^2} \qquad \text{A/m}^2$$

and

The conductivity of the wire material is denoted as σ (copper has $\sigma = 5.8 \times 10^7$ S/m), and the wire material is assumed to be nonmagnetic, $\mu_r = 1$. At dc, $f = 0$, the skin depth is infinite, showing that *a dc current is distributed uniformly over the wire cross section.*

and

as R. For line currents we consider the current I to be contained in a differential cylinder of length dl and cross-sectional area ds. If the current density is uniformly distributed over the cylinder cross section (as will be the case for dc currents [3]), the total current is $J\,ds = I$, so that $J\,dv = J\,ds\,dl = I\,dl$,

so now he is saying things are uniform for DC currents.

BUT,

filament on the axes of those wires. For isolated direct currents (dc) not in proximity to other currents, the current is, in fact, uniformly distributed over the wire cross section. However, for a current that is in close proximity to other currents, the current in the wire will not be distributed uniformly over the wire cross section. Nearby currents will cause the current to be concentrated on the side of the wire nearest the neighboring current, a phenomenon known as the *proximity effect*. Proximity effect is usually not pronounced unless the two currents are within about four radii of each other (i.e., one wire will just fit between the two). This is investigated in Section 4.6. High-frequency currents

AND

We have made the approximation that $s - r_w \cong s$ since the wires are assumed to be widely spaced. Hence, the approximate per-unit-length inductance of the line for widely spaced wires which assumes a uniform current distribution over the wire cross sections is

$$l_{\text{approximate}} = \frac{\mu_0}{\pi} \ln \frac{s}{r_w} \quad \text{H/m} \tag{4.73}$$

If the wires are closely spaced as shown in Fig. 4.16(b), the currents will be concentrated toward the facing sides, and the result above for the per-unit-length inductance in (4.73) is an approximation since that relied on the currents being uniformly distributed over the wire cross sections. It can be shown (see [3,8]) that the magnetic fields are as though the total currents are concentrated as filaments but separated by a distance $D \leq s$ as shown in Fig. 4.16(b). The exact per-unit-length inductance for this result can be shown to be [3,8]

$$l_{\text{exact}} = \frac{\mu_0}{\pi} \ln \left[\frac{s}{2r_w} + \sqrt{\left(\frac{s}{2r_w}\right)^2 - 1} \right] \quad \text{H/m} \tag{4.74}$$

But then later he flops back again to no DC prox effect:

Here he is explicitly making the focal point claim for the currents, not just the charges! You see his statement right there! But the references concern the inductance formula perhaps and not the fact I am looking for. I note that his (4.74) disagrees with my Chapter 6 result! Perhaps that is because I used a uniform current density. So I am still in dire need of his reference 3 or 8 to get more on this!

Here he really is saying there is a DC proximity effect! I swear that is what he is saying in the above paragraph!

This author Clayton Paul talks about inductance rather than the current, and shows that the proximity effect causes the inductance to be what I say it is from K in lines doc, the fancy formula similar for K.

If the wires are closely spaced as shown in Fig. 4.16(b), the currents will be concentrated toward the facing sides, and the result above for the per-unitlength inductance in (4.73) is an approximation since that relied on the currents being uniformly distributed over the wire cross sections. It can be shown (see [3,8]) that the magnetic fields are as though the total currents are concentrated as filaments but separated by a distance $D \le s$ as shown in Fig. 4.16(b). The exact per-unit-length inductance for this result can be shown to be [3,8]

$$l_{\text{exact}} = \frac{\mu_0}{\pi} \ln \left[\frac{s}{2r_w} + \sqrt{\left(\frac{s}{2r_w}\right)^2 - 1} \right] \quad \text{H/m} \tag{4.74}$$

There is his clear statement with no comment on frequency ω. And he says that the currents act as if concentrated at the bipolar focal points. I only showed this was true of the surface charge. He gives two references which are:

[note that inductance is both a DC and AC effect. Paul's statements seem valid to me in terms of surface charge, but maybe not so valid in terms of conductor current. Later when I look up one of his books, he discusses proximity effect there ONLY in terms of surface charge!]

[Comment 5.15.14. I now know that in the extreme skin effect, $J_z(a,\theta)$ is proportional to $n(\theta)$, and so Paul is correct in this situation when he says the currents act if they were at the focal points in the sense that lines through the focal points would be the "center of J_z current" for what that is worth. Perhaps the B fields from this current are as if they were at the focal points. For small ξ, this would mean a stronger B field in the gap than on a far part of the surface.]

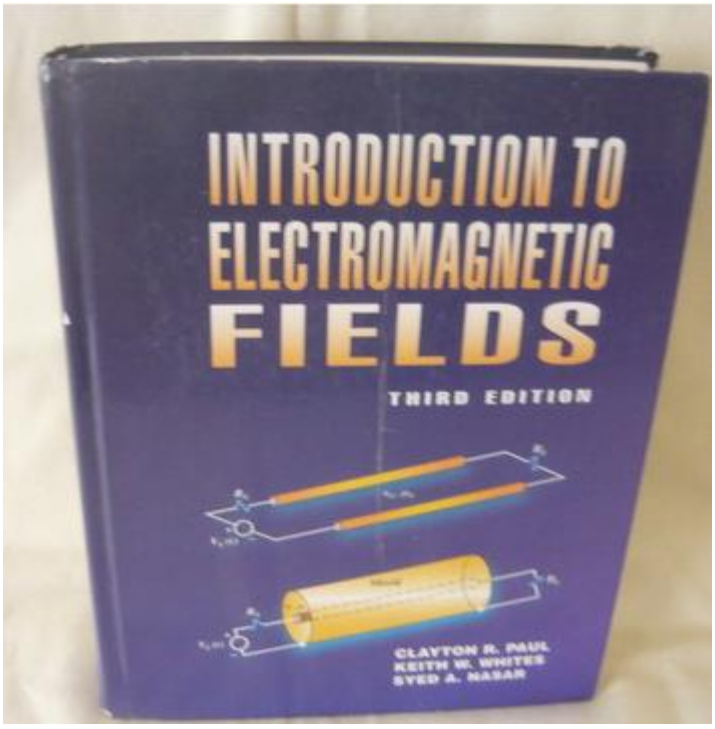
I was able to get Paul's book from Russia, so I can look up the refs which I think are missing in google. This book is very much "up my alley", maybe the closest thing I have seen yet to what I do. Here are his two references (they are in fact at the end of the book and were in google after all)

[3] C.R. Paul and S.A. Nasar, *Introduction to Electromagnetic Fields*, McGraw-Hill, New York, second edition, 1987, and third edition, 1998.

[8] C.R. Paul, *Analysis of Multiconductor Transmission Lines*, second edition, Wiley-Interscience, Hoboken, NJ, 2008.

Both refer to his own work (again, similar to me).

[3] I get a partial web copy, index says proximity effect is on page 424. By the way, NONE of my E&M books has this word in the index. I cannot find a download of this item. But the amazon cover picture sure seems relevant



Web scan for this book? It is very well protected probably by the other two authors. I have the TOC but no phrases show anywhere on the web.

What about Ref 8 ? I have some TOC text and that led to a google book. Here are some quotes from this Ref 8

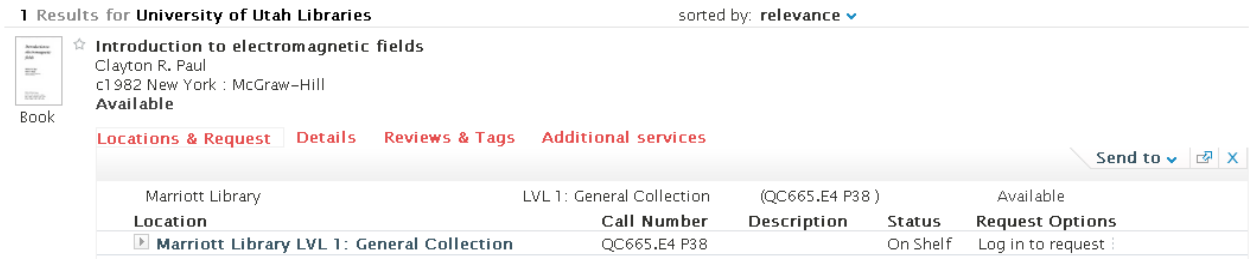
> *assume the current is symmetric about the axis of the wire* [4, 8]. At dc the current is uniformly distributed over the cross section, whereas at higher frequencies the current

As in the Partial book, he continues to hem and haw on my question, but he never presents any calculation of the asymmetry due to the prox effect.

An older 1982 version of this book is at Marriott and is on the shelf right now! He gives ref 10 for proximity effect, which is this

> [10] V. Belevitch, Theory of the proximity effect in multiwire cables, *Philips Research Reports*, **32** (Part I), 16–43, (Part II), 96–177, 1977.

This thing is nowhere to be found, fine.



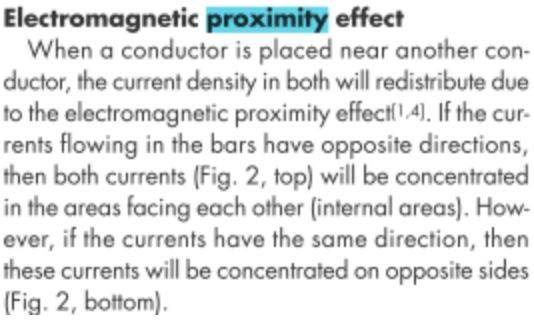
[8] This one IS at Marriott. But only as "online access", as if they don't really have the book. Then of course the online access is "not available" with no reason given. Perhaps they just have it up during a course. So in fact Marriot does NOT have this book at all.



so I could go over there and take a look I suppose. // Nothing there.

I continue the general proximity effect web search

This one says it only happens at AC:



Electromagnetic proximity effect

When a conductor is placed near another conductor, the current density in both will redistribute due to the electromagnetic proximity effect[1,4]. If the currents flowing in the bars have opposite directions, then both currents (Fig. 2, top) will be concentrated in the areas facing each other (internal areas). However, if the currents have the same direction, then these currents will be concentrated on opposite sides (Fig. 2, bottom).

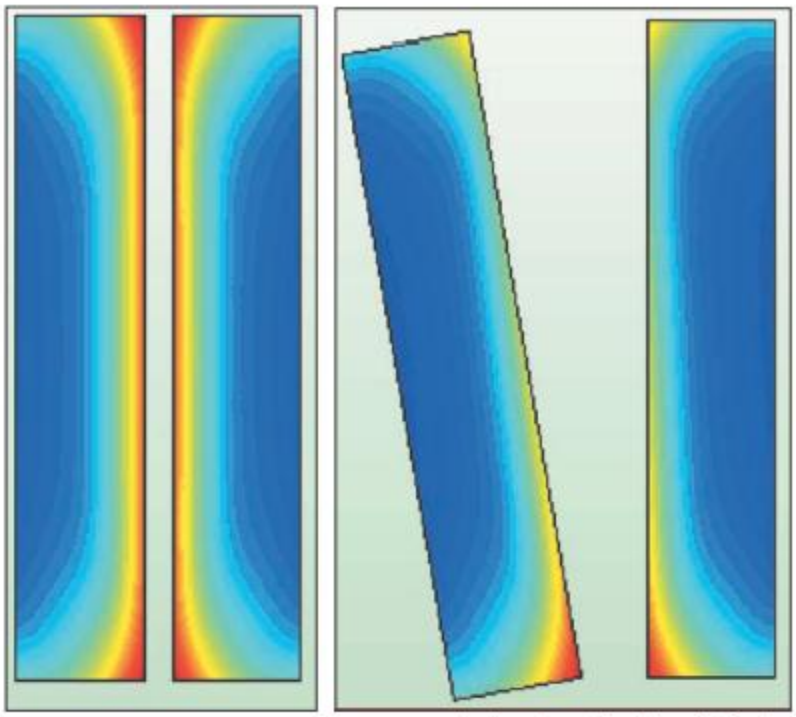


Fig. 3 — Current density redistribution due to electromagnetic proximity effect in symmetrical (left) and non-symmetrical (right) systems (results of numerical computer modeling).

4. A.E. Slukhotskii and S.E. Ryskin, *Inductors for Induction Heating*, Energy Publications, St. Petersburg, Russia, 1974.

"DC proximity effect" shows only 7 hits! So that Marriott book is very valuable, I will go look at it. We are right now in spring break, limited hours are 7AM to 6 PM. Fine.

(b) Library Trip looking for the Paul book

I found the 1982 book, it had proximity effect in the index p 321, **but** that pointed only to a discussion of the surface charge being asymmetric, which is not the issue in question. It said nothing about currents. I looked in the index of about 20 nearby "electromagnetism" books and only one had "proximity" in its index, but it was the charge thing again. I bet the newer Paul book has the same thing.

(c) Continue web search.

Maybe this is the wrong search handle. Current crowding seems semiconductor in nature.

"proximity effect" = 450,000 hits

"proximity effect" conductors = 43,000 hits

"proximity effect" conductors "current distribution" = 40,000 hits

"proximity effect" conductors "current distribution" DC = 36,000 hits

I have found some hits and will continue. I want someone who actually calculates the effect. I will note references as I find them

[4] Proximity effect in wires and thin tubes.
Trans. AIEE, vol. 42, 1923, p. 850-859.
H.B. DWIGHT.

[5] Recherches sur la répartition du courant
dans les barres rectangulaires.
Arch, Elektr. XVII, 1927, p. 537-589.
H. SCHWENKHAGEN.

I have located a web readable version of the Dwight paper, first page only.

Proximity Effect in Wires and Thin Tubes

BY H. B. DWIGHT
Member, A. I. E. E.
Canadian Westinghouse, Ltd., Hamilton, Ont.

The proximity effect in a return circuit of two wires was calculated for low frequencies in 1920 by H. L. Curtis (Reference 13). A complete solution was published by J. R. Carson in 1921, (Reference 14), and an alternative solution is given in this paper, (Formula VII).

[1] R. W. P. King, Fundamental Electromagnetic Theory, Dover Publications, New York, Chapter 5, 1963.

[2] S. Ramo, J. R. Whinnery and T. VanDuzer, Fields and Waves in Communication Electronics, John Wiley and Sons, New York, Chapter 5, 1965, pp. 286-303.

[3] H. A. Wheeler, "Formulas for the Skin Effect", Proc. I. R. E., 30:412-424, 1942.

[4] A. E. Kennelly, F. A. Laws, and P. H. Pierce, "Experimental Researches on Skin Effect in Conductors", Trans. A. I. E. E., 34: 1953-2018, 1915.

[5] A. E. Kennelly, H. A. Affel, "Skin-Effect Resistance Measurements of Conductors to 100 Kilocycles", Proc. I. R. E., 4:523-574, 1916.

[6] J. R. Carson, "Wave Propagation over Parallel Wires: The Proximity Effect", Phil. Mag., Ser. 6, 41:607-633, 1921.

[7] H. B. Dwight, "Skin Effect and Proximity Effect in Tubular Conductors", Trans. A. I. E. E., 41:189-198, 1922.

[8] H. B. Dwight, "Proximity Effect in Wires and Thin Tubes", Trans. A. I. E. E., 42: 850-859, 1923.

Search for Kennelly: I have a Kennelly google book "skin effect in conductors" full view I think. They not that the resistance increases with proximity, that is their point of view on this issue. It suggests AC only but has no theory.

Someone else:

There are two kinds of eddy-current effects: Skin effect and proximity effect. When an ac current flows in a conductor, the current density tends to decrease from the surface to the center of the conductor and thus increase the power loss when the operating frequency gets high. This is called skin effect. The proximity effect is similar but it is caused by the current carried by a nearby conductor. The current in the nearby conductor causes a time-varying field and induces a circulating current inside the conductor. Both the skin effect and the proximity effect cause the current density to be nonuniform in the cross-section of the conductor and cause higher winding loss at higher frequency [17].

In this paper we focus exclusively on proximity-effect loss, because, in a multilayer winding, it strongly dominates over skin effect loss. Thus we examine the effect of an external AC field applied to a winding.

How is Proximity Effect Analyzed?

As shown, proximity effect totally dominates wire losses for many common cases. Oddly enough, there is little obtainable literature on proximity effect. An examination of many magnetic and power supply books, and programs revealed absolutely no coverage. As a result, even experienced magnetic and power engineers do not consider proximity effects.

1.) P. L. Dowell, "Effects of Eddy Currents in Transformer Windings," Proc. IEE, Vol 113 No. 8, August 1966

4.) KO Systems, "Proxy, A Proximity and Skin Effect Analysis Program," 10437 Laramie Ave, Chatsworth, CA, 818 341 3864

PROXIMITY EFFECT FACTOR(y_p):

The proximity effect also increases the effective resistance and is associated with the magnetic fields of two conductors which are close together. If each carries a current in the same direction, the halves of the conductors in close proximity are cut by more magnetic flux than the remote halves. Consequently the current distribution is not even throughout the cross-section, a greater proportion being carried by the remote halves. If the currents are in opposite directions, the halves in close proximity will carry the greater density of current.

1) For TWO CORE and TWO SINGLE CORE cables:



$$y_p = x_p^4 / (192 + 0.8\, x_p^4) * (d_c / S)^2 * 2.9$$

where

$$x_p^2 = 8 . \pi . f . 10^{-7} . k_p / R_{dc_t}$$

f = Frequency (Hz)

k_p = Factor determined by conductor construction
1 for circular, stranded, compacted and sectored
0.8 if above conductors are dried and impregnated

d_c = Diameter of conductor (mm)

PROXIMITY EFFECT

Conductors spaced close to one another and carrying alternating current will have the current distribution in each conductor altered by mutual reactance. This results in increased resistance known as proximity effect.

The increase is commonly designated as Y_{ep} in the formula $R_{ac} = R_{dc} \cdot (1 + Y_{es} + Y_{ep})$

In most cases if conductor spacing exceeds 10 times the conductor diameter proximity effect will be less than 1% and can be neglected

The following 1935 paper seems pretty trustworthy, I downloaded a PDF of it":

The Bell System Technical Journal

Vol. XIV *April, 1935* *No. 2*

Cable Crosstalk—Effect of Non-Uniform Current Distribution in the Wires

By R. N. HUNTER and R. P. BOOTH

When wires are close to each other as they are in cable, the mutual inductance coupling between pairs is not a simple number, constant at all frequencies. Because of the non-uniform and non-symmetrical distribution of current over the cross-sections of the conductors the "effective mutual inductance" is of the form $M = M_a + jM_b$ where both M_a and M_b vary with frequency. This paper discusses the results of certain measurements which have been made of the effective mutual inductance between straight wires and between cable pairs over a wide range of frequencies extending up to a million cycles. This is of interest in connection with crosstalk problems in cable carrier telephone systems.

FOR many years it has been recognized that non-uniform distribution of current over the cross-section of a conductor reduces the efficiency of transmission in either power or communication circuits. With direct current or with alternating current of very low frequency the current is distributed almost uniformly. As the frequency increases, the current distribution becomes more and more non-uniform.

Note the ambiguity of the last sentence. Is the distribution "almost uniform" at DC ? But here we are later on:

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CASE II—CURRENT UNIFORMLY DISTRIBUTED IN A SOLID CYLINDRICAL DISTURBING WIRE

Consider next the case where the total disturbing current is uniformly distributed over the cross section of a solid cylindrical wire. Such a distribution exists exactly with direct current only, but is closely approximated at very low frequencies. Since the magnetic

So they are clearly stating what I want to see: current is uniform at DC, case closed! But wait, maybe they are talking about one conductor in isolation. No, I don't think so. They go on to discuss the AC case in great detail with wonderful drawings of the current distribution.

Status 2:30PM. I made no progress today despite lots of web search and library trip. My "new set of rules" found today seems to indicate a DC non-uniform field, but nobody else sees this except author Clayton Paul in one place but I don't have his 1987 book and it has been purged from the web. It has a 1987 and 1998 edition, but I doubt it really deals with the issue at hand. Author just self-promoting.

I have this sinking feeling that Paul is wrong and so am I and there is no DC proximity effect. So something is wrong with my derivation.

3. Try again perhaps using uniform J_z as zeroth approx:

[this is perhaps my first attempt at "eddy current theory" which now appears in Appendix P. The work below is pretty inscrutable as it is stated and I don't think of any use.]

Just writing for the record:

$$\text{curl } \mathbf{E} = -\partial_t \mathbf{B} \quad \Leftrightarrow \quad \oint_C \mathbf{E} \bullet d\mathbf{s} = -\partial_t \left[\int_S \mathbf{B} \bullet d\mathbf{S} \right] \quad . \qquad (1.1.36)$$

$$\text{curl } \mathbf{E} = -j\omega \mathbf{B} \quad \Leftrightarrow \quad \oint_C \mathbf{E} \bullet d\mathbf{s} = -j\omega \left[\int_S \mathbf{B} \bullet d\mathbf{S} \right] \quad . \qquad (1.1.36)$$

Go back to this:

$$\text{curl } \mathbf{E} = \hat{\mathbf{r}} \left[r^{-1} \partial_\theta E_z - \partial_z E_\theta \right] + \hat{\boldsymbol{\theta}} \left[\partial_z E_r - \partial_r E_z \right] + \hat{\mathbf{z}} \left[r^{-1} \partial_r (r E_\theta) - r^{-1} \partial_\theta E_r \right]$$

$$\text{curl } \mathbf{E} = -j\omega \mathbf{B} = \hat{\mathbf{r}} \left[-j\omega B_r \right] + \hat{\boldsymbol{\theta}} \left[-j\omega B_\theta \right] + \hat{\mathbf{z}} \left[-j\omega B_z \right]$$

This is EXACT! Now there are three different equations:

$$-j\omega B_r = r^{-1} \partial_\theta E_z - \partial_z E_\theta$$
$$-j\omega B_\theta = \partial_z E_r - \partial_r E_z$$
$$-j\omega B_z = r^{-1} \partial_r (r E_\theta) - r^{-1} \partial_\theta E_r \quad [\text{all OK}]$$

Maybe better to do this in Cartesian

curl $\mathbf{E} = \hat{\mathbf{x}}\,(\partial_y E_z - \partial_z E_y) + \hat{\mathbf{y}}\,(\partial_z E_x - \partial_x E_z) + \hat{\mathbf{z}}\,(\partial_x E_y - \partial_y E_x)$

curl $\mathbf{E} = -j\omega\mathbf{B}$ **[** recognize this as (P.1.5) in my eddy current Appendix P!]

$-j\omega B_x = \partial_y E_z - \partial_z E_y$

$-j\omega B_y = \partial_z E_x - \partial_x E_z$

$-j\omega B_z = \partial_x E_y - \partial_y E_x$ [all OK]

Now suppose we use our static H_x, H_y fields found in my plotting program deal (based on uniform J_z in each conductor) AND, suppose we assume without justification that E_z is the dominant field inside, Then

$-j\omega B_x(x,y) = \partial_y E_z(x,y)$

$-j\omega B_y(x,y) = -\partial_x E_z(x,y)$

$0 = \partial_x E_y(x,y) - \partial_y E_x(x,y)$

So what exactly do I do with these first two equations:

$\partial_y E_z(x,y) = -j\omega B_x(x,y)$

$\partial_x E_z(x,y) = j\omega B_y(x,y)$

where the right sides are fully known.

[At this point I first interpreted the above as a system of two PDE's! Later I realized it was just $\nabla E_z = -j\omega\mathbf{B}$, but not until I went off on this spawned question and looked in Polyanin (downloading his 1st order PDE pdf this time, so he has two PDE pdf's). I then wrote up this "inverse gradient" problem as a doc, and that along with a certain nice pdf is now stored in math/calculus]

I was intending to use my plot routine's functions H_x and H_y and then use that to compute E_z, but now that seems a fruitless task. The integrals will be difficult, but you should just end up with E_z = uniform inside the wires, since that is what you assumed to compute the B field. So I won't go off on that spawned tangent, I just let it be. At least I learned about the "inverse tangent operator".

4. The Eddy Current Interpretation

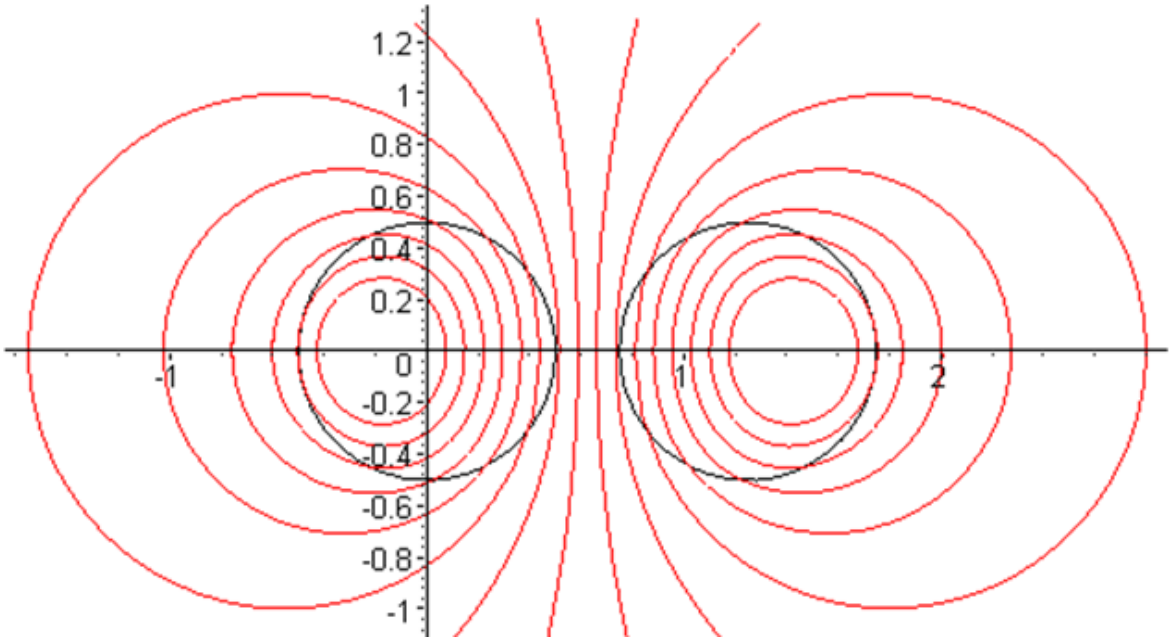
This is a way to show that you expect J_z asymmetry only to appear when $\omega > 0$.

You start with your two cylinders at DC, each one carries I, and you assume J_z is uniform. Here is an argument for why it should be uniform.

(a) argument for uniform J_z at DC.

[Notice how this next paragraph *brings up* the notion of a radial Hall effect, requiring me later to do a long digression on that subject!]

Imagine you are an electron flowing in one of these two closely spaced cylinders at DC. Your only awareness of the other cylinder is that your local B field is "wrong" -- it is not what it would be if you were in an isolated cylinder wire. Be that as it may, at each point inside the left cylinder where you are flowing in the z direction, there is a static B field. It is the sum of the B fields of the two wires, and we know its general direction from our famous picture,



As you flow, you are first feel a **v** x **B** Lorentz force perhaps pushing you do the left. But after a very short while, a tiny Hall effect surface charge will appear on the surface of your conductor and will arrange itself in a complicated manner such that it exactly cancels the Lorentz force on every flowing electron inside the fat conductor. This would of course be an interesting spawned problem, to compute that surface charge! If the charge is wrong, electrons continue to deflect and continue to build up charge at the surface (radial pumping!) until that surface is just right, and then there is no more deflection, and every electron goes in the z direction only. Now the claim is that in this final stable static situation, there is no reason for J_z to be more toward the left edge versus the right edge of the conductor. That Hall surface charge has no effect on the J_z other than exactly balancing out the static Hall field. So if you buy this argument, you end up with a uniform J_z and some tiny charges on the surfaces of each conductor, and on each conductor the total such charge (per unit length) is zero. You can see from the red field lines above that the field has a different direction at different location in the cylinder, so the Hall charge generated E field has to have a very strange configuration.

Bug in this argument: When I draw the Hall E field that would be required, it seems to point outward from the approximate center in all directions. This, there is a region inside the conductor that seems to have div E ≠ 0. But there can be no charge inside the conductor, so we have a nice new Paradox!

[this is resolved in my radial Hall effect appendix N stuff]

Ignoring this bug for a moment, here is the eddy current argument.

(b) eddy current argument for skin effect and proximity effect

[this is now all presented in Appendix P]

You now raise $\omega \rightarrow 0$ and you imagine some thin math loop in the left conductor. It is long in the z direction, and the thin edges are horizontal in the above picture. If there were only one conductor present, one could analyze the changing flux through this loop and that would give rise to a symmetrical left/right variation in the field E_z and this would (hopefully) lead to the skin effect.

Now you bring in the other conductor and the B field changes. It is now stronger on the right side of the left conductor than on the left side, so flux change is larger on the right, so $\partial_x E_z$ is larger there and this is supposed to cause the "proximity effect" non-uniform J_z. But since there is only changing flux at AC, there is no such effect when $\omega = 0$. So this then is the "eddy current argument" for the proximity effect which occurs with closely spaced wires at AC. I guess you are supposed to add this eddy current induced E_z field to the uniform one.

I can only assume that this effect would be included in my "general theory" of current inside a round wire. It should just "fall out" from the equations which are exact solutions of all the Helmholtz stuff.

(c) Mystery with Lorentz force and isolated cylinder wire

[this is the radial Hall effect subject]

Draw picture on whiteboard of the B field inside a round wire where current flows toward the viewer. Imagine current of plusons instead of electrons. B field is CCW. Consider an arbitrary pluson in motion in the z direction. The Lorentz **v** x **B** force is always toward the center of the wire. So every pluson wants to deflect toward the center of the wire. How can a surface Hall charge stop this deflection? You would need a line of physical charge existing along the cylinder axis and a counter charge on the outer surface in order to straighten out the deflections in order to end up with J_z only in the z direction. But I thought I said you could not have free charge inside a conductor? So I have spawned a brand new complete mystery . I now have to go spend 4 hours trying to find this situation discussed on the web under some unknown search handle. Otherwise I have a live mystery which can infect my thinking on other subjects because something is missing! That could then invalidate everything else I do. So off we go to the web!

"Lorentz force inside a round wire" no hits

This is for me a completely mind-blowing question. The simplest questions are often the hardest questions I have found. The question is here is completely trivial. What happens with a typical electron in the current carrying wire. Does it see the B field of all the other electrons? Does it not see this B field for some reason due to the macroscopic average business? Suppose you "shoot" an electron through a region of space with a uniform B field, what does it do? I don't even know! The Lorentz force is perp to both v and B. For a uniform **B** field going into the plane of paper, a pluson which is launched tangent to just the right circle will go in a circle in the plane of paper, were mv^2/r balances the L force. This is the cyclotron story. In Purcell's Hall picture page 218, electrons *start off* doing a circular path, that is, their deflection is the start of a circular path, but then the E field comes on and the deflection is stopped.

Of course there are collisions and we are talking about the average path of an electron.

I did some web searching 3/12/14. Yes, you really need to do the quantum theory of solids and all that stuff, but I want something simple. My solution is that yes, initially all the electrons deflect toward the center (say), but this builds up a free electron density there which then creates a field relative to the outer surface which then stops the deflection. I doubt anyone could ever measure such a thing because you cannot put a voltage prove at the center of a solid cylinder. That is just going to have to be my personal resolution of this mystery until I run into some better explanation. Surely the drifting electrons see the B field made by all the other electrons. So I am claiming then a "cylindrical Hall effect". Needless to say, there are no web hits on that phrase. Well there are, but they have to do with Hall Thrusters which is not really what I am interested in.

March 13, 2014

I verified today with Maple two things concerning my Appendix D E field solutions with the coefficients evaluated at per my two boundary conditions:

(1) they satisfy div **E** = 0

(2) they satisfy $(\nabla^2 + \beta^2)$ **E** = 0 vector Helmholtz.

Given the BC's, my solutions are correct, I just don't think there is any error in them. I used the Maple fancy cyl coordinates diff op operators for the first time to do this.

Question: At DC, do I expect the E fields inside a wire to satisfy these two equations?

(1) All electrostatics situations have div = **E** where there is no free charge.

(2) At DC, $\beta^2 = 0$ so the Helmholtz is just the vector Laplace equation ∇^2 **E** = 0. This is not familiar to me. Well, write ∇^2 = grad(div **E**) – curl (curl **E**) . We know that div **E** so this says

$$\nabla^2 \mathbf{E} = -\text{curl}(\text{curl}\ \mathbf{E}) = 0.$$

Now at ω = 0 we look at curl **E** = - ∂_t**B** and say curl **E** = 0 at DC, so there it is, ∇^2 **E** = 0 .

Plan A. How about a pure DC solution to the problem. As a start, suppose I do this;

(1) assume uniform J_z in each of two adjacent cylinders
(2) compute **H** everywhere (I think I have already done this)
(3) Then use curl **H** = **J** to compute J, and see if it agrees with the starting uniform value.

Confusion: Somehow this has to give J = 0 outside the conductors. How is that going to work?

OK, my existing calculation of H is located in "the surface currents issue" doc, Section 6 (b), and this is already loaded into "B field two round wires.mws" . So all I have to do is use the Maple Cartesian curl operator I used earlier today. Interestingly, I have no idea what mws file that was done in, even though it was earlier today! It is not in any of my usual working areas! Where did I enter all those fm values? I can't even find the doc I did it in!

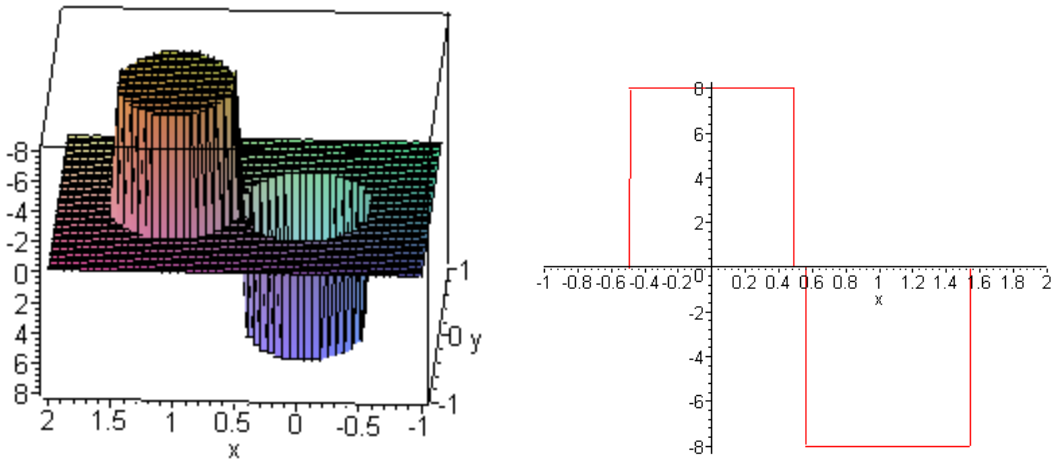
Time out to search for this morning's lost work. First, I seek a doc which has curl(curl in it.

Desktop? no
Trans lines folder: no

Witzend: the mws file is here (but not the doc)
D:\Work\My Interests\Physics\Transmission Lines\Appendix D EB round

I guess my only comment on this is a shown above right in this doc now in red.

Resume: I added code to "B field two round wires". The result is quite conclusive:



You get back exactly what you started with, for J everywhere!!! There is no "tilt" of J_z, even if I set b = 1.01 which brings them very close. And b = 1.01 and both radii 1/2 I get $\xi = 0.144$ which is plenty small to show any effect if it existed.

Conclusion: <u>This is a *self-consistent* magnetostatics problem!</u> Start with constant $I = 2\pi$ and then

$J_z = 2\pi/\pi a_1^2 = 2/(a_1)^2 = 8$ [OK, magnetostatics works I am saying]

Then compute the exact **H** field everywhere, then use curl **H** = **J** to get **J**, and you get exactly what you started with! Is this a fact of superposition? I guess it is, but the AC problem for some reason does not allow superposition. If my radial Hall voltage were needed to stabilize the current, that E field would have no effect on this little self-consistent problem! I think solutions are unique, so this MUST BE the only solution to this DC problem.

Friday 3/14/14

Two questions:
(1) If I start with my asymmetric predicted current at $\omega = 0$, what B lines does it produce?
(2) What happened to the King gauge condition in my whiteboard flow chart

Item (1) : My current B line program is based on these fields taken from "surface currents":

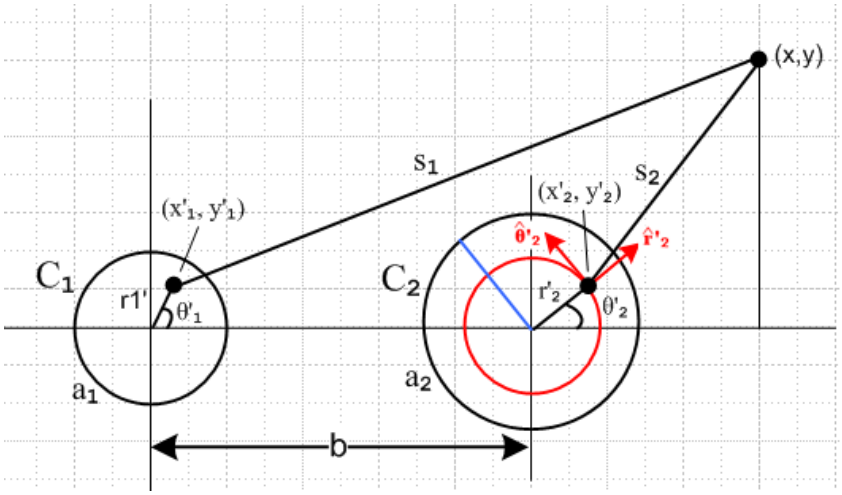
$$H_1(r_1) = (I/2\pi)[\ \theta(r_1>a_1)\ (1/r_1)\ + \theta(a_1>r_1)\ (r_1/a_1^2)]$$
$$H_2(r_2) = -(I/2\pi)[\ \theta(r_2>a_2)\ (2/r_2)\ + \theta(a_2>r_2)\ (r_2/a_2^2)]$$

$$H_x = -\ (y/r_1)H_1(r_1) - (y/r_2)\ H_2(r_2)$$
$$H_y = (x/r_1)\ H_1(r_1) + ((x-b)/r_2)\ H_2(r_2)$$

This is a direct superposition deal : I compute **H** for each wire assuming uniform J_z, I do the vector addition, and there you are. For my asymmetric current, things are much harder. I would have to use my 2D Biot Savart from lines doc which says

$$\mathbf{H}(x,y) = \int d^2\mathbf{x}'\ \frac{1}{2\pi R^2}\ \mathbf{J}(\mathbf{x}')\ x\ \mathbf{R} \qquad\qquad \mathbf{R} \equiv \mathbf{x} - \mathbf{x}' \qquad\qquad (B.2.24)$$

$$J_z\ =\ \sigma I\ R_{dc}\ [\ 1 + \sum_{m=1}^{\infty} (-1)^m\ e^{-m|\xi_1|}\ (r/a)^m\ [\ (m+1)]\ \cos(m\theta)\ ]$$



For the left conductor

$$\mathbf{J}(\mathbf{x}')\ x\ \mathbf{s}_1\ = [J_z\ \hat{\mathbf{z}}\ ]\ x\ [(x-x'_1)\ \hat{\mathbf{x}}\ + (y-y'_1)\ \hat{\mathbf{y}}\ ]\ = J_z\ \{(x-x'_1)\ \hat{\mathbf{y}}\ -\ (y-y'_1)\ \hat{\mathbf{x}}\ \}\ \theta(r'_1 < a_1)$$

$$\mathbf{H}_{left}(x,y) = \int d^2\mathbf{x}'\ \frac{1}{2\pi s_1^2}\ \theta(r'_1 < a_1)\ J_z\ \{(x-x'_1)\ \hat{\mathbf{y}}\ -\ (y-y'_1)\ \hat{\mathbf{x}}\ \}$$

$$= \int r'_1 dr'_1 d\theta'_1\ \frac{1}{2\pi[(x-x'_1)^2 + (y-y'_1)^2]}\ \ \theta(r'_1 < a_1)\ J_z(r'_1, \theta'_1)\ \{(x-x'_1)\ \hat{\mathbf{y}}\ -\ (y-y'_1)\ \hat{\mathbf{x}}\ \}$$

$$= \int r'_1 dr'_1 d\theta'_1\ \frac{1}{2\pi[(x-r'_1\cos\theta'_1)^2 + (y-r'_1\sin\theta'_1)^2]}\ J_z(r'_1, \theta'_1)\ \{(x-\ r'_1\cos\theta'_1)\ \hat{\mathbf{y}}\ -\ (y-\ r'_1\sin\theta'_1)\ \hat{\mathbf{x}}\ \}$$

So then

$$[\mathbf{H_{left}}(x,y)]_x = \int r'_1 dr'_1 d\theta'_1 \frac{(x - r'_1\cos\theta'_1)}{2\pi[(x-r'_1\cos\theta'_1)^2 + (y-r'_1\sin\theta'_1)^2]} J_z(r'_1, \theta'_1)$$

$$[\mathbf{H_{left}}(x,y)]_y = - \int r'_1 dr'_1 d\theta'_1 \frac{(y - r'_1\sin\theta'_1)}{2\pi[(x-r'_1\cos\theta'_1)^2 + (y-r'_1\sin\theta'_1)^2]} J_z(r'_1, \theta'_1)$$

where

$$J_z(r'_1, \theta'_1) = \sigma I R_{dc} [\, 1 + \sum_{m=1}^{\infty} (-1)^m e^{-m|\xi_1|} (r'_1/a)^m [\, (m+1)] \cos(m\theta'_1) \,]$$

Not surprisingly, the integration is ugly, but might be doable analytically. Numerically we would have to do a double integral for every point in the x-y plane. I suspect this calculation is much easier in bipolar coordinates. I have two conjectures:

Conjecture 1: The current in each conductor acts as if concentrated at the bipolar focal point.
[I know this is true now in extreme skin effect]

Conjecture 2: One would find that the H lines align with the cylinder surface, which would mean that A_z was a constant on the cylinder surface.

Digression: Let's now review Appendix M which says $A_t << A_z$. One of the assumptions I make there can be written this way

$f_m \approx g_m \approx h_m$ in "scale"

I think somewhere I showed this was valid for the skin depth regime but I would have to do a 10 minute search to find that info. Right now I wonder what happens for small ω. I found that

$f_m = (2/a\beta)\,(m+1)(r/a)^m$ $\quad m > 0$
$f_0 = (4/a\beta)$ $\quad m = 0$ $\quad \beta = e^{3\pi j/4}\sqrt{\sigma\mu}\;\omega^{1/2}$

$g_m = (1/ar)\,(r^2 + a^2)(r/a)^m$ $\quad m > 0$
$g_0 = 2(r/a)$ $\quad m = 0$

$h_m = (1/ar)\,(r^2 - a^2)(r/a)^m$ $\quad m > 0$
$h_0 = 2(r/a)$ $\quad m = 0$

It seems fair to say that $g_m \approx h_m$ in "scale", but f_m recall blows up as $\omega \rightarrow 0$. But lets keep going:

Second summary of the E field solutions for m = 0 and ω → 0 *(D.2.33)*

$J_z(r,0) = \sigma I R_{dc}$

$J_r(r,0) = \sigma(j/2)\, I\, R_{dc}\ (r\beta_d)$

$J_\theta(r,0) = \sigma(1/2)\, I\, R_{dc}\ (r\beta_d)$

What we see in this limit is that J_r and J_θ are smaller than J_z by factor $(r\beta_d/2)$ which in fact is very small if we are in the transmission line limit. This the claim of Observation (1) in Appendix M is borne out in the limit $\omega \rightarrow 0$, but not quite in the manner in which it was described there. I think the cancellation argument continues to be valid, so the conclusion of small A_t seems to survive!

Item (2) : We now switch gears to see how the King gauge fits into our whiteboard theory architecture which I don't yet have as hard copy. The King gauge says

$$\text{div}\ \mathbf{A} = -\mu\varepsilon j\omega\varphi - \mu\sigma\varphi = -j\omega\mu(\varepsilon+\sigma/j\omega)\varphi = -j\omega\mu\xi\varphi = -j(\beta^2/\omega)\ \varphi \qquad \text{King gauge} \qquad (1.5.5)$$

Write this as both inside and outside conductors in this manner

$\partial_x A_x + \partial_y A_y + \partial_z A_z = -j(\beta^2/\omega)\ \varphi$ inside

$\partial_x A_x + \partial_y A_y + \partial_z A_z = -j(\beta_d^2/\omega)\ \varphi$ outside

I have never addressed the question of potential φ inside the conductor, only in the dielectric. So let's stick with the dielectric so

$\partial_x A_x + \partial_y A_y + \partial_z A_z = -j(\beta_d^2/\omega)\ \varphi$ dielectric

Suppose now according to Appendix M we could ignore the first two terms, just suppose. Then you would have

$$\partial_z A_z = -j(\beta_d^2/\omega)\ \varphi$$

or

$$-j\beta_d A_z = -j(\beta_d^2/\omega)\ \varphi$$

or

$$-jA_z = -j(\beta_d/\omega)\ \varphi$$

or

$$A_z = (\beta_d/\omega)\ \varphi$$

or

$$A_z = (1/v_d)\ \varphi$$

as I have found elsewhere, but don't want to search now. If this condition were true, it would say that the equipotential lines of A_z must match the boundary of the cylinder cross section, since that is the case for φ in the dielectric. This then is the basis for my Conjecture 2 above. Somehow my "theory" gives an asymmetric current and A_z lines that match the boundary, whereas the known DC solution instead gives a uniform current J_z and A_z lines that do NOT match the boundary.

Elsewhere I crudely said let D be a transverse dimension and then

$$T1 \equiv \partial_x A_x + \partial_y A_y \approx (A_x + A_y)/D$$

$$T2 \equiv \partial_z A_z \approx \beta_d A_z = (2\pi/\lambda)\, A_z$$

$$T1/T2 = (A_x + A_y)/D \;/\; (2\pi/\lambda)\, A_z = (A_x + A_y)/A_z * (1/2\pi) * (\lambda/D)$$
$$= \text{small} \qquad \text{large}$$

I don't recall this result earlier. It shows that the derivatives might offset the potential ratio and make the above approximation invalid. Now for DC and J_z = uniform, I know of a surety that the A_z lines do NOT line up with the conductor boundaries, so $A_z = (1/v_d)\, \varphi$ cannot be exactly true. This is a long subject I have dealt with elsewhere. Maybe in the skin effect limit we get D replaced by δ, and then we have

$$T1/T2 = (A_x + A_y)/A_z * (1/2\pi) * (\lambda/\delta)$$

but then this is even worse since the last factor is then even larger than before.

Conclusion: $A_z = (1/v_d)\, \varphi$ is probably not valid for any ω. Earlier in "computation of Az for two cylinders" I claimed this was valid in the skin limit and there justified the King approach to things.