Reflection Scenario v 2 PhL 6.20.14

The reflection model here is improved over last time, but it does not kill off the J_z asym at low ω, see item 4 General Comments below. Far below is an early proof that E tracks n(θ). Then I am off trying to blame it all on the radial Hall effect, but then somehow that idea collapsed. I eventually concluded that radial Hall is too small an effect to cause J_z asym. (10.8.14)

Here I quote details figured out on 6.19.14 in "reflection scenario", but starting deep into the document at a place titled "Resume this development on 6.19.14" which is on page 16.

[When I wrote this, I was imagining "gold" to be a perfect conductor. I later learned that it is a worse conductor than copper, but I leave it as gold here with the idea of perfect conductor.]

[Quantity β_d is now called k in Appendix D]

[no loss because the reflected wave has same amplitude as the incident wave]

1. We imagine a "gold block" sitting at z = L which causes the reflection. The basic solution superposition is this:

$$E_r(r,m,z,t) = (j/4)\,\eta_m\, I\, R_{dc}\, (a\beta_d)\, g_m \{ e^{j(\omega t-\beta_d[z-L])} - e^{j(\omega t+\beta_d[z-L])} \}$$

$$= (j/4)\,\eta_m\, I\, R_{dc}\, (a\beta_d)\, g_m \{ 2j \sin(-\beta_d[z-L]) \}\, e^{j(\omega t)}$$

So get rid of the time to define

$$E_r(r,m,z) = (j/4)\,\eta_m\, I\, R_{dc}\, (a\beta_d)\, g_m \{ 2j \sin(-\beta_d[z-L]) \} \ .$$

So then we have

$$E_z(r,m,z) = (1/4)\,\eta_m\, I\, R_{dc}\, (a\beta')\, f_m \{ 2 \cos(-\beta_d[z-L]) \}$$
$$E_r(r,m,z) = (j/4)\,\eta_m\, I\, R_{dc}\, (a\beta_d)\, g_m \{ 2j \sin(-\beta_d[z-L]) \}$$
$$E_\theta(r,m,z) = (1/4)\,\eta_m\, I\, R_{dc}\, (a\beta_d)\, h_m \{ 2j \sin(-\beta_d[z-L]) \} \ .$$

The justification is as follows:

Argument #1: since E_r and E_θ are transverse to the gold block, and since they are 0 inside the gold block, and since E_t must be continuous, we need then to have the sine form.

Argument #2: The left going wave has $\beta_d \rightarrow -\beta_d$ and this is why I show the minus sign between the terms in $E_r(r,m,z,t)$ above. [???]

Fact: Looking at $E_z(r,m,z)$ above, we have asym J_z and this is in the model right from the very start. So nothing I do below is going to make asym J_z "go away" at ω = 0. Recall that for $\omega \rightarrow 0$.

$f_m = (r/a)^m (m+1) (2/\beta' a) \qquad f_0 = 4/(a\beta')$

and therefore

$$J_z(r,m,z) = \sigma (1/4) \eta_m I R_{dc} (r/a)^m (m+1) (2\{ 2 \cos(-\beta_d[z-L]) \} \qquad m \neq 0$$

Note added 8/6/14 in review:

$$\frac{J_z(r,m,z)}{J_z(r,0,z)} = \frac{\eta_m f_m}{f_0} = \frac{\eta_m (r/a)^m (m+1) (2/\beta' a)}{4/(a\beta')} = (1/2) \eta_m (r/a)^m (m+1) \qquad m \neq 0$$

and this shows that the $m \neq 0$ moments never go away. This does assume $\beta' a$ is very small.

2. Using $J_z = \sigma E_z$ with $E_z$as above, it is possible to compute $I_{tot}(z)$, the total current in the wire. I do that in the original document and I will summarize that calculation here.

$$J_z(r,m,z) = \sigma (1/2) \eta_m I R_{dc} (a\beta') f_m \cos(\beta_d[z-L])$$

$$J_z(r,\theta,z) = J_z(r,m=0,z) + 2\Sigma_{m=1}^{\infty} J_z(r,m,z) \cos(m\theta)$$

The last terms give nothing in the I_{tot} integration so we then have

$$I_{tot}(z) = \int dA \; J_z(r,m=0,z) = 2\pi \int_0^a dr \, r \, J_z(r,m=0,z)$$

$$= 2\pi \int_0^a dr \, r \, [\sigma (1/2) I R_{dc} (a\beta') f_0(r) \cos(\beta_d[z-L])]$$

$$= 2\pi [\sigma (1/2) I R_{dc} (a\beta') \cos(\beta_d[z-L]) \int_0^a dr \, r \, f_0(r)$$

But then as I show in detail in the original doc,

$$\int_0^a dr \, r \, f_0(r) = 2 a/\beta'$$

so then

$$I_{tot}(z) = 2\pi [\sigma (1/2) I R_{dc} (a\beta') \cos(\beta_d[z-L]) \; 2 a/\beta'$$

$$= 2\pi [\sigma (1/2) I 1/(\pi a^2 \sigma) (a) \cos(\beta_d[z-L]) \; 2 a$$

$$= 2 I \cos(\beta_d[z-L])$$

So this tells us the current at any point in the shorted cylinder line, and the current is max when z = L where it has the value 2I where I is the parameter which appears in the App D solutions. Notice that I is no longer the same as i(0) because i(z) is now $I_{tot}(z)$. As expected, we have $I_{tot}(z)$ being a standing wave.

If we use the no-loss β_d, we find that at $\omega = 0$ $\beta_d = 0$ and then

$$I_{tot}(z, \omega=0) = 2I$$

3. I think the right model for $n(\theta)$ for the reflection superposition is this

$$n_{tot}(\theta,z,t) = n(\theta) \left[e^{j(\omega t - \beta_d [z-L])} - e^{j(\omega t + \beta_d [z-L])} \right]$$

$$= n(\theta)\, 2j \sin(-\beta_d (z-L)) e^{j\omega t}$$

$$n_{tot}(\theta,z) = n_1(\theta)\, 2j \sin(-\beta_d (z-L))$$

where $n(\theta)$ is the single-wave $n(\theta)$ as computed in Chapter 6 for the two round wires. The total charge then per unit length is going to be

$$q_{tot}(z) = 2j \sin(-\beta_d (z-L)) \int d\theta\, a\, n(\theta) = 2j \sin(-\beta_d (z-L))\, q(0)$$

where $q(z=0)$ is the single-wave charge/length. Since the capacitance C is fixed by geometry regardless of what waves are going on, we conclude that

$$V_{tot}(z) = q_{tot}(z)/C = 2j \sin(-\beta_d (z-L))\, q(0)/C = 2j \sin(-\beta_d (z-L))\, V(0)$$

where V(0) was called just V in appendix D for the single wave. So let's use this form

$$V_{tot}(z) = 2j \sin(-\beta_d (z-L))\, q(0)/C$$

as a description of $V_{tot}(z)$ on our reflecting waves line! It vanishes at z = L as expected, and it is a standing wave.

We can then define the impedance at any point on the line to be

$$Z_{tot}(z) = V_{tot}(z)/I_{tot}(z) = 2j \sin(-\beta_d (z-L))\, q(0)/C \;/\; 2\, I \cos(\beta_d [z-L])$$

$$= -2j\, q(0)/(2IC) \tan(\beta_d [z-L]) = q(0)/(jIC) \tan(\beta_d [z-L])$$

and this has a familiar look from ancient times, it has zeros and poles going along the line.

At the left end of our line we have: [first argument is z]

$$I_{tot}(0;\omega) = 2\, I \cos(\beta_d L)$$

$V_{tot}(0;\omega) = 2j \sin(\beta_d L)\, q(0)/C$ // this is the "driving voltage" for the reflection problem

$Z_{tot}(0;\omega) = q(0)/(jIC)\tan(\beta_d[-L]) = [\, j\, q(0) / IC]\ \tan(\beta_d L)$

If we assume that $\beta_d \rightarrow 0$ as $\omega \rightarrow 0$, then in the DC limit with no-loss assumed we get

$I_{tot}(0;0) = 2\, I$
$V_{tot}(0;0) = 0$ // this is the "driving voltage" for the reflection problem
$Z_{tot}(0;0) = 0$

We get $V_{tot}(0;\omega) = 0$ because this is a perfect line with no loss! So perhaps better to use our loss model version of β_d which says $\beta_d = (\omega - j\omega_c)/v_d$. Then our limit is $\beta_d \rightarrow -j\omega_c/v_d$ which recall is a small quantity. Then

$\cos(\beta_d L) \rightarrow \cos(-j\omega_c/v_d\, L) \approx 1$ since L is not going to be light seconds long.

$\sin(\beta_d L) \rightarrow \sin(-j\omega_c/v_d\, L) \approx -j\omega_c/v_d\, L$ since $\sin x = x$

$\tan(\beta_d L) \rightarrow -j\omega_c/v_d\, L$ as well.

Then we find that

$I_{tot}(0;0) = 2\, I$
$V_{tot}(0;0) = 2j \sin(\beta_d L)\, q(0)/C = 2j\, [-j\omega_c/v_d\, L]\, q(0)/C = 2\, [(\omega_c/v_d)\, L]\, [q(0)/C\,]$
$Z_{tot}(0;0) = [\, j\, q(0) / IC]\ \tan(\beta_d L) = [\, j\, q(0) / IC]\, [\ -j\omega_c/v_d\, L] = [\, q(0) / IC]\, [\, \omega_c/v_d\, L]$

So now at least we have some kind of DC limit for these quantities.

If I now force $V_{tot}(0;0)$ to be some kind of V_{drive} (perhaps from a low int imp generator), that gives a value for q(0) from the middle line above. Recall that I was the single-wave current. In that single wave scenario we had

$i(z) = q(z)\, v$ (4.11.19a)

So I then argue that $q(0) / I = 1/v_d$ and then I can restate a lot of results above

$V_{tot}(z;\omega) = -\, j\, 2I\, [1/(Cv_d)]\ \sin(\beta_d(z-L))$
$I_{tot}(z;\omega) = 2I\ \cos(\beta_d[z-L])$
$Z_{tot}(z;\omega) = -j\, [1/(Cv_d)] * \tan(\beta_d[z-L])$ dimensions good!

$V_{tot}(0;\omega) = j\, 2I\, [1/(Cv_d)]\ \sin(\beta_d L)$ // $= V_{drive}$
$I_{tot}(0;\omega) = 2I\ \cos(\beta_d L)$
$Z_{tot}(0;\omega) = j\, [1/(Cv_d)] * \tan(\beta_d L)$

$$V_{tot}(0;0) = \; j\,2I\,[1/(Cv_d)]\,[-j\omega_c/v_d\,L] \; = (\,2I/C)\,[\omega_c/v_d^2]\,L$$
$$I_{tot}(0;\,0) = \quad 2I$$
$$Z_{tot}(0;0) = \; [1/(C)]\;[\omega_c/v_d^2]\,L$$

Now in the loss model we have

$$\omega_c \; = \; (R_{DC}C)\,(v_d^2/2) \qquad\qquad \Rightarrow \qquad \omega_c/v_d^2 = \; (R_{DC}C)/2$$

so we can rewrite

$$V_{tot}(0;0) = (\,2I/C)\,[(R_{DC}C)/2]\,L \; = (\,I)\,[(R_{DC})]\,L \; = I\,/\,(LR_{DC})$$
$$I_{tot}(0;\,0) = \quad 2I$$
$$Z_{tot}(0;0) = \; [1/(C)]\;[(R_{DC}C)/2]\,L \; = \; [(R_{DC})/2]\,L \; = (LR_{DC}/2)$$

where recall that R_{DC} is the sum of both wires. So these last results seem reasonable, not sure of factors of 2. BUT, we still have asym J_zI am pretty sure!

4. General Comments

The superposed reflection solution is perhaps accurate to describe high ω action on such a shorted line with two cylinders. However, the J_z distribution remains asymmetric all the way down to ω = 0 and we know this result is incorrect. This tells us that something in the theory is not accurate at low frequencies.

I have always felt that everything in Appendix D was accurate down to DC. So I think the solutions obtained there for E are all good down to ω = 0. I assume as ansatz a wave form and I use the two BC's and I get all those solutions one sees there in various places.

Since Appendix D is correct down to ω = 0, where does the problem lie?? I do think that n(θ) on the two cylinder line is accurate down to DC. I do think the CPBC is valid all the way down as well.

Now it is true that the Chapter 4 transmission line theory is NOT valid down to ω = 0. In particular, A_z is not constant on the cylinders. The fields E and B are not at right angles. So something strange happens out in that dielectric. [I think it is T(z) which makes e^{-jkz} not work at low ω, but also the transverse A_z equation is invalid as stated since A_z not constant on the surface for low ω. 10.8.14]

In our reflection model, we do at least get $I_{tot}(z;\,\omega)$ → a DC current 2I, so this DC result at least is what I want to see. But the issue is the non-uniformity of the current!!! The Appendix D equations say J_z is non-uniform, and I am saying that the App D equations are good down to DC! (confirmed once again below).

I have a very loose bolt somewhere, and I simply cannot find it, despite 4 months of looking for it maybe 10 hours a day! The "reflection scenario" seems to me to be a completely viable model of the shorted two cylinders, but it gives the wrong ω = 0 limit. Maybe I can make a list of possible locations of the error.

1. The expo ansatz at the start of App D is no good at low ω because it predicts the wrong result even though that result satisfies all Max and Helm equations and the two BC's. [well, the ansatz is no good at low ω because the TL equations are no good there.]

2. Something I am overlooking in App D restricts App D results to be invalid at low ω.

3. Something is just wrong with the "reflection scenario" as a model for the shorted cylinders.

4. I have a math error in App D so E fields I get are in fact wrong, through they satisfy Max and Helm.

5. Somehow n(θ) goes away at DC, the surface charge "runs away" or what have you.

6. In the CPBC which says $E_r(r=a,m) = (j\omega/\sigma)\ N_m$, at ω = 0 this BC stops doing anything, and says $E_r(r=a,m) = 0$, so the mechanism causing asymmetry goes away. Addressed in 6.5 (d).

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I have said that losses build up at low ω, but have not made this too clear. Certainly below ω_c we have no wave and all exponential decay! So I am getting something like this for a solution

$$E_z(r,m,z) = (1/4)\ \eta_m\ I\ R_{dc}\ (a\beta')\ f_m\ \{\ 2\ \cos(\beta_d[z-L])\ \}$$

But what does this mean when $\beta_d \approx$ =- $j\omega_c/v_d$. Then the above says

$$E_z(r,m,z) = (1/4)\ \eta_m\ I\ R_{dc}\ (a\beta')\ f_m\ \{\ 2\ \cosh(\omega_c/v_d[z-L])\ \}$$

So yes, there is expo increase going backwards from z = L to z = 0. But maybe it is better to view the sum of waves again

$$\{\ e^{j(\omega t-\beta_d[z-L])} - e^{j(\omega t+\beta_d[z-L])}\ \}$$

I think suddenly that this might be wrong in the lossy situation. The right-moving wave decays before it hits the gold wall, and so the amplitude of the reflection is not unity after all. So at low ω, we are trying to deal with strongly decaying waves, and I guess the theory above no longer works in this case.

The odd thing is to just consider the right-going wave with its basic $e^{j(\omega t-\beta_d z)}$. At low ω, this has a strong amplitude decay, though that depends on $(\omega_c/v_d)L$. It might NOT be such strong decay. But you have to remember that the DC situation is "all loss". The Belden example had a decay distance of a few km, so in fact decay is NOT rapid.

Question: I like to write $i(z) = i(0)\ e^{j(-\beta_d z)}$. But how can the current decay in the ω = 0 resistor where the current has to be the same constant value at all z?? So this form for i(z) seems very wrong near ω = 0. Where did this equation come from?? It first appears in (4.7.3) and I state the decay in (4.9.2) . Well the loss involves $R_{dc}C$ and this product stays the same all the way to $\omega \rightarrow 0$. You still get loss because you still have some C left. But at absolute ω = 0, the C can have no effect.

Still, I think this is a significant question. Whatever leads to the above equation has something wrong as you approach $\omega \rightarrow 0$ since we MUST find in that limit that i(z) = constant.

Question: As you go to the ω=0 limit, we expect to see V(z) "drop off" as we go down the line, but we don't expect to see i(z) drop off. They are different in this limit, and somehow that difference is getting lost. If I say that E_i all drop off, then J_z has to drop off, and then i(z) drops off.

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Idea done before: Suppose you take Appendix D solutions in the two directions without evaluating the constants. Then you get

$$E_{zR}(r,m) = -j(\beta'/\beta_d)\frac{K_{mR}}{2} J_m(x)$$

$$E_{zL}(r,m) = -j(\beta'/\beta_d)\frac{K_{mL}}{2} J_m(x)$$

In order to get a *standing* wave you are forced to having the two K's equal!!!
Same for the a coefficients. So then we can say

$$E_z(r,m,z) = [-j(\beta'/\beta_d)\frac{K_{mR}}{2} J_m(x)]\{2\cos(-\beta_d[z-L])\}$$

$$E_r(r,m,z) = [a_m x^{-1} J_m(x) + \frac{K_m}{2} J_{m+1}(x)]\{2j\sin(-\beta_d[z-L])\}$$

$$jE_\theta(r,m,z) = [-a_m x^{-1} J_m(x) + (\frac{K_m}{2} + \frac{a_m}{m}) J_{m+1}(x)]\{2j\sin(-\beta_d[z-L])\}.$$

$$n(\theta,z) = n(\theta)\{2j\sin(-\beta_d(z-L))\}$$

$$N_m(z) = N_m\{2j\sin(-\beta_d(z-L))\}$$

NOW let's apply the two BC's. The E_θ one requires that

$$[-a_m x_a^{-1} J_m(x_a) + (\frac{K_m}{2} + \frac{a_m}{m}) J_{m+1}(x_a)] = 0$$ // just as in (2) of lines p 298

The CPBC must hold at each value of z,

$$E_r(r=a,m,z) = (j\omega/\sigma) N_m(z)$$

or

$$[a_m x_a^{-1} J_m(x_a) + \frac{K_m}{2} J_{m+1}(x_a)]\{2j\sin(-\beta_d[z-L])\} = (j\omega/\sigma) N_m\{2j\sin(-\beta_d(z-L))\}$$

or

$[a_m\ x_a^{-1}\ J_m(x_a) + \frac{K_m}{2}\ J_{m+1}(x_a)] = \ (j\omega/\sigma)\ N_m$ // just as in (1) of lines page 298

So the K and a are going to come out exactly the same, although now I am applying it to the total superposition wave.

So try as I might, I cannot blame the "failure" of the reflection scenario on a bad CPBC.

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The div E tracking proof at low frequencies.

Start off as in (6.5.14) and near the surface we get

$$E_z(r,\theta) \approx (1/j\beta_d)\ (1/r)\ \partial_r\ (r\ E_r(r,\theta)) \quad . \qquad \text{// near r = a} \qquad (6.5.15)$$

$$E_z(a,\theta) \approx (1/j\beta_d)\ (1/a)\ [\partial_r\ (r\ E_r(r,\theta))]^{r=a-} \quad . \qquad \text{// near r = a} \qquad \text{STARE} \qquad (6.5.15)$$

So this says probably that E_z is asym if E_r is asym. But, the CPBP says

$$E_r(r=a,\theta) = (j\omega/\sigma)\ n(\theta) \qquad \textit{(D.2.24)} \qquad (6.5.14)$$

This clearly says that, as $\omega \to 0$, although $n(\theta)$ maintains its shape, we get $E_r(r=a,\theta) \to 0$. Since my expected limit is that $E_r(r,\theta) = 0$ every where inside at $\omega = 0$, certainly derivatives are 0. Then the above line says that $E_z(a,\theta) \to 0$, assuming we use the loss model for β_d. But I expect to have E_z = come constant uniformly inside the wire, not $E_z = 0$. So what is going wrong here? Recall that ansatz says:

$$E_z(r,\theta,z) = E_z(r,\theta)\ e^{j(\omega t - \beta_d [z-L])}$$

With the loss model, we get $\beta_d \to -j\omega_c/v_d$ = a constant as $\omega \to 0$. Therefore, $E_z(r,\theta,z)$ = constant is RULED OUT by the ansatz, right at the start! So it is impossible to obtain E_z = constant in any limit of Appendix D results !!! Since E_z = constant is the correct $\omega = 0$ limit, Appendix D cannot take us there because it assumes at the start that E_z = constant is ruled out.

Wrong! My issue is not constant in z, it is constant in θ. Well, go back and stare some more:

$$E_z(a,\theta) \approx (1/j\beta_d)\ (1/a)\ [\partial_r\ (r\ E_r(r,\theta))]^{r=a-} \quad . \qquad \text{// near r = a} \qquad \text{STARE} \qquad (6.5.15)$$

Again, as $\omega \to 0$ we seem to get $E_r(a,\theta) = 0$ just below the surface from the CPBC. This seems to be causing $E_z(a,\theta) \to 0$. This is the issue. This is the answer that is wrong. The true result has $E_z(a,\theta) \to E_{z0}(z)$ which we allow as may decay in z. But the above is saying $E_z(a,\theta) \to 0$. I think this is the core of my problem somehow. For the infinite line, we really do have $E_z \to 0$. But my issue now is with the reflection scenario, where we still have div **E** = 0 just below the round wire surface.

June 21, 2014

I realized last night that, amazingly enough, it is the "radial Hall effect" that is going to resolve the little paradox I state above. The conclusions of the radial Hall section are these:

1) At DC, inside the wire there is a radial field $E_r(r,\theta) = E_s(r/a)$ where E_s is a constant. However, there is no radial current J_r that goes along with it. Thus, we have $E_r(r,\theta) = E_s(r/a)$ with $J_r(r,\theta) = 0$. The normal Ohm's Law is thus violated as outlined in App N.

2) The CPBC *really* says this

$$J_r(r=a,\theta) = (j\omega)\, n(\theta)$$

since it comes from div J = $-\partial_t\rho$. The CPBC is about J_r, and not about E_r. Thus, we $\omega \rightarrow 0$, we end up with $J_r(a,\theta) = 0$ which is consistent with the radial Hall effect result.

3) The paradox noted above had this form

$$E_z(a,\theta) \approx (1/j\beta_d)\,(1/a)\,[\partial_r\,(r\,E_r(r,\theta))]^{r=a-} \quad . \qquad \text{// near r = a} \qquad \text{STARE} \qquad (6.5.15)$$

My paradox was that I felt $E_r \rightarrow 0$ from the CPBC as $\omega \rightarrow 0$ and thus $E_z \rightarrow 0$, but I know $E_z \neq 0$.

To resolve this paradox, we have to "back up" and re-examine div E = 0 near (6.5.15). First of all, we have to set div E = ρ/ε_0 where ρ is the constant Hall free charge. Then doing everything else the same we get

$$\partial_r\,(r\,E_r(r,\theta,z)) + r\,\partial_z E_z(r,\theta,z) \approx \rho/\varepsilon_0 \qquad \text{// near r = a}$$

$$\partial_r\,(r\,E_r(r,\theta)) - j\beta_d\,r\,E_z(r,\theta) \approx \rho/\varepsilon_0 \qquad \text{// using } \partial_z \rightarrow -j\beta_d\text{, see (D.1.16), then cancel } e^{j\beta_d z} \text{ factors}$$

$$j\beta_d\,r\,E_z(r,\theta) \approx \partial_r\,(r\,E_r(r,\theta)) - \rho/\varepsilon_0$$

$$E_z(r,\theta) \approx (1/j\beta_d r)\,\partial_r\,(r\,E_r(r,\theta)) - (1/j\beta_d r)\rho/\varepsilon_0$$

But that paradox hopefully goes away when we write $J_r \rightarrow 0$ from the CPBC and $E_r(r,\theta) = E_s(r/a)$. Then we get

$$E_z(a,\theta)^{\text{1st term}} \approx (1/j\beta_d a)\,[\partial_r\,(r\,E_r(r,\theta))]^{r=a-} = (1/j\beta_d a)\,[\partial_r\,(r\,E_s(r/a)]^{r=a-}$$

$$= (1/j\beta_d a)\,(E_s/a)\,[\partial_r\,r^2]^{r=a-} = (1/j\beta_d a)\,(E_s/a)\,2a$$

$$= (E_s/a)\,(1/j\beta_d)\,2 = 2E_s\,(1/ja\beta_d)$$

The first term is a constant, and to this we now add the second term where

$$\rho = \varepsilon_0(2E_s/a) \ . \qquad \text{(N.7.16)}$$

So with both terms we get

$$E_z(a,\theta) \approx (1/j\beta_d r)\, \partial_r (r\, E_r(r,\theta)) - (1/j\beta_d r)\rho/\varepsilon_0$$

$$= 2E_s\, (1/ja\beta_d) - (1/j\beta_d a)\, 1/\varepsilon_0[\varepsilon_0(2E_s/a)]$$

$$= (1/ja\beta_d)\, [\, 2E_s - 2E_s] = 0$$

oops! Something went wrong. In Appendix N I compute ρ from div**E** but I set $\partial_z E_z = 0$ since the wire is infinitely long and uniform. But in Chap (6.5) I have $\partial_z E_z = -j\beta_d E_z \neq 0$. This latter expression allows for slow expo decay of E_z in the TL if I use my little loss model. But at ω = 0, the capacitance has no effect in the TL and I have G = 0 all the time, so then I expect E_z NOT to decay. On the other hand, I know that for ω = 0 in this case, we really do have $E_z = 0$ because there is no current for the ∞TL. So maybe my oops is OK for the single wave scenario.

Let's try the above for the reflection scenario. We then have

$$\partial_r (r\, E_r(r,\theta,z)) + r\, \partial_z E_z(r,\theta,z) \approx \rho/\varepsilon_0 \qquad \text{// near r = a}$$

But now

$$E_r(r,m,z) = (j/4)\, \eta_m\, I\, R_{dc}\ (a\beta_d)\, g_m\, \{\, 2j \sin(-\beta_d[z-L])\, \}$$
$$E_z(r,m,z) = (1/4)\, \eta_m\, I\, R_{dc}\ (a\beta')\, f_m\, \{\, 2 \cos(-\beta_d[z-L])\, \}$$

Just writing this out gives

$$(j/4)\, \eta_m\, I\, R_{dc}\ (a\beta_d)\, \{\, 2j \sin(-\beta_d[z-L])\, \}\partial_r (r\, g_m(r))$$

$$+ r\, \partial_z\, (1/4)\, \eta_m\, I\, R_{dc}\ (a\beta')\, f_m\, \{\, 2 \cos(-\beta_d[z-L])\, \} \approx \rho/\varepsilon_0$$

In this case, we have

$$\partial_z\{\, 2 \cos(-\beta_d[z-L]) = \{-2\, \beta_d \sin(\, \beta_d[z-L])\, \}$$

and then the equation reads

$$(j/4)\, \eta_m\, I\, R_{dc}\ (a\beta_d)\, \{\, 2j \sin(-\beta_d[z-L])\, \}\partial_r (r\, g_m(r))$$

$$+\ a\, (1/4)\, \eta_m\, I\, R_{dc}\ (a\beta')\, f_m\, \{-2\, \beta_d \sin(\, \beta_d[z-L])\, \} \approx \rho/\varepsilon_0$$

or

$$- (1/4)\,\eta_m\, I\, R_{dc}\ (a\beta_d)\ \{\ 2\sin(-\beta_d[z-L])\ \}\partial_r\ (r\ g_m(r))$$

$$-\ \ a\ (1/4)\,\eta_m\, I\, R_{dc}\ (a\beta')\ f_m\ \{2\ \beta_d \sin(\ \beta_d[z-L])\ \}\quad \approx \rho/\varepsilon_0$$

or

$$- (1/4)\,\eta_m\, I\, R_{dc}\ \{\ 2\sin(-\beta_d[z-L])\ \}\ [\ \ (a\beta_d)\ \partial_r\ (r\ g_m(r)) + (a\beta')\ f_m\ ]\ \approx\ \rho/\varepsilon_0$$

Well, let's pause on this for more comments.

Comment 1. In Appendix D, I use β_d in the opening ansatz. This is really only β_d in the lossless case, so the name β_d is a misnomer really. But going ahead with this symbol β_d and its ansatz, we grind through Appendix D and we get the answers we get there as functions of β_d. This analysis ignores the possibility of a tiny Hall charge density ρ inside the round wire since we assume div **E** = 0. The analysis also assumes that $J_r = \sigma E_r$ but I know that this is wrong close to DC since at DC we have $J_r = 0$ and $E_r \neq 0$ inside the wire. This last fact is consistent with the true CPBC which says $J_r(r=a,\theta) = (j\omega)\ n(\theta)$. The Hall surface charge is also ignored. Also, if I try to activate my loss model, I know it is only valid for small loss since it uses the TL equations. So all these things are reasons that the Appendix D results might be wrong at $\omega = 0$. Finally I have some possible reasons!

Is there some easy way to "repair" Appendix D so it becomes more accurate at low ω?

Appendix D only uses E fields in the opening sections. So the first summary is still OK in terms of the unknown K_m and a_m. The first deviation occurs at (D.2.24) where I first use $J_r = \sigma E_r$. So this would be a place where some kind of repair might be installed. From Appendix N we have

$$J_z = \sigma\ (\ E_z + \omega_c\tau\ E_r)\ \frac{1}{1+(\omega_c\tau)^2} \qquad \omega_c \equiv\ (qB/m)$$

$$J_r = \sigma\ (E_r - \omega_c\tau\ E_z)\ \frac{1}{1+(\omega_c\tau)^2} \qquad \mathbf{B} = B\ \hat{\boldsymbol{\theta}}$$

$$J_\theta = \sigma E_\theta\ . \qquad \sigma =\ (nq^2\tau/m) \qquad \text{(N.7.8)}$$

but this is only for DC! For general low ω AC, I don't know the connection between the J's and the E's.

OK, so I don't know how to do the repair, but I do have a REASON why the results won't be right at low ω. This then would explain the failure of the reflection model at low ω to give the correct two cylinders limit.

I am now ready to attempt an explanation of why my theory does not work for low ω. Then the question will be: at what ω does the theory "drop out" ? Well, one answer would be that it fails when $J_r = \sigma E_r$ fails. Or maybe when E_r becomes on the order of $E_{r,Hall}$. We know that

$E_{r,Hall}(r,\theta) = E_s(r/a)$

$E_r(r,m) = (j/4)\,\eta_m\, I\, R_{dc}\ (a\beta_d)\, g_m$

Maybe simplify this to say

$E_{r,Hall}(r,\theta) \sim E_s(r/a)$ $\qquad$ $E_s \equiv E_r(a) = \dfrac{\mu_0 I^2}{2\pi^2 a^3 nq}$ volts/m

$E_r(r,\theta) \sim (j/4)\, I\, R_{dc}\ (a\beta_d)\, g_0$ $\qquad$ $g_o \approx 2\,(r/a) = 2$

so then we get at r = a,

$\dfrac{\mu_0 I^2}{2\pi^2 a^3 nq} \ll (j/4)\, I\, R_{dc}\ (a\beta_d)\, 2$ $\qquad$ valid

Thus, the limit will be I dependent.

$\dfrac{\mu_0 I}{\pi^2 a^3 nq} \ll R_{dc}\ (a\beta_d) = 1/(\sigma\pi a^2)\,(a\beta_d) = \beta_d/(\sigma\pi a)$

$\dfrac{\mu_0 I}{\pi a^2 nq} \ll \beta_d/(\sigma)$

If we use the no-loss version of β_d then

$\dfrac{\mu_0 I}{\pi a^2 nq} \ll \omega/(v_d\sigma)$

$\omega >> \dfrac{\mu_0 I\, v_d \sigma}{\pi a^2 nq}$

$2\pi f >> \dfrac{\mu_0 I\, v_d \sigma}{\pi a^2 nq}$

$f >> \dfrac{\mu_0 I\, v_d \sigma}{2\pi^2 a^2 nq}$

I will be amazed if I have the units right.

dimRHS = henry/m * amps * m/sec * 1/ohm * 1/m * $1/m^2$ * m^3 * 1/ Cou

= henry * amps * 1/sec * 1/ohm * 1/Cou

=ohm-sec * amps * 1/sec * 1/ohm * 1/Cou

= amps * 1/Cou = sec^{-1} yes!

Let's try some numbers for Belden 8281 cable

$\mu_0 = 4\pi \times 10^{-7}$ henry/m

I = 1 amp

$v_d = 2$ e 8 m/sec

$\sigma = 5.81 \times 10^{7}$ mho/m

a = 394e-6 m ?

$n = 8.5 \times 10^{28}$ electrons/m^3

$q = 1.6 \times 10^{-19}$ Coul

Here is Maple on this:

```
mu0 := 4*Pi*1e-7;
```

$$\mu 0 := .4\ 10^{-6}\ \pi$$

```
vd := 2e8;
```

$$vd := .2\ 10^{9}$$

```
sigma := 5.81e7;
```

$$\sigma := .581\ 10^{8}$$

```
a := 394e-6;
```

$$a := .000394$$

```
#a := 1e-2;    # power line
n := 8.5e28;
```

$$n := .85\ 10^{29}$$

```
q := 1.6e-19;
```

$$q := .16\ 10^{-18}$$

```
i := 1;
```

$$i := 1$$

```
#i := 100;  #power line
f := evalf(mu0*i*vd*sigma/(2*Pi^2*a^2*n*q));
```

$$f := 350392.5786$$

The result is 350 KHz. Recall that f_c for this same cable was about 7800 Hz. This is much higher than I wanted to see it be!

What does this give for power line with a = 1e-2 m (1 cm) and I = 100A? I get 54 KHz which is not at all what I wanted to see!

Question: The loss model says $\beta_d = (\omega - j\omega_c)/v_d$ compared to $\beta_d = \omega/v_d$ for the no loss model. If I go back to the above Hall condition

$$\frac{\mu_0 I}{\pi a^2 nq} \ll \beta_d/(\sigma)$$

and if I use the loss thing on the RHS, then it says

$$\frac{\mu_0 I}{\pi a^2 nq} \ll \omega_c/(v_d\sigma)$$

and this then puts a low limit on current I guess for which App D works. But then I get the results in my examples that I need $\omega_c >>$ 350 KHz for Belden.