Starting Over 3_14_14 PhL 3.14.14
updated 3.17.14

This is a long rambling document. I give a status report, then make some conjectures, then I try various "Ideas" to solve the low ω mysterious asymmetry of J_z problem. Some ideas led to new Appendices and various ideas are incorporated into lines doc. But the mystery remains. The escape hatch Z_0 idea is discovered here in Idea K!

Idea A. Something is screwy with the BC between the inside and outside for a wave? Losses?
Idea B. The Debye currents bugaboo.
Idea C. Try simpler case of parallel plates. I did a whole huge effort on this, no big conclusion.
Idea D. I compute E_z for strong skin effect. This improves what I had in Sec 6.5 so I updated!
Idea E. Atomic forms, Helm vs Laplace for small ω, all makes sense.
Idea F. Try do $\omega \to 0$ first THEN compute a_m and K_m App D. Made no difference.
Idea G. Idea that n(θ) "gets into "$J_z(r,\theta)$ through the cpbc. This is now in Section 6.5.
Idea H. Search for others claiming non-uniform J_z at DC, including King and Matick.
Idea I. How explain uniform J_z at DC? Leads to notion of Radial Hall Effect, more web scan.
Idea J. Given the weird ω=0 J_z asym, what B field does it create? I state a result.
Idea K. Here I discover the Escape Hatch: since $Z_0 \to \infty$ we must have $I \to 0$ at $\omega = 0$

1. Status Report

I have been unsuccessful after working for a month trying to "repair" lines doc. There are so many equations and so many field variables, I cannot get them under control. I cannot seem to get any firm stakes in the sand, everything moves every day. I seem far from justifying the King Ch 4 approach, for example.

I cling very firmly to "the capacitor problem" as a starting point, only because most supporting data items use this theory to find things like Z_0 for a transmission line. The capacitor problem provides a "stake in the sand" only for the following field variables:

φ in the dielectric

I want to say that this determines **E** in the dielectric, but for ω > 0 we have

$$\mathbf{E} = -\nabla\varphi - j\omega\mathbf{A}$$

At DC, you can ignore the **A** term and go on to compute **E** and then n(θ) as I have done. But at AC, I don't seem to know anything about **A**, so this pathway to n(θ) is just not valid. For the capacitor problem, the above equation may be written

$$\mathbf{E}_t = -\nabla_t\varphi - j\omega\mathbf{A}_t$$

I do know that $\mathbf{A}_t$ is the integral of $\mathbf{J}_t$ over both conductors, but I don't know $\mathbf{J}_t$ so this gives no grip on the field $\mathbf{A}_t$. In Appendix D I do get these Helmholtz/divE =0 solutions before boundary conditions:

First summary of the E field solutions (D.2.21)

$$E_z(r,m) = -j\,(\beta'/\beta_d)\,\frac{K_m}{2}\,J_m(x) \qquad x = \beta' r \qquad \textit{(D.1.27)}$$

$$E_r(r,m) = a_m\,x^{-1}\,J_m(x) + \frac{K_m}{2}\,J_{m+1}(x) \qquad \beta'^2 = \beta^2 - \beta_d^2 \qquad \textit{(D.2.11)}$$

$$jE_\theta(r,m) = -a_m\,x^{-1}\,J_m(x) + \left(\frac{K_m}{2} + \frac{a_m}{m}\right) J_{m+1}(x) \qquad \textit{(D.2.15)}$$

and all I can say is that this "suggests" that there might be E_r and E_θ fields inside the wire, which of course means there might be J_r and J_θ inside the wire. But I don't know the constants K_m and a_m so I have little to say about J_r and J_θ. At DC I think both are 0. Notice how indirect the path is here:

$\mathbf{A}_t$ = Helm Integral ($\mathbf{J}_t$) but $\mathbf{J}_t$ = unknown

I do know that

$E_\theta = -(1/a)\partial_\theta\varphi - j\omega A_\theta$ at conductor outside surface

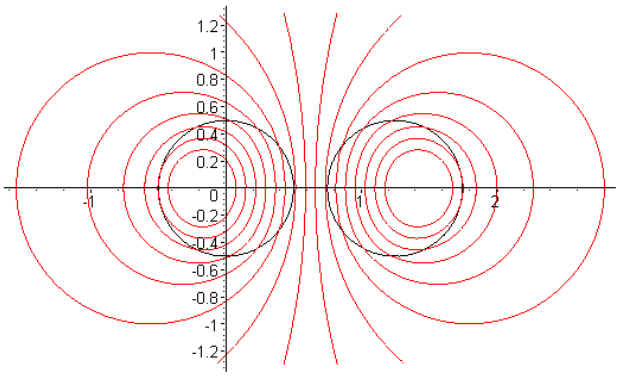
and I do know that $\partial_\theta\varphi = 0$ from the capacitor problem. So this leaves me with

$E_\theta = -j\omega A_\theta$ at conductor outside surface

I have been somehow assuming this is the same as $E_\theta = 0$ based on some vague ansatz that A_θ is "small", but in retrospect, I see no support at all for that claim. This has been an Achilles Heel of the theory for me since 1991 and for the last 6 months of resumed effort on this problem.

The capacitor problem comes into the picture when you look at the $\omega = 0$ "end" of the transmission line problem. You have a transverse PDE in the low-loss case which says $\nabla^2\varphi = 0$ with the two boundary conditions. This PDE system IS "the capacitor problem" and that gives φ. Perhaps we are treating a transmission line as "quasi static".

I would like to have some sort of similar PDE equation for **A** or maybe A_z. That is to say, some sort of magnetic "starting point" which goes with the electric starting point of the capacitor problem. At DC we do seem to have a physical starting point of uniform current in each conductor. I have shown that this starting point leads to a **B** field as shown here:



These are the **B** field lines. It happens that they do not line up with the conductor boundaries. This is similar to the plot Purcell makes on page 209 where the reader assumes constant J_z in the bus bars. The point I make is that the B lines do not match the conductor boundaries, so the fact that this happens in my picture above is not surprising.

Now there is a fixed relationship between **B** and **A** which is this:

$$\text{curl } \mathbf{A} = \mathbf{B} \qquad \Leftrightarrow \qquad \oint_C \mathbf{A} \bullet d\mathbf{s} \;=\; \int_S \mathbf{B} \bullet d\mathbf{S} \,. \tag{1.1.39}$$

I have often done my "thin math loop" analysis as follows: Place a thin rectangular math loop such that the long direction is into paper, and such that the loop lies right on a B field line such that there is then no B flux going through the loop. For such a math look, $\oint_C \mathbf{A} \bullet d\mathbf{s} = 0$. Since the loop is differentially thin, and since we assume A is finite everywhere. this means that the two sizes of the loop must give equal and opposite contributions. This says that A_z on the two sides of the loop must be the same. This in turn means that these red B lines are also equipotential surfaces for A_z. We thus arrive at the conclusion that at the DC "end" of the transmission line problem, the conductors do NOT have constant A_z on their surfaces. This is of course in direct conflict with King's analysis with his function W(z). This then puts me in a difficult position if I am trying to be a presenter of King's development! I think this basically says that his theory is invalid at low frequencies at least, where the capacitor problem is "most valid".

In addition to the argument just made, I have computed A_z directly for the DC case (assuming uniform current density) in "two cylinder calculations.doc", and the lines of constant A_z are the same as the lines of B, reaffirming the conclusion just stated.

Fact #1. The King theory of W(z) *does not* extend down to low frequencies near DC. This theory requires that A_z be constant on the conductor cross section, and that is clearly not true in the above picture, and it is clearly not going to true at .0001 Hz or 1 Hz. [correct]

Conjecture #1. The King theory applies at ω such that $\delta < a/10$, where a is a transverse dimension of the conductor and δ is the skin depth. This then puts a *lower* limit on the ω of applicability. [An *upper* limit on ω is provided by the transmission line limit and perhaps other factors.] [sounds right]

Fact #2. The argument that $H_n = 0$ just above the surface requires that $H_n = 0$ at some small distance below the surface, such as below δ. Distance δ must be small so that the sides of the Gaussian box don't admit significant **B** flux. This is all based on div **B** = 0 and so $\int \mathbf{B} \bullet d\mathbf{S} = 0$ for the little box. If $H_n = 0$ at the surface, then A_z lines are tangent to the surface, and then the King W(z) theory works! [yes]

Conjecture #2. First, the King Gauge says

$$\text{div } A = -\mu\varepsilon j\omega\varphi - \mu\sigma\varphi = -j\omega\mu(\varepsilon+\sigma/j\omega)\varphi = -j\omega\mu\xi\varphi = -j(\beta^2/\omega)\ \varphi \qquad \text{King gauge} \qquad (1.5.5)$$

which we can write as: $\partial_x A_x + \partial_y A_y + \partial_z A_z = -j(\beta_d^2/\omega)\ \varphi$ // dielectric

If we can ignore the first two terms, and if we assume z wave motion, this equation says

$$A_z = (1/v_d)\ \varphi$$

and this is very consistent with the King W(z) and V(z) theory. So the *conjecture is* that if you go below the ω boundary where δ is no longer small, you no longer have $|\ \partial_x A_x + \partial_y A_y\ | << \beta_d |A_z|$. And the conjecture also claims that above this ω boundary, we DO have this inequality. [all in Sec 3.7 now]

Conjecture #3. I suspect that in the skin effect limit if we consider $\mathbf{E}_t = -\nabla_t\varphi - j\omega\mathbf{A}_t$, it can be shown that the last term $-j\omega\mathbf{A}_t$ can be neglected compared to the first term $-\nabla_t\varphi$.

[I have a proof of this now in Section 3.7 for extreme skin limit]

Notice that both conjectures #2 and #3 assume in some sense that "transverse potential is small".

Conjecture #4. Even in the skin effect limit, there must be some notion of an "active region" along a conductor surface which has some kind of well-defined Z_s that can go into the King model. [I have solved this problem in Chapter 4 where Z_s is a certain average.]

Exercise: It would be nice to actually *demonstrate* the validity of these conjectures in the two-cylinder case about which we think we know a lot.

Problem #1. Why are the Appendix D equations for **E** wrong at low ω ? It would seem that Conjecture #3 is satisfied in this limit so that $\mathbf{E}_t = -\nabla_t\varphi$ is valid and therefore BC#2 is valid. I have no conjecture to answer this question! Is there something wrong with BC#1 ?

Idea A. Appendix D assumes $e^{j(\omega t - \beta_d z)}$ wave action. The use of β_d here implies no loss. But no loss implies very good conductor. That in turn implies small δ and therefore high ω. Although this β_d wave based solution for **E** and **B** satisfies Maxwell's equations in the dielectric perfectly, it does not satisfy the phase match in z BC at the conductor surface and that is why it fails [just an idea]. So there is then some boundary condition OTHER THAN BC#1 and BC#2 that I always use. Maybe things could be rescued if

I replaced the parameter β_d with something else which includes loss effects. // I did this in doc "including loss in" and it did not pan out. It does not force all moments m≠0 of E_z to vanish at ω = 0.

Idea B. Try including the Debye surface currents in the charge pump BC, maybe things near DC are dominated by surface charges. // I did this in a new section in doc "the surface currents", but it did not pan out. I computed new K_m and a_m constants, but since $(\beta'\delta_{debye}) << 1$ at all reasonable ω, the original constants came back.

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Idea C. Let's write a version of Appendix D for a parallel plate transmission line where we can use Cartesian geometry and not worry about the "y" dimension. Maybe this will shed light on the problem. // I did this in a single shot, and the conclusion is that at DC ω→0 you get J_z = constant!! There is no DC asymmetry in that simpler problem. [see "App D for par plates" doc and supporting mws and vsd]

Idea D. Examine the $E_z(r,m)$ solution for strong skin effect where x = β'r is large. I do this right here, and steal a result from "the current asymmetry" doc for the large x limit of Bessel,

$$\beta = (j-1)(1/\delta) \qquad x = \beta r = (j-1)(r/\delta)$$

$$J_m(x) = (2/\pi x)^{1/2} \cos(x - m\pi/2 - \pi/4) = (2/\pi x)^{1/2} (1/2)\, e^{+r/\delta}\, e^{j[(r/\delta)+m\pi/2+\pi/4]}$$

$$= (1/\sqrt{2}\,\pi\beta r)^{1/2}\, e^{+r/\delta}\, e^{j[(r/\delta)+m\pi/2+\pi/4]}$$

which shows m appears only in the phase. Then consider

$$f_m = \left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right]$$

Now

$$\frac{J_m(x)}{J_{m+1}(x_a)} = \frac{(1/\sqrt{2}\,\pi\beta r)^{1/2}\, e^{+r/\delta}\, e^{j[(r/\delta)+m\pi/2+\pi/4]}}{(1/\sqrt{2}\,\pi\beta a)^{1/2}\, e^{+a/\delta}\, e^{j[(a/\delta)+(m+1)\pi/2+\pi/4]}} = (a/r)^{1/2}\, e^{-(a-r)/\delta}\, e^{j[(r-a)/\delta-\pi/2]}$$

$$\frac{J_m(x)}{J_{m-1}(x_a)} = (a/r)^{1/2}\, e^{-(a-r)/\delta}\, e^{j[(r-a)/\delta+\pi/2]}$$

Then

$$f_m = \left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right] = (a/r)^{1/2}\, e^{-(a-r)/\delta}\, e^{j[(r-a)/\delta}\, [e^{-j\pi/2} - e^{+j\pi/2}]$$

$$= (a/r)^{1/2}\, e^{-(a-r)/\delta}\, e^{j[(r-a)/\delta}\, [-j - j]$$

$$= -2j\,(a/r)^{1/2}\, e^{-(a-r)/\delta}\quad e^{j[(r-a)/\delta}\quad = \text{independent of m !}$$

and there you see the classic skin effect including its phase. Note that it was crucial to keep track of the m dependent phase. If we ignored it, we would have said $f_m = 0$. The total solution is then

$$E_z(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\ (a\beta)\ [-2j\,(a/r)^{1/2}\, e^{-(a-r)/\delta}\ e^{j[(r-a)/\delta}]$$

$$= (1/4)\,\eta_m\, I\, R_{dc}\ ((j-1)(a/\delta))\ [-2j\,(a/r)^{1/2}\, e^{-(a-r)/\delta}\ e^{j[(r-a)/\delta}]$$

All the $E_z(r,m)$ are identical and only η_m makes them different! Thus I would claim that

$$E_z(r,\theta) = \sum_{m=-\infty}^{\infty} E_z(r,m)\, e^{jm\theta} = E_z(r,0) + \sum_{m\neq 0} \eta_m\,(\text{stuff})\, e^{jm\theta}$$

$$= (\text{stuff}) + \sum_{m=1}^{\infty} \eta_m\ (\text{stuff})\cos(m\theta)$$

$$= (\text{stuff})\ [\ 1 + \sum_{m=1}^{\infty} \eta_m \cos(m\theta)\ ]$$

$$= (1/4)\, I\, R_{dc}\ ((j-1)(a/\delta))\ [-2j\,(a/r)^{1/2}\, e^{-(a-r)/\delta}\ e^{j[(r-a)/\delta}]\ [\ 1 + \sum_{m=1}^{\infty} \eta_m \cos(m\theta)\ ]$$

where

$$= [\ 1 + \sum_{m=1}^{\infty} \eta_m \cos(m\theta)\ ] = 1 + 2 \sum_{m=1}^{\infty} (-1)^m\, e^{-m|\xi_1|} \cos(m\theta) = \text{shape of } n_1(\xi_1,\theta)$$

So in the deep skin effect limit, we have the n(θ) shape around azimuth, and expo decay in r, and this is exactly what I expect it to be for high frequency. [correct]

Idea E. Let's check atomic forms. I peruse Appendix D. I start with

$$(\nabla^2 + \beta^2)\, E_z = 0 \qquad \text{a true 3D Helm scalar Helm equation}$$

$$[\partial_r^2 + (1/r)\,\partial_r + (1/r^2)\,\partial_\theta^2 + \partial_z^2 + \beta^2\ ]\, E_z(r,\theta,z,t) = 0\ . \qquad \text{// cylindricals} \qquad (1)$$

Moon and Spencer page 15 suggests these atomic forms (I replace their p and ψ by my m and θ)

$$e^{jm\theta}\ \sin(\sqrt{\beta^2+q^2}\ z)\ J_m(jqr) \qquad \text{// recall that } \beta'^2 = \beta^2 - \beta_d^2 \text{ from (D.2.2)}$$

Now replace jq by β' and we then have, reading from M&S, $\beta^2 - \beta'^2 = \beta_d^2$

$$e^{jm\theta}\ \sin(\sqrt{\beta^2-\beta'^2}\,z)\,J_m(\beta' r) = e^{jm\theta}\ \sin(\beta_d z)\,J_m(\beta' r) = e^{jm\theta}\,J_m(\beta' r)\,e^{-j\beta_d z}$$

so OK, my solution contains exactly the right atomic forms for this Helm equation (just checking).

Now, as $\omega \rightarrow 0$ we know that $\beta \rightarrow 0$ so we should then have $\nabla^2 E_z = 0$ which is Laplace. When I look at the 3D Laplace atoms in my cylindrical area, I see this

$$e^{\pm kz}\,[\,J_m(kr), N_m(kr)]\ e^{\pm im\theta}$$

How does this jibe with the $\omega \rightarrow 0$ limit of my solution, which is this

$$E_z(r,\theta) = I\,R_{dc}\,[\,1 + \sum_{m=1}^{\infty} (-1)^m\ e^{-m|\xi_1|}\ (r/a)^m\,[\,(m+1)]\cos(m\theta)\,]$$

The only thing that seems "right" here is the $\cos(m\theta)$ factor.

But trace what happened. We started with 3D Helmholtz, I assumed an atomic form that was totally correct, and I ended up with

$$[\nabla_{2D}^2 + \beta'^2]\,E_z(r,\theta) = 0$$

which is a 2D Helmholtz in polars. Now I take $\omega \rightarrow 0$ and $\beta' \rightarrow 0$ (since both β and $\beta_d \rightarrow 0$), and then that leaves me with $[\nabla_{2D}^2]\,E_z(r,\theta) = 0$ which is 2D Laplace in polars. But I know the atomic forms for the 2D Laplace to be $r^{\pm m}\cos(m\theta)$ from good old Stak, and this is exactly what you see above for $E_z(r,\theta)$. So I think all is well regarding atomic forms. Very generally, I know that in my limit of interest, we must have

$$E_z(r,\theta) = A + \Sigma_{m=1}^{\infty} B_m\,r^m\,\cos(m\theta)$$

to be correct at $r = 0$. It's just that I was expecting $B_m = 0$ as $\omega \rightarrow 0$. Instead I got

$$E_z(r,m) = \{\,(1/4)\,\eta_m\,I\,R_{dc}\,(a\beta')\,[\frac{1}{J_{m+1}(x_a)} - \frac{1}{J_{m-1}(x_a)}]\,\}\ J_m(x)$$

$$\approx (1/2)\,\eta_m\,I\,R_{dc}\,(m+1)(r/a)^m$$

so the coefficient is then

$$B_m = (1/2)\,\eta_m\,I\,R_{dc}\,(m+1)(1/a)^m$$

Easy viewing comes from recasting the above as

$$\frac{E_z(r,m)}{E_z(a,m)} = \frac{J_m(x)}{J_m(x_a)}$$

Then the limit $(r/a)^m$ is inescapable.

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Idea F. What happens if you take the ω→0 limit first, then solve for coefficients? That is to say, suppose we are at this point in Appendix D, prior to coefficient evaluation,

$$E_z(r,m) = -j(\beta'/\beta_d)\frac{K_m}{2} J_m(x) \qquad x = \beta' r \qquad \textit{(D.1.27)} \qquad \text{(D.2.21)}$$

$$E_r(r,m) = a_m x^{-1} J_m(x) + \frac{K_m}{2} J_{m+1}(x) \qquad \beta'^2 = \beta^2 - \beta_d^2 \qquad \textit{(D.2.11)}$$

$$jE_\theta(r,m) = -a_m x^{-1} J_m(x) + \left(\frac{K_m}{2} + \frac{a_m}{m}\right) J_{m+1}(x) \qquad \textit{(D.2.15)}$$

And here are the BC's,

$$E_r(r=a,m) = (j\omega/\sigma) N_m \qquad \text{(D.2.26)}$$

$$E_\theta(r=a,m) = 0 \qquad \text{(D.2.27)}$$

Write this out like so:

$$a_m x_a^{-1} J_m(x_a) + \frac{K_m}{2} J_{m+1}(x_a) = (j\omega/\sigma) N_m \qquad (1)$$

$$-a_m x_a^{-1} J_m(x_a) + \left(\frac{K_m}{2} + \frac{a_m}{m}\right) J_{m+1}(x_a) = 0 \ . \qquad (2)$$

Now first take this to the small ω limit without going all the way, replacing the J functions,

$J_m(x) = (x/2)^m / m!$ = this is the small-x leading term and m > 0 here only let's say

$$a_m x_a^{-1} (x_a/2)^m / m! + \frac{K_m}{2} (x_a/2)^{m+1} / (m+1)! = (j\omega/\sigma) N_m \qquad (1) \text{ low } \omega$$

$$-a_m x_a^{-1} (x_a/2)^m / m! + \left(\frac{K_m}{2} + \frac{a_m}{m}\right) (x_a/2)^{m+1} / (m+1)! = 0 \ . \qquad (2) \text{ low } \omega$$

or

$$a_m x_a^{-1} (x_a/2)^m / m! + \frac{K_m}{2} (x_a/2)^{m+1} / (m+1)! = (j\omega/\sigma) N_m \qquad (1) \text{ low } \omega$$

$$-a_m x_a^{-1} (x_a/2)^m / m! + \left(\frac{K_m}{2} + \frac{a_m}{m}\right) (x_a/2)^{m+1} / (m+1)! = 0 \ . \qquad (2) \text{ low } \omega$$

Now use

$$x_a^{-1}\ (x_a/2)^m = (1/2)(2/x_a)\ (x_a/2)^m = (1/2)\ (x_a/2)^{m-1}$$

so

$$a_m/2\ (x_a/2)^{m-1}/m! + \frac{K_m}{2}\ (x_a/2)^{m+1}/(m+1)! = (j\omega/\sigma)\ N_m \qquad (1)\ \text{low}\ \omega$$

$$-a_m/2\ (x_a/2)^{m-1}/m! + \left(\frac{K_m}{2} + \frac{a_m}{m}\right)\ (x_a/2)^{m+1}/(m+1)! = 0\ . \qquad (2)\ \text{low}\ \omega$$

Next, replace $x_a = \beta' r$ to get

$$a_m/2\ (\beta' r/2)^{m-1}/m! + \frac{K_m}{2}\ (\beta' r/2)^{m+1}/(m+1)! = (j\omega/\sigma)\ N_m \qquad (1)\ \text{low}\ \omega$$

$$-a_m/2\ (\beta' r/2)^{m-1}/m! + \left(\frac{K_m}{2} + \frac{a_m}{m}\right)\ (\beta' r/2)^{m+1}/(m+1)! = 0\ . \qquad (2)\ \text{low}\ \omega$$

Next, replace β' by its low ω limit which I already computed somewhere. Have to be very careful right here!

Branch F.1. (ignoring loss). From "current asym" I write

$$\beta = (j-1)(1/\delta) \qquad \delta \equiv \sqrt{2/(\omega\mu\sigma)} \qquad \beta = (j-1)\sqrt{\mu\sigma/2}\ \omega^{1/2}$$

$$\beta_d = \beta_{d0}\ [1 - j\,(1/2)\tan_L] \approx \beta_{d0} = (\omega/v_d)$$

$$\beta'^2 = \beta^2 - \beta_d^2 = (j-1)^2(\mu\sigma/2)\omega - \omega^2/v_d^2 = \omega\,[\,(j-1)^2(\mu\sigma/2) - \omega/v_d^2] \rightarrow \omega\,[\,(j-1)^2(\mu\sigma/2)$$

$$\beta' \rightarrow (j-1)\sqrt{\mu\sigma/2}\ \omega^{1/2} = \beta$$

So in the small ω limit, β_d can be ignored here. We then rewrite our two BC's:

$$a_m/2\ ((j-1)\sqrt{\mu\sigma/2}\ \omega^{1/2} r/2)^{m-1}/m! + \frac{K_m}{2}\ ((j-1)\sqrt{\mu\sigma/2}\ \omega^{1/2} r/2)^{m+1}/(m+1)! = (j\omega/\sigma)\ N_m$$

$$-a_m/2\ ((j-1)\sqrt{\mu\sigma/2}\ \omega^{1/2} r/2)^{m-1}/m! + \left(\frac{K_m}{2} + \frac{a_m}{m}\right)\ ((j-1)\sqrt{\mu\sigma/2}\ \omega^{1/2} r/2)^{m+1}/(m+1)! = 0$$

and now all the ω's are exposed. but do them more explicitly

$$a_m/2\ ((j-1)\sqrt{\mu\sigma/2}\, r/2)^{m-1}\ \omega^{(m-1)/2}/m! + \frac{K_m}{2}\ ((j-1)\sqrt{\mu\sigma/2}\, r/2)^{m+1}\ \omega^{(m+1)/2}/(m+1)! = (j\omega/\sigma)\ N_m$$

$$-a_m/2\ ((j-1)\sqrt{\mu\sigma/2}\, r/2)^{m-1}\ \omega^{(m-1)/2}/m! + \left(\frac{K_m}{2} + \frac{a_m}{m}\right)\ ((j-1)\sqrt{\mu\sigma/2}\, r/2)^{m+1}\ \omega^{(m+1)/2}/(m+1)! = 0$$

The general form of these equations is the following

$$+A(\omega)\ \omega^{(m-1)/2} + B(\omega)\ \omega^{(m+1)/2} = (j\omega/\sigma)\ N_m$$

$$-A(\omega)\,\omega^{(m-1)/2} + C(\omega)\,\omega^{(m+1)/2} = 0$$

Add these equations to get

$$B(\omega)\,\omega^{(m+1)/2} + C(\omega)\,\omega^{(m+1)/2} = (j\omega/\sigma)\,N_m$$

or

$$[B(\omega) + C(\omega)]\,\omega^{(m+1)/2} = (j\omega/\sigma)\,N_m$$

In order to have the LHS behave as ω^1 we have to have

$$[B(\omega) + C(\omega)] = (j/\sigma)N_m\,\omega\,\omega^{-(m+1)/2} = (j/\sigma)N_m\,\omega^{-(m-1)/2}$$

We then find that N_m "stays in the problem" because it is now installed in coefficient B+C.

Now we have really

$$A(\omega) = a_m/2\ \ ((j\text{-}1)\sqrt{\mu\sigma/2}\ a\ /2)^{m-1}/\ m!$$

$$B(\omega) = \frac{K_m}{2}\ \ ((j\text{-}1)\sqrt{\mu\sigma/2}\ a\ /2)^{m+1}\ /\ (m+1)!$$

$$C(\omega) = \left(\frac{K_m}{2} + \frac{a_m}{m}\right)\ \ ((j\text{-}1)\sqrt{\mu\sigma/2}\ a\ /2)^{m+1}\ /\ (m+1)!$$

so they are linearly related. I am sure that if we now solve for the coefficients, we just duplicate the solution found in Appendix D.

Branch F.2. (including loss). Go back now to our starting position,

$$a_m/2\ \ (\beta' r\ /2)^{m-1}\ /\ m! + \frac{K_m}{2}\ \ (\beta' r\ /2)^{m+1}\ /\ (m+1)! = (j\omega/\sigma)\,N_m \qquad \text{(1) low } \omega$$

$$-\,a_m/2\ \ (\beta' r\ /2)^{m-1}\ /\ m! + \left(\frac{K_m}{2} + \frac{a_m}{m}\right)\ \ (\beta' r\ /2)^{m+1}\ /\ (m+1)! = 0\ . \qquad \text{(2) low } \omega$$

If we include loss, we know from "including loss" doc that

$$\beta'^2 \rightarrow\ \ 2j\ \ [(R_{dc1} + R_{dc2})C\ ]\ (v_d/2) \qquad \equiv \beta'^2_0\ \not\rightarrow\ \ 0$$

In this case, if we take the $\omega \rightarrow 0$ limit of the above two equations, we get just this

$$a_m/2\ \ (\beta' a\ /2)^{m-1}\ /\ m! + \frac{K_m}{2}\ \ (\beta' a\ /2)^{m+1}\ /\ (m+1)! = 0\ \ \text{[wrong!]} \qquad \text{(1) low } \omega$$

$$-\,a_m/2\ \ (\beta' a\ /2)^{m-1}\ /\ m! + \left(\frac{K_m}{2} + \frac{a_m}{m}\right)\ \ (\beta' a\ /2)^{m+1}\ /\ (m+1)! = 0\ . \qquad \text{(2) low } \omega$$

.

where β' stands for this limit 2j [$(R_{dc1}+ R_{dc2})C$] (v_d/2). NOW for the first time ever we have something new! The first equation says that

$$\frac{K_m}{2} = c\ a_m/2 \qquad \text{for c = a constant I could write out}$$

Then the second equation says

$$c_1\ a_m + c_2\ c\ a_m/2 + c_2 \frac{a_m}{m} = 0 \qquad \Rightarrow \qquad a_m = 0 \quad \Rightarrow K_m = 0$$

Well, this is just a Cramer's rule problem with a null right side vector and that is the solution. Here then is what we find:

$$E_z(r,m) = -\,j\,(\beta'/\beta_d)\,\frac{K_m}{2}\,J_m(x) = 0 \quad \text{[wrong]} \qquad x = \beta' r \qquad \textit{(D.1.27)} \qquad \text{(D.2.21)}$$

$$E_r(r,m) = a_m\ x^{-1}\,J_m(x) + \frac{K_m}{2}\,J_{m+1}(x) = 0 \qquad \beta'^2 = \beta^2 - \beta_d^2 \qquad \textit{(D.2.11)}$$

$$jE_\theta(r,m) = -\,a_m\ x^{-1}\,J_m(x) + \left(\frac{K_m}{2} + \frac{a_m}{m}\right) J_{m+1}(x) = 0 \qquad \textit{(D.2.15)}$$

and then in the DC limit ALL moments vanish for m > 0 , something I have been looking for now for several days. But does this kill off the m = 0 as well??

Let's go back and examine m = 0 separately:

$$a_0/2\ (\beta' r/2)^{-1}/0! + \frac{K_0}{2}\ (\beta' r/2)^1/(1)! = (j\omega/\sigma)\,N_0 \qquad \text{(1) low } \omega$$

$$-\,a_0/2\ (\beta' r/2)^{-1}/0! + \left(\frac{K_0}{2} + \frac{a_0}{0}\right)\ (\beta' r/2)^1/(1)! = 0\ . \qquad \text{(2) low } \omega$$

Recall from the box that

$$E_r(r,m) = a_m\ x^{-1}\,J_m(x) + \frac{K_m}{2}\,J_{m+1}(x) \ \rightarrow\ a_m\ x^{-1}\ (x/2)^m/m! + \frac{K_m}{2}\ (x/2)^{m+1}/(m+1)!$$

so then

$$E_r(r,0) \ \rightarrow\ a_0\ x^{-1} + \frac{K_0}{2}\ (x/2)^1$$

With no doubt at all, this tells us that $a_0 \equiv 0$, no way around that. So our two m = 0 BC's are then

$$\frac{K_0}{2}\ (\beta' a/2)^1 / (1)!\ = (j\omega/\sigma)\, N_0 \qquad \text{(1) low } \omega$$

$$+\left(\frac{K_0}{2} + \frac{a_0}{0}\right)\ (\beta' a/2)^1 / (1)!\ = 0\ . \qquad \text{(2) low } \omega$$

But we know that

$$N_0 = (\beta_d/2\pi\omega a)\ I\ =\ I/(2\pi a v_d)\ = \text{a constant} \quad [\text{ wrong}] \qquad \text{(D.2.31)}$$

We have to use the new β_d here and then we get $(j\omega/\sigma)\, N_0 \rightarrow$ constant, not 0.

The first equation can be written differently:

$$\frac{K_0}{2}\ (\beta' a/2)^1\ = (j\omega/\sigma)\ (\beta_d/2\pi\omega a)\ I\ = (j/\sigma)\ (\beta_d/2\pi a)\ I\ = I\beta_d(j/2\pi a\sigma)$$

While ω is still small but not zero, divide by β_d to get

$$\frac{K_0}{2}\ (\beta'/\beta_d)\ (a/2) = \ (j/2\pi a\sigma)I$$

This says that

$$\frac{K_0}{2}\ (\beta'/\beta_d)\ \ =\ \ (j/2\pi a\sigma)(2/a)I = (j/\pi\sigma a^2)I$$

Now take limit and we find that

$$\lim_{\omega\rightarrow 0}\left[\frac{K_0}{2}\ (\beta'/\beta_d)\right]\ \ =\ (j/\pi\sigma a^2)I$$

Now since

$$E_z(r,m) = \ -\ j\ (\beta'/\beta_d)\ \frac{K_m}{2}\ J_m(x)$$

we get

$$E_z(r,0) = \ -\ j\ (\beta'/\beta_d)\ \frac{K_0}{2}\ J_0(x)\ \ =\ -j\ (j/\pi\sigma a^2)I\ \ J_0(x)$$

$$= (1/\pi\sigma a^2)I\ J_0(x)$$

$$= I\ R_{dc}\ J_0(x) \qquad\qquad J_0(x)\ = (x/2)^0 / 0!\ \ = 1$$

$= I R_{dc}$

which is the correct result.

So perhaps, just maybe, this resolves our problem. So far this is just Idea F, but I will try to write this up in a separate doc and see if it survives more scrutiny.

// I did this, and under increased scrutiny, this idea has failed, so once again we are back to Square Zero. See "DC limit of App....".

Idea G. In a pure math sense, given the Helmholtz equation and div E = 0 and the boundary conditions, exactly what is the math *mechanism* that is causing the external charge density n(θ) with its peaked shape to find its way into the DC $E_z(r,\theta)$ distribution? After all n(θ) is on the outside surface, whereas $E_z(r,\theta)$ is inside, and seemingly in a non-related direction!

First of all consider the charge pump boundary condition.

$$E_r(r=a,\theta) = (j\omega/\sigma)\, n(\theta) \,. \qquad \text{(D.2.24)}$$

$$E_r(r=a,m) = (j\omega/\sigma)\, N_m \,. \qquad \text{(D.2.25)}$$

This does establish some kind of connection between the exterior n(θ) and something on the interior. Since we find in the DC limit that $E_r(r=a,m) = 0$, and then since both LHS and RHS of the above equation is 0, it is hard to see how 0 = 0 can transmit those η_m moments to the interior. But as usual, we need to think of ω = 0.1 so it is not zero, and then the connection still exists.

Note: Due to this BC, $E_r(a,\theta)$ has the same strong peak that n(θ) has. The *shape* of the peak is the same at any frequency.

But how does E_r affect E_z ? Of the four underlying equations, only div E = 0 mixes E_z with some other field! It says

$$\partial_r (r\, E_r(a,\theta)) + \partial_\theta E_\theta(a,\theta) + r\, \partial_z E_z(a,\theta) = 0 \,.$$

My other BC says that $E_\theta(a,\theta) = 0$ since I am in the low frequency capacitor limit let's say. This says φ is constant on the circle. Then of course $\partial_\theta E_\theta(a,\theta) = 0$ of necessity, and then we have

$$\partial_r (r\, E_r(a,\theta)) + \partial_z E_z(a,\theta) = 0.$$

$$\partial_r (r\, E_r(a,\theta)) - j\beta_d\, E_z(a,\theta) = 0.$$

$$j\beta_d E_z(a,\theta) = \partial_r (r\, E_r(a,\theta))$$

$$E_z(a,\theta) = -j\, (1/\beta_d)\, \partial_r (r\, E_r(a,\theta)) = -j\, (v_d/\omega)\, \partial_r (r\, E_r(a,\theta))$$

Now you see from BC#1 that $E_r(a,\theta) = (j\omega/\sigma)\, n(\theta)$ that mag of $E_r(a,\theta) \rightarrow 0$ as $\omega \rightarrow 0$, and so probably we expect $\partial_r\,(r\, E_r(a,\theta)) \rightarrow 0$ in magnitude as well. But $(v_d/\omega) \rightarrow \infty$ and these offset each other allowing the function $E_z(a,\theta)$ to retain the peak in θ and a finite value as $\omega \rightarrow 0$.

So here is the answer to the question: It is the charge pump BC which causes the shape of $E_r(a,\theta)$ to have the same shape as $n(\theta)$. It is then divE = 0 together with BC $E_\theta(a,\theta) = 0$ which then moves this shape into $E_z(a,\theta)$. It is not that mysterious after all.

Idea H. Suppose it were really true that J_z is non-uniform at DC. You would think *someone* in the world would know about that!

King? In his page 24 discussion of the "closely space round conductors", he still states the idea that there is some $W_z = A_{1z} - A_{2z}$ which does not depend on θ, so he is assuming that the conductor surfaces really are A_z equipotentials. Page 25 he comes right out and says it: "A_z has a constant but different value on each conductor surface". Bang! He claims $A_z = K \ln(s_2/s_1)$ and then shows the contours are circles. He then goes on and on to obtain the usual two-cylinder solution (all in Cartesians). He then makes his page 30 statement that no one has ever computed Z_s for the two cylinder problem, but he gives a "high frequency formula". I keep browsing King's early pages and I NEVER see any mention of skin effect or that his results are only valid in the skin effect regime. The word "skin" does not appear in King's book until page 361 where he is talking about a coaxial resistor. This is the only appearance. [Another term people use is "perfect conductor"]

Matick? He Chapter 8 is exactly on "transmission line parameters" so he must say *something*! He says that at DC or low ω a "true TEM" cannot exist due I guess to losses (p 310). But, he says, as you increase ω and get into the skin depth regime. But they he says if you are in the TL limit, a true TEM exists at any ω. After this opening text, he dives into the coax line which does not exhibit my problem. But then 8.3 is the "two wire line". In this very short section he does comment on "proximity effect" causing a "redistribution of charge and current on the conductor surfaces", but he does not mention whether this is at all frequencies or not. He then does some stripline examples. Then in 8.5 we have more interesting text: he claims that the surfaces stay equipotential up to 10^{19} Hz, which is no doubt my same calculation, and this is my diffusion to the surface problem! So he is confirming that the basic capacitor approach works up to very high ω ! I need confirmation from time to time! Aha. Now: "for very low frequencies, a uniform current distribution can be assumed throughout the conductor cross sectional area." He claims you can compute the current distribution using Maxwell's Equations, but he does not do this, but refers to Sec 4.5 where he does show (as I recently did show) that as δ gets larger, you lose your expo decay and J_z becomes uniform, albeit very small! And that is the end of Matick.

Idea I. How do you explain either uniform or non-uniform J_z in a pure DC analysis?

I.1 The Radial Hall Effect // I make this up, have never it anywhere

For the single wire, electrons deflect at first toward the center line. I think we then get a radial Hall effect where there is a thin line of polarization charge at the conductor center since electrons crowd to that area. Then at the surface there is a positive "ion" charge density. This radial E field then causes J_z electrons to have no deflection at all. Then the question is: does this cause some sort of $J_z(r)$ radial variation? The situation is super thin charge on the line and on the surface, and in the bulk of the conductor, there is no deflection. Charges enter at one end all across the cross section, they do their collision thing and drift

between collisions with no transverse force. So I guess the argument is that "there is no reason" for J_z to be radially non-uniform once the radial Hall field has been "set up". In effect the Hall field is set up by a temporary radial current which shifts electrons from the surface to that center line, but this then stops. So I guess this explains why J_z is uniform at DC in a round wire.

I.2 Bringing in the second wire. Since the first wire is "in balance" with respect to its own field, we need only consider the effect of the field of the second wire. We don't need details on that "external" field other than it will have a gradient across the first wire. We then find that the Lorentz force is different as a function of position in the first wire. I have not proven it, but I think a Hall surface charge distribution could exist which makes the net transverse force be 0 at every point in the first wire cross section. If something were not balanced, more electron charge would build up to correct for it. Even if there were some fancy non-uniform J_z , the magnitude of J_z at some point would have no effect on deflection, unless of course there were a smaller v where there is a smaller J_z. Hmm, *that* would be a mechanism. This is too hard a problem for me, I definitely need help on it.

I.3 Web search on this subject.

"direct current proximity effect"	nothing
"dc proximity effect"	nothing
"low frequency proximity effect"	not much
"current distribution inside a wire"	some
"current distribution inside the wire"	
"current distribution in the wire"	

I keep trying this kind of search, and I never find anything.

Idea J. What does the B field look like?

Let's assume that my fields are correct at all frequencies because Helm and divE = 0 are correct at all frequencies and so are the Maxwells. I have seen the strange shape of the DC current J_z distribution. What does the corresponding B field look like? Are the conductor surfaces by any change equipotentials? Well, I did this in my first "very low freq" doc, and I got this result

$$\omega B_\theta(r,m) = -(j/2a)\,\eta_m\, I\, R_{dc}\, m(m+1)(r/a)^{m-1}$$

which I did two different ways with the same answer. This says $B_\theta \rightarrow \infty$ which is totally strange.

Idea K. Look more closely at I !! If you look at the network model for the transmission line at $\omega = 0$, you get nothing but little resistors top and bottom, and you get this formula for Z_0

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \rightarrow \sqrt{\frac{R}{G}} = \sqrt{\frac{R}{0}} = \infty$$

So in my thought experiment with the two cylinders, if you hold V constant at the source and vary ω, you find that as $\omega \rightarrow 0$, you must have $I \rightarrow 0$ since $I = V/Z_0$!! This idea solves two problems:

(1) It makes $E_z(r,m) \rightarrow 0$ and then I get a uniform J_z maybe, instead of my crazy pattern

(2) It makes $B_\theta \rightarrow I/\omega$ which has a chance of being finite.

So maybe this is my low-ω model for Z_0 (remember that my charge pump condition assumes G = 0!)

$$Z_0 \approx \sqrt{\frac{R_{dc}}{j\omega C}} = \sqrt{R_{dc} Z_c}$$

Then

$$I = V/Z_0 = V \sqrt{\frac{j\omega C}{R_{dc}}} = e^{i\pi/4}\, V\, \sqrt{C/R_{dc}}\; \omega^{1/2}$$

Then look back at our E_z equation which are

$$E_z(r,m) = (1/2)\, \eta_m\, I\, R_{dc}\, (m+1)(r/a)^m \qquad \text{for } m > 0$$
$$E_z(r,0) = I\, R_{dc} \qquad \text{for } m = 0$$

Then these say

$$E_z(r,m) = (1/2)\, \eta_m\, e^{i\pi/4}\, V\, \sqrt{C/R_{dc}}\; \omega^{1/2}\, R_{dc}\, (m+1)(r/a)^m \qquad \text{for } m > 0$$
$$E_z(r,0) = e^{i\pi/4}\, V\, \sqrt{C/R_{dc}}\; \omega^{1/2}\, R_{dc} \qquad \text{for } m = 0$$

and *both* these go to zero. So this does not kill the m>0 terms relative to the m = 0 term.

So OK, I agree, at ω = 0, we must have I = 0. But for small ω, we still have the problem that, although I is small, the ratio of $E_z(r,m)/ E_z(r,0) = I\, R_{dc}$ has a problem, In fact

$$E_z(r,m)/ E_z(r,0) = (1/2)\, \eta_m\, (m+1)(r/a)^m$$

so once again, the strange distribution refuses to go away.